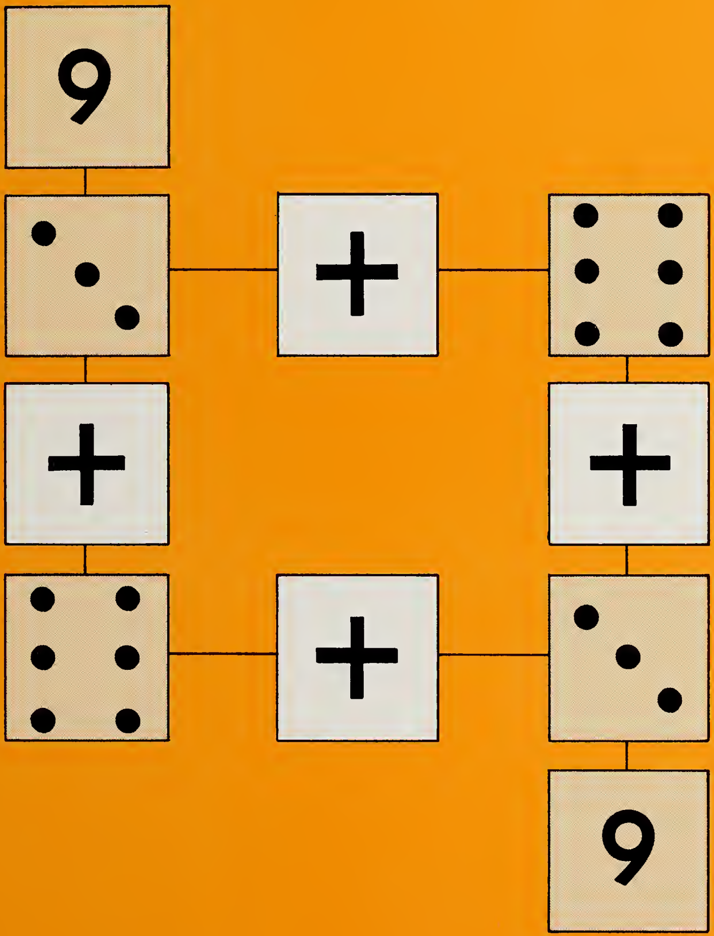
AL 2 1991-550
University of Alberta Library
0 1620 3409838 2

MATHEMATICS 9 (M)



ALBERTA DISTANCE LEARNING CENTRE
ALBERTA EDUCATION
BARRHEAD, ALBERTA

CANADIANA

OCT 25 1991

Mathematics 9

Lessons 1-30

MODIFIED

Distance Learning

Alberta
EDUCATION

Mathematics 9 (Modified)
Student Module
Lessons 1-30
Alberta Distance Learning Centre
ISBN No. 0-7741-0435-X

ALL RIGHTS RESERVED

Copyright © 1991, the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education 11160 Jasper Avenue, Edmonton, Alberta, T5K 0L2.
All rights reserved. Additional copies may be obtained from the Learning Resources Distributing Centre.

No part of this courseware may be reproduced in any form including photocopying (unless otherwise indicated), without the written permission of Alberta Education.

Every effort has been made both to provide proper acknowledgement of the original source and to comply with copyright law. If cases are identified where this has not been done, please notify Alberta Education so appropriate corrective action can be taken.

UNIVERSITY LIBRARY
UNIVERSITY OF AL

PLEASE READ THIS INTRODUCTION BEFORE BEGINNING LESSON 1.

This is the third in the series of junior high modified mathematics courses offered by the Alberta Distance Learning School. These courses are designed to prepare students for the general mathematics course, Mathematics 15, at the senior high level. They do not provide sufficient background material for the matriculation mathematics course, Mathematics 10.

General Instructions

1. This course consists of 30 lessons which contain teaching notes and exercises. No textbook is needed.

2. Remove the staples from your course and transfer the pages to a looseleaf binder. (Instead of a binder you may use large rings or string.)

3. Check the page numbers in each lesson to see that your course is complete. When you send in Lesson 1, ask for any pages that are missing from your course. (List the lesson and page numbers.)

4. You will need a compass, ruler and protractor to do some of the exercises in the lessons.

5. In order to finish the course in good time, you should send in one lesson a week.

Instructions for Doing Lessons

1. Begin work on Lesson 1 and do the exercises and lessons in the proper order.

2. Your lessons should be done in ink but pencil is allowed for graph work and diagrams.

3. Time must be spent studying the teaching notes and examples so that you will have a good background to handle the exercises.

4. Feel free to ask questions about teaching notes and exercises you do not understand.

5. Some of the exercises in the lesson are of the self-correcting type. Answers for these exercises are located at the end of each lesson. It is important that you work through these exercises very carefully as they prepare you for the assignments that follow.

6. As soon as you have completed each lesson and checked your work carefully, mail the entire lesson for correction. Be sure that you have included the completed Lesson Record Form for each lesson that you have enclosed in the envelope.

Instructions for Corrected Lessons

1. When each corrected lesson is returned to you, study all the teacher's comments and corrections. Try to make sure that you do not make the same kinds of errors in future lessons.

2. If you do not understand some of the corrections on your returned lesson, ask about them in the next lesson you send in.

3. If a lesson is returned to you with the word "Incomplete" or "Repeat" written on it, you must send back the entire lesson with any additional work asked for by your teacher. Then a grading for that lesson can be given.

4. After you have reviewed each corrected lesson, return it to your looseleaf binder or rings.

Course Outline

Lesson	Topic
1	Place Value
2	Multiplication of Whole Numbers
3	Division
4	Numbers
5	Greatest Common Factor
6	Least Common Multiples
7	Angles and Polygons
8	Triangles and Quadrilaterals
9	Graphs
10	Review
11	Fractions and Mixed Numerals
12	Decimals
13	Multiplication of Decimals
14	Division of Decimals
15	Terminating and Repeating Decimals
16	Per Cent
17	Formulas
18	More Formulas
19	Perimeter, Area and Volume
20	Review
21	Conversion of Measures
22	Integers
23	Addition of Integers
24	Multiplication of Integers
25	Subtraction and Division of Integers
26	Properties of Integers
27	Formal Equation Solving
28	Problem Solving
29	Sets
30	Review

Digitized by the Internet Archive
in 2016

https://archive.org/details/mathematics9holt130albe

ADVANCE NOTICE: TESTING AND COURSE EVALUATION

To complete this course, you must write a supervised test set by the Alberta Correspondence School before your registration expires. (Information about expiry dates is in the Information Bulletin you received with your application.) In order to pass this course you must score at least a D on the test and have a final mark of at least a C. (The test will make up 40% of your final mark. The other 60% will be based on course work, as evaluated by your correspondence teacher.) If you are dissatisfied with your mark, you may request to write an appeal paper. Such requests must be made within 30 days.

(a) **Classroom students** (taking any courses in an Alberta school):

If you are a classroom student taking correspondence courses, you do **not** need to submit an application for your final test. Test papers are sent automatically to the principal after you have completed 26 satisfactory lessons. (For a 15-lesson course Test papers are sent automatically to the principal after you have completed 12 satisfactory lessons.) You must, however, submit all lessons before you write the test.

Questions about the **scheduling** of final tests should be directed to your principal.

(b) **Non-classroom students** (studying exclusively by correspondence):

For a 30-lesson course, your final test application will be mailed to you with your corrected Lesson 20, and you must return the completed application promptly. The test will be sent to your supervisor after you have completed **26 satisfactory lessons.** (For a 15-lesson course, your final test application will be mailed to you with your corrected Lesson 10 and you must return the completed application promptly. The test will be sent to your supervisor after you have completed 12 satisfactory lessons.) You must, however, submit all lessons before you write the test.

Note: For the purpose of writing final tests, students who live outside Alberta come under the same regulations as those in category (b).

A LESSON RECORD FORM MUST BE COMPLETED FOR EVERY LESSON SUBMITTED FOR CORRECTION, AS ILLUSTRATED BELOW

A Lesson Record form with the **correct** label attached **must** be enclosed with **every lesson** submitted for correction, as illustrated below.

Correct use of these labels will ensure prompt processing and grading of your lessons.

The enclosed **Lesson Labels** must be checked for spelling and address details.

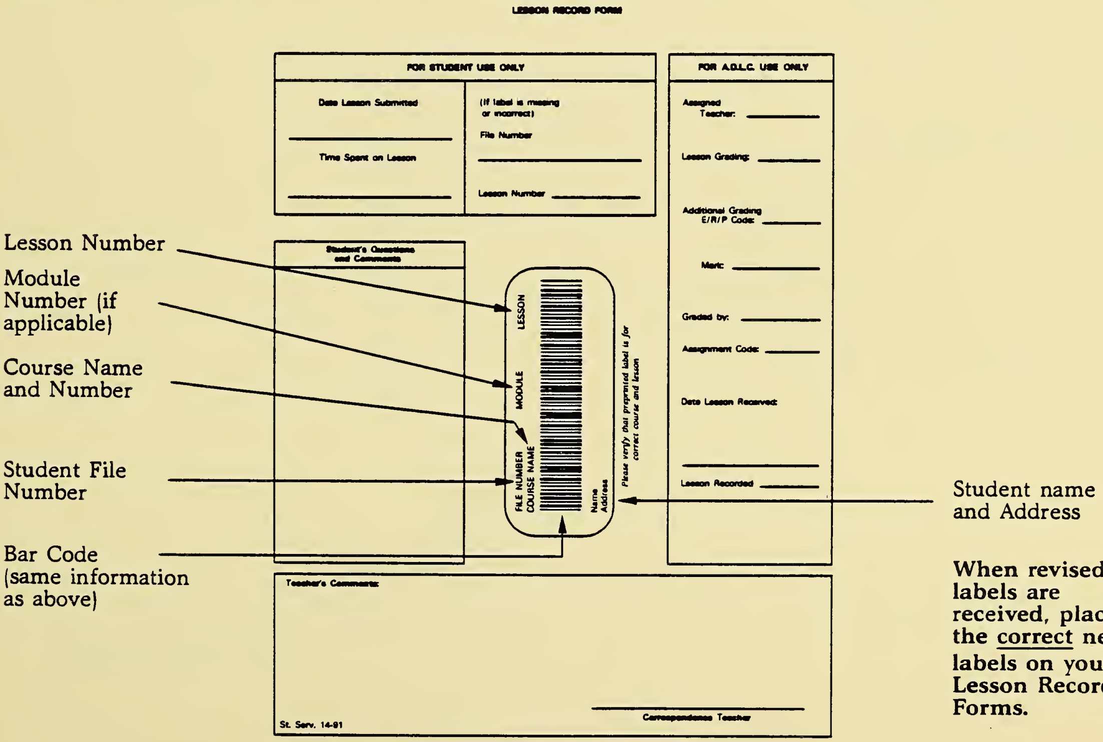
Please advise the Alberta Distance Learning Centre promptly of any changes in name, address, school, or any other details and we will issue a revised set of labels. Your file number is permanently assigned and **must** be included on all correspondence with the Alberta Distance Learning Centre. If the proper label and Lesson Record Form is not attached to each lesson as indicated it will delay your lessons being processed and credited to you.

Lesson labels are to be attached to the **lesson record forms** in the space provided for student name and address.

Check carefully to ensure that the **subject name, module number** and **lesson number** on each label corresponds exactly with the lesson you are submitting.

Labels are to be **peeled** off waxed backing paper and **stuck** on the lesson record form.

Only **one** label is to be placed on each lesson.



When revised labels are received, place the correct new labels on your Lesson Record Forms.

DO NOT MARK OR COVER BAR CODING.

CHANGE OF ADDRESS

If the address on your lesson record form differs from the address you supplied on your registration application, please explain. Indicate whether the different address is your home, school, temporary or permanent change of address.

St. Serv. 3-88

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: __________

Graded by: ________

Assignment Code: _____

Date Lesson Received:

____

Lesson Recorded _____

Student's Questions and Comments

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

How to Succeed

You have shown that you are an achiever, that you are a rather exceptional person, as you have chosen to study by correspondence. You have discovered an alternative form of learning which many students feel is highly appropriate for them. You may believe as they do that correspondence study is closer to the preferred form of learning than traditional classroom instruction.

Now that you have indicated you want to learn by correspondence, we want to do all we can to help you succeed. Correspondence study has one thing in common with any other skill you might like to learn — from basketball to computer technology — it takes regular practice and guidance from a coach to help you achieve your goals. If you work on your course regularly, you will get the practice you need and, if you submit your lessons regularly, you will get any assistance you need from your teacher.

You will find the course work easier if you set aside some time each day for study and you attempt only part of the lesson at each session. Try to finish one lesson in the course each week and send the lesson for correction as soon as you have finished it.

If you follow these suggestions your correspondence teacher will have ample time to help you and you will be more likely to succeed in your studies. Start your course right away, today, and keep working on it until you reach your goal. You will find you will need determination and perseverance, but with support from your teacher and from your family and friends, you will be successful in your correspondence studies.

We at the Alberta Distance Learning Centre will do all we can to help you achieve your learning objective. Write to us or call us at 674-5333 if you need any special help with your course work.

Garry Popowich

Garry Popowich
Director

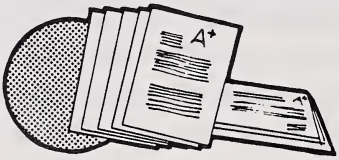
Formula for Success in Correspondence Studies

If X = Success

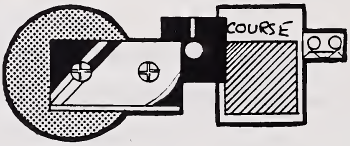
then X = a + b + c + d + e

When

a = Eager Student

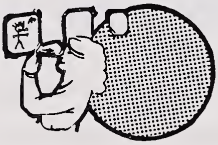


b = High Quality Instructional Materials



c = Hard Work

d = Concerned Teacher



e = Regular Contact

Place Value

Topic One: Names of numbers

Each digit in a numeral is in a specific position. The position of the digits affects the name of the number.

When deciding on the name for a number, we begin by working from the right to the left (⟵). The digits (beginning at the right) are separated into groups of three. Each of these groups is called a PERIOD. Each period has a special name.

e.g. In 273,461, the second group of three digits 273 forms the thousands' period.

Only the period farthest to the left can contain less than three digits.

Each digit in a numeral holds a place. The names of the places (beginning at the right and working to the left) are.

ones, tens, hundreds,	units' period
thousands, ten thousands, hundred thousands,	thousands' period
millions, ten millions, hundred millions,	millions' period
billions, ten billions, hundred billions,	billions' period
trillions, ten trillions, hundred trillions.	trillions' period

To name a number, we must begin at the left. Read the digits in the period farthest to the left, then name the period. Do the same for the other periods until we reach the right end of the numeral. The digits are usually separated into groups of three by commas. This makes it easier to read the name of the number.

If there is one group of three digits, no special name is required.

e.g. In 579, there is only one group of three digits. The name of this period is units. The number could be read five hundred seventy-nine units or just five hundred seventy-nine (5 hundred 79).

If there are two groups of three digits, the period farthest to the left is thousands.

e.g. In 356,521, there are two groups of three digits. The name of the period farthest to the left is thousands. This number is read three hundred fifty-six thousand, five hundred twenty-one or 3 hundred 56 thousand, 5 hundred 21.

If there are three groups of three digits, the period farthest to the left is millions.

e.g. In 85,090,080, there are three groups of digits. The name of the period farthest to the left is millions. This number is read eighty-five million, ninety thousand, eighty or 85 million, 90 thousand, 80.

If there are four groups of three digits, the period farthest to the left is billions.

e.g. In 2,965,461,332, there are four groups of digits. The name of the period farthest to the left is billions. This number is read two billion, nine hundred sixty-five million, four hundred sixty-one thousand, three hundred thirty-two or 2 billion, 9 hundred 65 million, 4 hundred 61 thousand, 3 hundred 32.

If there are five groups of three digits, the period farthest to the left is trillions.

e.g. In 25,985,234,096,741, there are five groups of digits. The name of the period farthest to the left is trillions. This number is read twenty-five trillion, nine hundred eighty-five billion, two hundred thirty-four million, ninety-six thousand, seven hundred forty-one or 25 trillion, 9 hundred 85 billion, 2 hundred 34 million, 96 thousand, 7 hundred 41.

We could continue on but for this year you will be required to know only these first five groups.

EXAMPLE: Read the name of each of the numbers.

1. 43,723,456,893

Solution

There are four groups of digits. Therefore the name of the period farthest to the left is billions.
forty-three billion, seven hundred twenty-three million, four hundred fifty-six thousand, eight hundred ninety-three or 43 billion, 7 hundred 23 million, 4 hundred 56 thousand, 8 hundred 93.

2. 214,703,916

Solution
three groups - millions
two hundred fourteen million, seven hundred three thousand, nine hundred sixteen or 2 hundred 14 million, 7 hundred 3 thousand, 9 hundred 16

Now we know how to read numbers. We must learn how to do the opposite - write the numbers using digits rather than words. Use a comma after each period. Use a zero if no other digit is given to hold a particular place in a number.

EXAMPLE: Write each number using digits.

1. sixty-nine billion, eight hundred forty-seven million, twenty-three thousand, two

Solution
In this number there is no digit given for hundred thousands, hundreds or tens. We will have to use zeros in these places.
69,847,023,002

2. seven hundred forty-six trillion, six hundred thirty-five billion, four hundred twenty-six thousand, three hundred thirteen

Solution
746,635,000,426,313
Since there are no digits in the millions' period, we had to use zeros for hundred millions, ten millions and millions.

Assignment 1

Write in words the name of each of the numbers.

1. 77,766,655

seventy-seven million, seven hundred sixty-six thousand, six hundred fifty-five

2. 1,024

3. 173, 000, 569, 013

4. 987, 461, 320, 706, 933

5. 59, 073

Use digits to write each of the numbers.

6. eighty-seven million, six hundred ninety-two thousand, one hundred ten

87, 692, 110

7. twenty billion, eight hundred forty million, five hundred thousand, eighty

8. one million, five thousand

9. six hundred sixty trillion, thirty million, nine thousand, two

10. fifty-two million, eighty-nine thousand, fourteen

In each of the statements there is a number. Write the number as you would read it.

11. Canada's population was 21,089,216 in 1974.

twenty-one million, eighty-nine thousand, two hundred sixteen

12. Light travels 5,880,000,000,000 miles in one year.

13. The land area of Canada is 3,851,809 square miles.

In each of the statements write the number using digits.

14. Alberta had a thirty million dollar surplus in the fiscal year of 1971.

30,000,000

15. Edmonton's population increased to four hundred thousand in 1971.

16. The area of the Pacific Ocean is over sixty million square miles.

17. Must all the periods in a number contain three digits? Explain your answer.

18. Is this number 243,308,72 written correctly? Explain your answer.

19. Why do some numbers contain one or more zeros?

Topic Two: Value of digits in a number

The position of the digits in a number affects the value of a number.

The number expressed by each digit is the product of the digit and the value of the place it holds.

e.g. 273 The number expressed by the digit 2 depends on the place it holds. 2 is in the hundreds' place. The number expressed by 2 is 2 × 100 or 200.

Since we can find the number expressed by each digit, we can write numerals as a sum. First write the number expressed by each digit as a product of the digit and the value of the place it holds. Then write the sum of the products. In other words the numeral will be written as a sum of products. When a numeral is written in this way, it is called the EXPANDED FORM of the numeral.

EXAMPLE. Write 356 in the expanded form.

Solution

3 in 356 expresses 3 × 100 or 300.
5 in 356 expresses 5 × 10 or 50.
6 in 356 expresses 6 × 1 or 6.
Therefore the expanded form of 356 is $(3 \times 100) + (5 \times 10) + (6 \times 1)$
or $356 = 300 + 50 + 6$.

Self-correcting Exercise 1

Name the digit in each of the numbers that expresses the number of ten thousands.

1. 16,149,200 ____________________
2. 2,801,683 ____________________
3. 786,259,420 ____________________
4. 873,126 ____________________
5. 289 ____________________

Name the number expressed by 4 in each of the numerals.

6. 296,314 ______________________________

7. 40,381 ______________________________

8. 146 ______________________________

9. 734,621,190,832 ______________________________

10. 33,412 ______________________________

Use the standard name of the number to express each of the numbers.

11. 3 TENS + 6 ONES ______________

12. 5 HUNDREDS + 6 TENS + 7 ONES ______________

13. 4 THOUSANDS + 0 HUNDREDS + 7 TENS + 3 ONES ______________

14. (8 x 1000) + (7 x 100) + (6 x 10) + (4 x 1) ______________

15. (5 x 1000) + (6 x 10) + (8 x 1) ______________

Write each number in the expanded form.

16. 347 ______________________________

____

17. 4,916 ______________________________

____

18. 3,033 ______________________________

____

19. 56,342 ______________________________

____

20. 20,518 ______________________________

____

Corrections are on page 23.

Topic Three. Increasing and decreasing numbers

To increase a number means to add to it.

To decrease a number means to subtract from it.

To increase or decrease a number by 1, the digit in the ones' place must change.

e.g. $$\left.\begin{array}{l}47238 + 1 = 47239\\ 47238 - 1 = 47237\end{array}\right\}$$

We need to consider only the digit in the ones' place.
8 + 1 = 9
8 - 1 = 7

To increase or decrease a number by 10, the digit in the tens' place must change.

e.g. $$\left.\begin{array}{l}47238 + 10 = 47248\\ 47238 - 10 = 47228\end{array}\right\}$$

We need to consider only the digit in the tens' place.
3 + 1 = 4
3 - 1 = 2

To increase or decrease a number by 100, the digit in the hundreds' place must change.

e.g. $$\left.\begin{array}{l}47238 + 100 = 47338\\ 47238 - 100 = 47138\end{array}\right\}$$

We need to consider only the digit in the hundreds' place.
2 + 1 = 3
2 - 1 = 1

To increase or decrease a number by 1000, the digit in the thousands' place must change.

e.g. $$\left.\begin{array}{l}47238 + 1000 = 48238\\ 47238 - 1000 = 46238\end{array}\right\}$$

We need to consider only the digit in the thousands' place.
7 + 1 = 8
7 - 1 = 6

To increase or decrease a number by 10000, the digit in the ten thousands' place must change.

e.g. $$\left.\begin{array}{l}47238 + 10000 = 57238\\ 47238 - 10000 = 37238\end{array}\right\}$$

We need to consider only the digit in the ten thousands' place.
4 + 1 = 5
4 - 1 = 3

A special change occurs when the digit $\underline{9}$ is increased by 1.

e.g. 4297 + 10 = 4307

Using the digit in the tens' place, 9 + 1 = 10. Since each place can contain only one digit, 10 TENS is regrouped to 1 HUNDREDS, 0 TENS. 0 is written in the tens' place. The digit in the hundreds' place is increased by 1.

A special change occurs when the digit 0 is decreased by 1.

e.g. 4032 - 100 = 3932

Using the digit in the hundreds' place, 0 - 1 has no answer. We must regroup 4 THOUSANDS to 3 THOUSANDS, 10 HUNDREDS. 10 - 1 = 9. 3 is written in the thousands' place. 9 is written in the hundreds' place.

Assignment 2

Complete each of the following sentences.

1. 346 + 10 = 356
2. 17,983 + 100 = ____________
3. 2,461 + 1 = ____________
4. 1,498,261 + 100,000 = ____________
5. 13 + 10 = ____________
6. 981 + 100 = ____________
7. 78,251 - 1000 = ____________
8. 409 - 10 = ____________
9. 38,271 - 100 = ____________
10. 4,673 - 1 = ____________
11. 7,483,296 - 100,000 = ____________
12. 2000 - 10 = ____________

Topic Four Exponential notation

Sometimes a product will contain a factor which is used more than once.

e.g. $12 = 2 \times 2 \times 3$ — In this product, the factor 2 is used more than once.

A short method can be used to write products that have a factor used more than once. This method is called EXPONENTIAL NOTATION. The factor that is used once or more than once is called the BASE.

e.g. $12 = 2 \times 2 \times 3$ — 2 is used more than once. 3 is used once.

A small number which represents how many times the factor is used in the product is written above and to the right of the factor. This small number is called the EXPONENT.

e.g. $12 = 2 \times 2 \times 3$ — 2 is used twice. 3 is used once. Therefore $12 = 2 \times 2 \times 3$ can be written $12 = 2^2 \times 3^1$. This is read "12 is equal to the product of the second power of 2 and the first power of 3".

EXAMPLE: Use $2^3 \times 5^1 \times 3^2$ for the following questions.

1. What is the exponent of 2^3? 5^1? 3^2?

Solution

The exponent of 2^3 is 3.

The exponent of 5^1 is 1.

The exponent of 3^2 is 2.

2. What is the base of 2^3? 5^1? 3^2?

Solution

The base of 2^3 is 2.

The base of 5^1 is 5.

The base of 3^2 is 3.

3. What does 2^3 mean? 5^1 mean? 3^2 mean?

Solution

2^3 means 2 has been used as a factor 3 times.

$2^3 = 2 \times 2 \times 2$

5^1 means 5 has been used as a factor 1 time.

$5^1 = 5$

3^2 means 3 has been used as a factor 2 times.

$3^2 = 3 \times 3$

4. What is the standard name of 2^3? 5^1? 3^2?

Solution

$2^3 = 2 \times 2 \times 2$

$= 4 \times 2$

$= 8$ The standard name of 2^3 is $\underline{8}$.

$5^1 = 5$ The standard name of 5^1 is $\underline{5}$.

$3^2 = 3 \times 3$

$= 9$ The standard name of 3^2 is $\underline{9}$.

5. What is the standard name of $2^3 \times 5^1 \times 3^2$?

Solution

$$
\begin{aligned}
2^3 \times 5^1 \times 3^2 &= (2\times 2\times 2)(5)(3\times 3) \quad \text{(Brackets indicate multiplication.)}\\
&= 8 \times 5 \times 9\\
&= 40 \times 9\\
&= 360
\end{aligned}
$$

The standard name of $2^3 \times 5^1 \times 3^2$ is $\underline{360}$.

6. Read $2^3 \times 5^1 \times 3^2$.

Solution

The product of the third power of two, the first power of five and the second power of three

It is interesting to note that any base with an exponent of zero is equal to 1.

e.g. $2^0 = 1$ 2 is used as a factor no times.
You know that 1 is a factor of every number.
Therefore we say that the zero power of any base is equal to the factor $\underline{1}$.

Self-correcting Exercise 2

Write each of the power using digits.

1. fourth power of six ______________________

2. first power of ten ______________________

3. sixth power of four ______________________

Write each power as a product.

4. 3^3 ______________________

5. 10^2 ______________________

6. 6^4 ______________________

Use the standard name to express each power.

7. 10^3 ______________________

8. 6^2 ______________________

9. 4^2 ______________________

Use exponential notation to write each product.

10. 2 × 2 × 3 × 3 ______________________

11. 10 × 10 × 10 × 10 ______________________

12. 2 × 5 × 2 × 7 × 11 × 7 ______________________

Corrections are on page 24.

Topic Five: Bases other than ten

To expand your understanding of place value, we will examine base six numerals.

The digits you have used in your lifetime are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These digits are members of the base ten number system. This number system is composed of ten different digits.

Base six is a number system composed of six digits. We will use 0, 1, 2, 3, 4, 5 as members of base six.

We must distinguish between base ten numerals and numerals of other bases. There are many ways of doing this. The numeral representing the base will be written below and to the right of the number.

e.g. 103_6 This number is read "one zero three, base six".

Let us review what is meant by a base 10 numeral. Consider the value of each digit in the number 986, 759.

	9	8	6	7	5	9
Name	hundred thousands	ten thousands	thousands	hundreds	tens	ones
Standard name	100, 000	10, 000	1000	100	10	1
Product of base	10×10×10×10×10	10×10×10×10	10×10×10	10×10	10	1
Power of base	10^5	10^4	10^3	10^2	10^1	10^0

This number may be written, using the expanded form, in many different ways.

e.g. $986,759 = (9\times10^5)+(8\times10^4)+(6\times10^3)+(7\times10^2)+(5\times10^1)+(9\times10^0)$

or $986,759 = (9\times10\times10\times10\times10\times10)+(8\times10\times10\times10\times10)+(6\times10\times10\times10)+(7\times10\times10)+(5\times10)+(9\times1)$

or $986,759 = (9\times100,000)+(8\times10,000)+(6\times1000)+(7\times100)+(5\times10)+(9\times1)$

or 986, 759 = 9 HUNDRED THOUSANDS + 8 TEN THOUSANDS + 6 THOUSANDS + 7 HUNDREDS + 5 TENS + 9 ONES

Each place to the left increases by one power of the base.

e.g. $10^2 = 10^1 \times 10^1$

$10^3 = 10^2 \times 10^1$

$10^4 = 10^3 \times 10^1$

Below is the value of each digit in a base 6 numeral. The digit farthest to the right is the first digit in the number. Use 213540_6.

	2	1	3	5	4	0
	sixth digit	fifth digit	fourth digit	third digit	second digit	first digit
Power of base	6^5	6^4	6^3	6^2	6^1	6^0
Product of base	6x6x6x6x6	6x6x6x6	6x6x6	6x6	6	1
Standard name	7776	1296	216	36	6	1

This number may be written, using expanded form, in many different ways.

e.g. $213540_6 = (2 \times 6^5) + (1 \times 6^4) + (3 \times 6^3) + (5 \times 6^2) + (4 \times 6^1) + (0 \times 6^0)$

or $213540_6 = (2 \times 6 \times 6 \times 6 \times 6 \times 6) + (1 \times 6 \times 6 \times 6 \times 6) + (3 \times 6 \times 6 \times 6) + (5 \times 6 \times 6) + (4 \times 6) + (0 \times 1)$

or $213540_6 = (2 \times 7776) + (1 \times 1296) + (3 \times 216) + (5 \times 36) + (4 \times 6) + (0 \times 1)$

Each place to the left increases by one power of the base.

e.g. $6^2 = 6^1 \times 6^1$
$6^3 = 6^2 \times 6^1$
$6^4 = 6^3 \times 6^1$

Self-correcting Exercise 3

Use 54189 to answer the following questions.

1. What is the base of this numeral? ______
2. Which digit expresses the number of ones? ______
3. Which digit is associated with the second power of the base? ______
4. With which power of the base is 5 associated? ______
5. What is the largest digit a number of this base could contain? ______
6. Write this number in words. ______

______

Use 2453_6 to answer the following questions.

7. What is the base of this numeral? ______

8. Which digit expresses the number of ones? ______

9. Which digit is associated with the second power of the base? ______

10. With which power of the base is 2 associated? ______

11. What is the largest digit a number of this base could contain? ______

12. Write this number in words. ______

______

Although we have not taken base 3, apply what you have learned from base 10 and base 6 to this base 3 numeral $\underline{2102_3}$.

13. Which digit expresses the number of ones? ______

14. Which digit is associated with 3^2 ? ______

15. With which power of 3 is 0 associated? ______

16. What is the largest digit a base 3 numeral could contain? ______

17. Write this number in words. ______

______

Corrections are on page 24.

Assignment 3

Now let's think about base 8 numerals.

1. Complete the chart by giving the value of each digit.

	fifth digit	fourth digit	third digit	second digit	first digit
Power of base			8^2		
Product of base		$8 \times 8 \times 8$		8	
Standard name	4096				1

2. If a base 8 numeral is read "six two five, base eight", it would be written, using digits, as ____________________

3. The largest digit that can be used in a base 8 numeral is

____

4. Write each of these numerals in an expanded form.

a. 763_8 $(7 \times 8^2) + (6 \times 8^1) + (3 \times 8^0)$

b. 100_8 ____________________

c. 52_6 ____________________

d. 3261 ____________________

e. 1534_6 ____________________

Topic Six: Converting to base 10 numerals

The numbers in different bases do not always mean much to us because we are so used to base 10 numerals. To make the numbers in different bases more meaningful, we can convert them to base ten numbers. To do this we must consider the value of each digit in the number. We write the number in the expanded form.

EXAMPLE: Convert 1532_6 to a base 10 number.

Solution

See the value of each digit in a base 6 numeral given on page 14.

	fourth digit (1)		third digit (5)		second digit (3)		first digit (2)
	$1(6^3)$	+	$5(6^2)$	+	$3(6^1)$	+	$2(6^0)$
=	1(216)	+	5(36)	+	3(6)	+	2(1)
=	216	+	180	+	18	+	2
=	416						

Thus 1532_6 = 416. 416 is a base 10 numeral.

Assignment 4

Write in words each of the numerals.

1. 325_6 three two five, base six
2. 7142_8 __________
3. 801 __________
4. 12_3 __________
5. 11_2 __________

Write in digits each of the numerals.

6. one six, base seven 16_7
7. three zero, base four __________
8. six two five, base eight __________
9. one five zero two, base six __________
10. one three nine, base ten __________

Write the numerals in expanded form.

11. 5700_8 $(5\times8^3)+(7\times8^2)+(0\times8^1)+(0\times8^0)$
12. 3405_6 __________
13. 2377_8 __________
14. 156 __________
15. 2102_6 __________
16. 437_8 __________
17. 2948 __________

Using the expanded form from p. 17, convert each numeral to a base 10 numeral.

18. 5700_8 = $(5\times8^3)+(7\times8^2)+(0\times8^1)+(0\times8^0)$

= $(5\times512)+(7\times64)+(0\times8)+(0\times1)$

= $2560 + 448 + 0 + 0$

= 3008

19. 3405_6 = $(3\times6^3)+(\underline{\qquad})+(\underline{\qquad})+(\underline{\qquad})$

= $(3\times216)+(\underline{\qquad})+(\underline{\qquad})+(\underline{\qquad})$

= $648 + \underline{\qquad} + \underline{\qquad} + \underline{\qquad}$

= $\underline{\qquad}$

20. 2377_8 = ______

21. 156 = ______

22. 2102_6 = ______

23. 2948 = ______

Topic Seven: Scientific notation

Sometimes numbers are very large.
e.g. 20,000,000,000

This number can be written in a much shorter form as 2×10^{10}. This is done by using what you have learned about exponential notation and the expanded form of a number. The number is written as a product of a number greater than or equal to 1 but less than 10 and a power of 10. This way of writing a number is SCIENTIFIC NOTATION.

EXAMPLE: Use scientific notation to express 3000.

Solution

$3000 = 3 \times 1000$

Now, use exponential notation to write 1000.
Since $1000 = 10 \times 10 \times 10$, we know that 10 is used as a factor 3 times in the number 1000. Therefore, 1000 can be written as 10^3.

Therefore $3000 = \underline{3 \times 10^3}$. (Scientific notation)

3 is a number between 1 and 10.

10^3 is a power of 10.

An easy way of deciding what the power of 10 will be is to count the zeros in the number.
e.g. 3000 has 3 zeros. Therefore the product will include the third power of 10 or 10^3.

Assignment 5

Use scientific notation to express each number.

1. 50 $5 \times 10 = 5 \times 10^1$
2. 10,000 ______
3. 2,000,000 ______
4. 600,000,000 ______

Use the standard name to express each number.

5. 7×10^7 $7 \times 10{,}000{,}000 = 70{,}000{,}000$
6. 9×10^1 ______
7. 3×10^5 ______
8. 4×10^{10} ______

Use scientific notation to express the number in each statement.

9. Alberta had a thirty million dollar surplus in the fiscal year of 1969.

$30{,}000{,}000 = 3 \times 10{,}000{,}000 = 3 \times 10^7$

10. Edmonton's population increased to four hundred thousand in 1971.

____

11. The area of the Pacific Ocean is over sixty million square miles.

____

12. Is scientific notation used to write the number 20×10^4?
Explain your answer.

____

____

Assignment 6

Write the standard name of each sum, difference, product or quotient.

1. Sums

a. $\overset{1}{2}4$ 17 41	b. 273 851 906	c. 425 673 829 439	d. 2613 4796 8571 4268 5528

2. Differences

a. $\overset{2}{\cancel{3}}\overset{11}{\cancel{1}}$ 28 3	b. 476 312	c. 4002 1983	d. 98237 46154

3. Products

a. 321 39 2889 9630 12519	b. 243 90	c. 497 280	d. 9462 831

4. Quotients

a. $$\begin{array}{r} 51 \\ 9\overline{)\,461} \\ 45^{x} \\ \hline 11 \\ 9 \\ \hline 2 \end{array}$$

b. $46\overline{)\,2830}$

c. $92\overline{)\,1002}$

d. $36\overline{)\,720}$

Assignment 7

For each problem:
- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. For breakfast Sally eats 1 slice of toast, Jane eats 3 slices, Father eats 4 and Mother eats 2. How many slices of toast must be prepared for breakfast?

Placeholder: Let n be the number of slices of toast prepared for breakfast.

Equation: $n = 1 + 3 + 4 + 2$

Answer: $n = 10$

Statement: 10 slices of toast must be prepared for breakfast.

Computation

$$\begin{array}{r} 1 \\ 3 \\ 4 \\ 2 \\ \hline 10 \end{array}$$

2. The number of students at the concert was 145, and the number of adults was 8. What was the total attendance?

Placeholder: Let x be the total number of persons

Equation: ______________________ Computation

Answer: ______________________

Statement: ______________________

____

3. George collects coins. After Joe gave him 5, he had 39 coins. How many coins did George have before Joe gave him the five coins?

Placeholder: Let y be ______________________

____

Equation: ______________________ Computation

Answer: ______________________

Statement: ______________________

____

4. In May, Ernie had 17 model cars. During the summer, he built some more cars. Then he had 32 model cars. How many model cars did Ernie build during the summer?

Placeholder: Let f be ______________________

____

Equation: ______________________ Computation

Answer: ______________________

Statement: ______________________

____

Key to Self-correcting Exercises in Lesson 1

Exercise 1, page 6

1. 4
2. 0
3. 5
4. 7
5. 0

6. 4 × 1 = 4
7. 4 × 10,000 = 40,000
8. 4 × 10 = 40
9. 4 × 1,000,000,000 = 4,000,000,000
10. 4 × 100 = 400

11. 30 + 6 = 36
12. 500 + 60 + 7 = 567
13. 4000 + 0 + 70 + 3 = 4073
14. 8000 + 700 + 60 + 4 = 8764
15. 5000 + 60 + 8 = 5068

16. (3 × 100) + (4 × 10) + (7 × 1)
or 300 + 40 + 7
or 3 HUNDREDS + 4 TENS + 7 ONES

17. (4 × 1000) + (9 × 100) + (1 × 10) + (6 × 1)
or 4000 + 900 + 10 + 6
or 4 THOUSANDS + 9 HUNDREDS + 1 TEN + 6 ONES

18. (3 × 1000) + (0 × 100) + (3 × 10) + (3 × 1)
or 3000 + 30 + 3
or 3 THOUSANDS + 3 TENS + 3 ONES

19. (5 × 10,000) + (6 × 1000) + (3 × 100) + (4 × 10) + (2 × 1)
or 50,000 + 6000 + 300 + 40 + 2
or 5 TEN THOUSANDS + 6 THOUSANDS + 3 HUNDREDS + 4 TENS + 2 ONES

20. (2 × 10,000) + (0 × 1000) + (5 × 100) + (1 × 10) + (8 × 1)
or 20,000 + 500 + 10 + 8
or 2 TEN THOUSANDS + 5 HUNDREDS + 1 TEN + 8 ONES

Exercise 2, page 12

1. 6^4
2. 10^1
3. 4^6

4. $3 \times 3 \times 3$
5. 10×10
6. $6 \times 6 \times 6 \times 6$

7. $10 \times 10 \times 10 = 1000$
8. $6 \times 6 = 36$
9. $4 \times 4 = 16$

10. $2^2 \times 3^2$
11. 10^4
12. $2^2 \times 5^1 \times 7^2 \times 11^1$

Exercise 3, page 14

1. 10
2. 9
3. 1
4. 10^4
5. 9
6. fifty-four thousand, one hundred eighty-nine

7. 6
8. 3
9. 4
10. 6^3
11. 5
12. two four five three, base six

13. 2
14. 1
15. 3^1
16. 2
17. two one zero two, base three

End of Lesson 1

JUNIOR HIGH MATHEMATICS QUESTIONNAIRE
Mathematics 9(M)

Name: ____________________

Address: ____________________

____

____

File #: ____________________ Telephone No. ____________________

Birthdate: Year __________ Month: ____________________ Day: __________

1. Details of last mathematics course completed successfully:

Course: ____________________ Year Completed: __________

School: ____________________ Final Mark: __________

2. What school are you attending now (if any)? ____________________

3. Are there other students in your school or district taking correspondence courses? __________

4. Do you study with any of them? __________

5. What other courses are you taking by correspondence? ____________________

____

____

6. Have you taken correspondence before? When? ____________________

Which course(s) did you study? ____________________

____

7. Why are you taking correspondence now? ____________________

____

8. Mention any special factors (handicaps, illness, family problems, etc) which may influence your progress in your studies.

____

____

____

____

In order to get to know you better, we would like you to tell us a little bit about yourself. Write a paragraph or two telling us about yourself, and your plans for the future, or anything else that you think is interesting.

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: ________

Graded by: ________

Assignment Code: _____

Date Lesson Received: ____________________

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

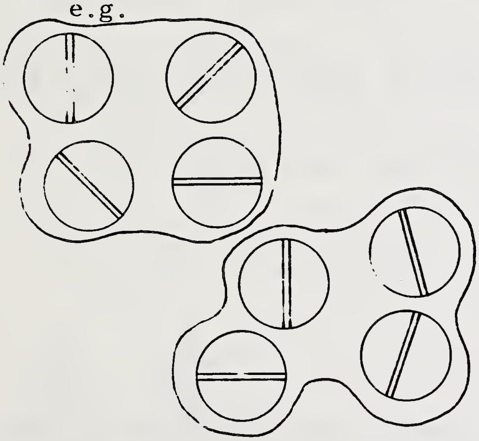
When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Multiplication of Whole Numbers

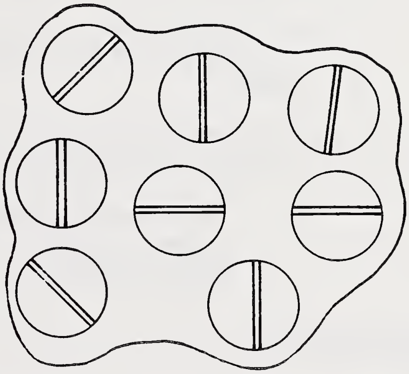
Topic One: Multiplication

MULTIPLICATION involves the combining of a number of groups each containing the same number of objects into one large group. This process of combining like groups can be shown by pictures.

e.g.



2 groups each containing 4 objects
2 groups of 4 balls



one large group containing 8 objects
1 group of 8 balls

The number of objects in the large group could be found by multiplying the number of like groups by the number of objects in each of these groups.

e.g. In this case, 2 like groups are combined and there are 4 objects in each of these groups.
$2 \times 4 = 8$
Therefore, there are 8 objects in the larger group.

In this lesson, we will be concerned with the multiplication of whole numbers. When multiplying, there are two important things to remember

1. A number remains unchanged when it is multiplied by 1.
   e.g. $2 \times 1 = 2$ or $1 \times 2 = 2$
   $3 \times 1 = 3$ or $1 \times 3 = 3$
   $9 \times 1 = 9$ or $1 \times 9 = 9$
   There is no change in the number.
2. When multiplying, we must consider the value of each digit in the multiplier.
   e.g. 247×21 2 in 21 is not just 2 but rather 20. 2 is in the tens' place. Therefore its value is 20.

Any multiplication problem consists of three parts: the two numbers you are multiplying and the result you obtain. The two numbers you are multiplying are factors. The factor containing the lesser number of digits is usually written after the "times" symbol or below the first factor when used in a vertical problem. The factor in this position is called the MULTIPLIER. The result of the multiplication problem is called the PRODUCT.

e.g. 21 × 9 = 189 ← product
multiplicand ↗ ↖ multiplier

In this example, the number 21 (multiplicand) is multiplied by the number 9 (the multiplier) in order to obtain the product (189).

This multiplication problem could also be written in the vertical form as follows:

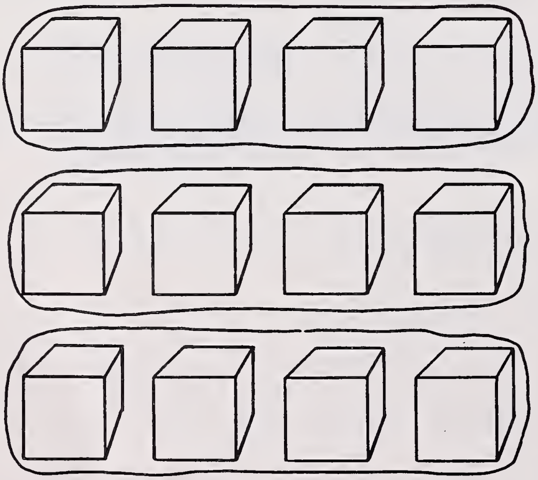
$$\begin{array}{rl} 21 & \leftarrow \text{multiplicand} \\ \underline{9} & \leftarrow \text{multiplier} \\ 189 & \leftarrow \text{product} \end{array}$$

Assignment 1

In each picture below, a number of groups each containing a number of objects is shown. You are to show mathematically how you would find the number of objects in the large group if the smaller groups were combined.

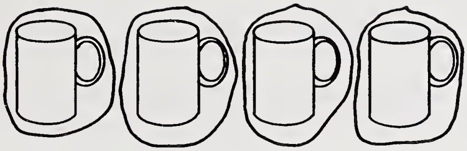
A number of groups each containing a number of objects	One large group containing all of the objects

1.



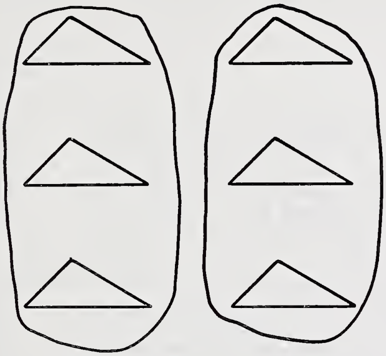
Number of groups	×	Number of objects in each group	=	Total number of objects in one large group
3	×	4	=	12

2.



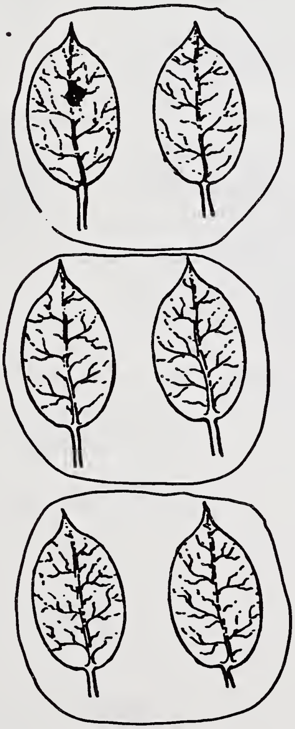
______ × ______ = ______

3.



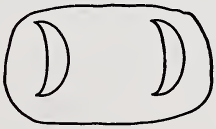
______ × ______ = ______

4.



______ × ______ = ______

5.



______ × ______ = ______

Find the product.

6. 8 × 1 = 8

7. 6 × 9 = ______

8. 9 × 6 = ______

9. 1 × 3 = ______

10. 10 × 1 = ______

What is the value of each digit in each number?

11. 47 7 is 7 ONES or just 7.

4 is 4 TENS or 40.

12. 283 ______

______

______

13. 1965 ______

______

______

______

Topic Two One-digit multipliers

Study the example below where you are shown how to multiply a number by a one-digit multiplier.

EXAMPLE: Multiply 423 by 6.

Solution

$$\begin{array}{r} 4\overset{1}{2}3 \\ \underline{6} \\ 8 \end{array}$$

Begin by multiplying 3 ONES by 6. This equals 18 ONES. Since each place in a number can contain only one digit, we must regroup 18 ONES to 1 TEN and 8 ONES. 8 is written in the ones' place. 1 is written above 2 to show that it must be added to the product of 2 TENS and 6.

$$\begin{array}{r} \overset{1}{4}\overset{1}{2}3 \\ \underline{6} \\ 38 \end{array}$$

The product of 2 TENS and 6 is 12 TENS. Remember to add 1 TEN (from 18 ONES) to 12 TENS. The result is 13 TENS which is regrouped to 1 HUNDRED and 3 TENS. 3 is written in the tens' place. 1 is written above 4 to show that it must be added to the product of 4 HUNDREDS and 6.

$$\begin{array}{r} \overset{1}{4}\overset{1}{2}3 \\ \underline{6} \\ 2538 \end{array}$$

The product of 4 HUNDREDS and 6 is 24 HUNDREDS. Remember to add 1 HUNDRED (from 13 TENS) to 24 HUNDREDS. The result is 25 HUNDREDS which is regrouped to 2 THOUSANDS and 5 HUNDREDS. 5 is written in the hundreds' place. Since we do not need to multiply any further, 2 is written in the thousands' place.

Thus 423 × 6 = 2538.

Assignment 2

Find the products.

1. $\begin{array}{r} \overset{2}{7}7 \\ \underline{4} \\ 308 \end{array}$

2. $\begin{array}{r} 95763 \\ \underline{9} \end{array}$

3. $\begin{array}{r} 768 \\ \underline{3} \end{array}$

4. $\begin{array}{r} 47092 \\ \underline{7} \end{array}$

5. $\begin{array}{r} 6636 \\ \underline{2} \end{array}$

6. $\begin{array}{r} 48612 \\ \underline{8} \end{array}$

7. $\begin{array}{r} 359 \\ \underline{5} \end{array}$

8. $\begin{array}{r} 1357 \\ \underline{6} \end{array}$

9. $\begin{array}{r} 812 \\ \underline{1} \end{array}$

Topic Three Multiplying by a number of tens

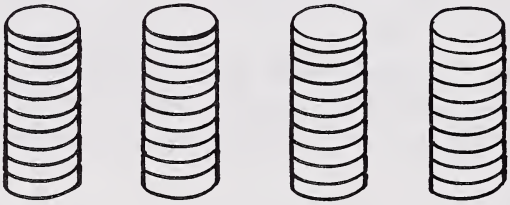
10, 20, 30, 40, 50, 60, 70, 80 and 90 are a number of tens.

10 is 1 TEN.
20 is 2 TENS.
30 is 3 TENS.
40 is 4 TENS.
50 is 5 TENS.
60 is 6 TENS.
70 is 7 TENS.
80 is 8 TENS.
90 is 9 TENS.

Notice that a number of tens is a number which has a digit other than zero in the tens' place and zero in the ones' place.

To multiply by a number of tens, you multiply the multiplicand by the digit in the tens' place of the multiplier. Put a zero at the right of this product. The product of a number and a number of tens ends in at least one zero.

EXAMPLE. Find the number of hockey pucks.



Solution

There are 4 groups of 10 pucks.

$$\left.\begin{array}{l} 4 \times 1 \text{ TEN} = 4 \text{ TENS} \\ 4 \times 10 \\ 40 \end{array}\right\} \text{These all name the same number.}$$

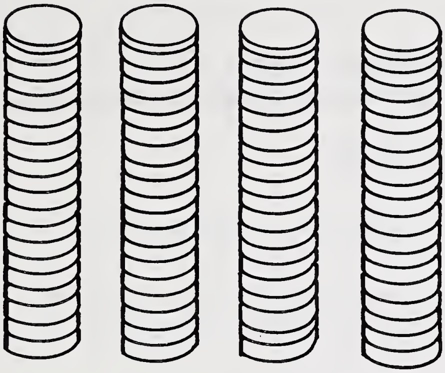
$4 \times 10 = 40$

$$\begin{array}{r} 4 \\ \underline{10} \\ 40 \end{array}$$

There are 40 pucks. (This number ends in one zero.)

Let's try an example in which the number of tens is not 1.

EXAMPLE: Find the number of hockey pucks.



Solution

There are 4 groups of 20 pucks. 20 can be regrouped to 2 TENS.

$$\left.\begin{array}{l} 4 \times 2 \text{ TENS} = 8 \text{ TENS} \\ 4 \times 20 \\ 80 \end{array}\right\} \quad \text{These all name the same number.}$$

$4 \times 20 = 80$

$$\begin{array}{r} 4 \\ \underline{20} \\ 80 \end{array}$$

There are 80 pucks. (This number ends in one zero.)

100, 200, 300, 400, 500, 600, 700, 800 and 900 are a number of hundreds. A number of hundreds is a number which has one digit other than zero in the hundreds' place and zeros in the tens' and ones' places.

To multiply by a number of hundreds, you multiply the multiplicand by the digit in the hundreds' place of the multiplier. Put two zeros to the right of this product. The product of a number and a number of hundreds ends in at least two zeros.

EXAMPLE: Multiply 21 × 200.

Solution

There are 21 groups of 200.

21 × 2 HUNDREDS = 42 HUNDREDS

21 × 200

4200

21 × 200 = 4200

(Number ends in two zeros.)

$$\begin{array}{r} 21 \\ \underline{200} \\ 4200 \end{array}$$

A number of thousands is a number which has a digit other than zero in the thousands' place and zeros in the hundreds', tens' and ones' places.

Find the product of the multiplicand and the digit in the thousands' place of the multiplier and put three zeros to the right of it.

EXAMPLE: Multiply 132 by 3000.

Solution

$$\begin{array}{r} 132 \\ \underline{3} \\ 396 \end{array} \qquad \begin{array}{r} 132 \\ \underline{3000} \\ 396{,}000 \end{array}$$

132 × 3000 = $\underline{396,000}$

To multiply by ten thousands, four zeros are required.
e.g. 47 × 50,000 = 2,350,000

Self-correcting Exercise 1

Find the products.

1. 39 × 30 = ________
2. 15 × 80 = ________
3. 67 × 800 = ________
4. 24 × 100 = ________
5. 49 × 5000 = ________
6. 368 × 6000 = ________
7. 50 × 70,000 = ________
8. 311 × 20,000 = ________
9. 400 × 386 = ________
10. 111 × 100,000 = ________

Corrections are on page 19.

Topic Four Two-digit multipliers

Study the example below where you are shown how to multiply a number by a two-digit multiplier.

EXAMPLE: Multiply 423 by 36.

Solution

423 × 36 is the same as 423 × 30 added to 423 × 6.

$$\begin{array}{r} 423 \\ \underline{6} \\ 2538 \end{array}$$

We must multiply 423 by 6. See the example on page 5 for explanations and method.

We can use the product of 423 and 3 to help us find the product of 423 and 30.

$$\begin{array}{r} 423 \\ \underline{3} \\ 1269 \end{array}$$

$$\begin{array}{r} 423 \\ \underline{30} \\ 12{,}690 \end{array}$$

To find the product of 423 and 30 from 423 and 3, we must put a zero to the right of the product of 423 and 3.

Thus the product of 423 and 36 is equal to the product of 423 and 30 plus the product of 423 and 6.

$$\begin{aligned} 423 \times 36 &= (423 \times 30) + (423 \times 6) \\ 423 \times 36 &= 12,690 + 2538 \\ 423 \times 36 &= \underline{\underline{15,228}} \end{aligned}$$

The products may be written one below the other so that the sum can be found directly.

$$\begin{array}{rl} 423 & \\ \underline{36} & \\ 2538 \leftarrow & \text{— Product of 423 and 6} \\ \underline{12690} \leftarrow & \text{— Product of 423 and 30} \\ 15{,}228 \leftarrow & \text{— Product of 423 and 36 (or sum of the above products)} \end{array}$$

Self-correcting Exercise 2

Find the products.

1. $$\begin{array}{r} 1137 \\ 96 \\ \hline \end{array}$$

2. $$\begin{array}{r} 4631 \\ 274 \\ \hline \end{array}$$

Corrections are on page 19.

Assignment 3

Use this multiplication problem to complete the following statements.

$$\begin{array}{r} {\scriptstyle 6\,7} \\ 279 \\ 8 \\ \hline 2232 \end{array}$$

1. 2 is written in the ones' place because _____ × _____ = _____.

2. 7 TENS times 8 equals ______________ which is regrouped to ____________________ and ____________________ .

3. 3 rather than 6 is written in the tens' place because ____ ____

4. 6 is written above 2 in 279 because ______________________ ____ ____

Use this multiplication problem to complete the following statements.

$$\begin{array}{r} 279 \\ \underline{28} \\ 2232 \\ \underline{5580} \\ 7812 \end{array}$$

5. 2232 is the product of ____________________ and ________________.

6. 279 was multiplied by ______________________ to obtain 5580.

7. 7812 is the ________________ of _______________ and __________________.

Find the products.

8.
$$\begin{array}{r} 918 \\ \underline{27} \\ 6426 \\ \underline{18360} \\ 24{,}786 \end{array}$$

9.
$$\begin{array}{r} 3687 \\ \underline{35} \end{array}$$

10.
$$\begin{array}{r} 61529 \\ \underline{174} \end{array}$$

11.
$$\begin{array}{r} 7286 \\ \underline{83} \end{array}$$

12.
$$\begin{array}{r} 82451 \\ \underline{96} \end{array}$$

13.
$$\begin{array}{r} 5892 \\ \underline{17} \end{array}$$

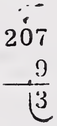
Topic Five: Multiplicands that contain a zero digit

We must remember that any number multiplied by zero equals zero or that zero multiplied by any number equals zero.

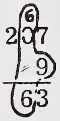
e.g. $2 \times 0 = 0$ or $0 \times 2 = 0$
$5 \times 0 = 0$ or $0 \times 5 = 0$ The products are all zero.
$9 \times 0 = 0$ or $0 \times 9 = 0$

EXAMPLE: Multiply 207 by 9.

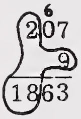
Solution



The product of 7 ONES and 9 is 63 ONES. 63 ONES is regrouped to 6 TENS and 3 ONES. 3 is written in the ones' place. 6 is written above 0 to show that it must be added to the product of 0 TENS and 9.



The product of 0 TENS and 9 is 0 TENS. 6 TENS (from 63 ONES) must be added to 0 TENS. The result is 6 TENS. 6 is written in the tens' place.



The product of 2 HUNDREDS and 9 is 18 HUNDREDS which is regrouped to 1 THOUSAND and 8 HUNDREDS. 8 is written in the hundreds' place and 1 is written in the thousands' place.

Thus $207 \times 9 =$ 1863.

Self-correcting Exercise 3

Write the standard name of each product.

1. $21 \times 0 =$ _______________ 2. $0 \times 13 =$ _______________

3. $21 \times 1 =$ _______________ 4. $1 \times 13 =$ _______________

5. $\begin{array}{r} 1706 \\ \underline{21} \end{array}$ 6. $\begin{array}{r} 9102 \\ \underline{307} \end{array}$

Corrections are on page 19.

Assignment 4

Find the products.

1. $\begin{array}{r} \overset{3}{2}7 \\ \underline{55} \\ 135 \\ 1350 \\ \hline 1485 \end{array}$

2. $\begin{array}{r} 3427 \\ \underline{601} \end{array}$

3. $\begin{array}{r} 6720 \\ \underline{12} \end{array}$

4. $\begin{array}{r} 3709 \\ \underline{93} \end{array}$

5. $\begin{array}{r} 4082 \\ \underline{450} \end{array}$

6. $\begin{array}{r} 95063 \\ \underline{508} \end{array}$

Topic Six: Rounding answers

Sometimes you are not required to give the exact answer to a problem. For example, you might be asked to round the answer to the nearer ten, hundred or thousand.

To round a number to the nearer ten we must decide whether the digit in the ones' place is less than half of 10 or more than half of 10.

e.g. 1 is less than half of ten.
2 is less than half of ten.
3 is less than half of ten.
4 is less than half of ten.
5 is exactly half of ten.
6 is more than half of ten.
7 is more than half of ten.
8 is more than half of ten.
9 is more than half of ten.

If the digit in the ones' place is less than half of 10, then the digit in the tens' place does not change.

e.g. 63 3 is less than half of 10. Therefore 6 does not change. 63 is nearer to 60 than to 70. 63 rounded to the nearer ten is 60.

If the digit in the ones' place is half or more than half of 10, then the digit in the tens' place is increased by 1.

e.g. 67 7 is more than half of 10. Therefore 6 is increased by 1. 67 is nearer to 70 than to 60. 67 rounded to the nearer ten is 70.

Self-correcting Exercise 4

Round each of the numbers to the nearer ten.

1. 27 ____________________
2. 335 ____________________
3. 164 ____________________
4. 1265 ____________________
5. 243 ____________________
6. 698 ____________________

For each number, name all of the numbers which, when rounded to the nearer ten, are rounded to the given number.

7. 210 ____
8. 330 ____
9. 700 ____
10. 490 ____

Corrections are on page 19.

To round numbers to the nearer hundred, we must consider the digit in the tens' place. If the digit is 1, 2, 3 or 4, the digit in the hundreds' place does not change. If the digit is 5, 6, 7, 8 or 9, the digit in the hundreds' place is increased by 1.

e.g. 362 The digit in the tens' place is 6. Therefore 3 must be increased by 1. 362 rounded to the nearer hundred is 400.

To round numbers to the nearer thousand, we must consider the digit in the hundreds' place. If the digit is 1, 2, 3 or 4, the digit in the thousands' place does not change. If the digit is 5, 6. 7. 8 or 9, the digit in the thousands' place is increased by 1.

e.g. 9388 The digit in the hundreds' place is 3. Therefore the digit in the thousands' place does not change.
9388 rounded to the nearer thousand is 9000.

You will notice that if a number is rounded
- to the nearer ten, it ends in at least one zero,
- to the nearer hundred, it ends in at least two zeros,
- to the nearer thousand, it ends in at least three zeros,
- to the nearer ten thousand, it ends in at least four zeros.

Assignment 5

Round each of the numbers to the nearer ten.

1. 13 10
2. 275 ______
3. 918 ______
4. 703 ______
5. 4231 ______

Round each of the numbers to the nearer hundred.

6. 73 100
7. 275 ______
8. 918 ______
9. 4231 ______
10. 6087 ______

Round each of the numbers to the nearer thousand.

11. 7225 7000

12. 1874 ______

13. 1168 ______

14. 9500 ______

15. 94,345 ______

Find the product. Below each product round the answer to the nearer hundred.

16.
$$\begin{array}{r} 98 \\ \underline{76} \\ 588 \\ \underline{6860} \\ 7448 \\ 7400 \end{array}$$
(rounded)

17.
$$\begin{array}{r} 2806 \\ \underline{58} \end{array}$$

18.
$$\begin{array}{r} 2955 \\ \underline{67} \end{array}$$

19.
$$\begin{array}{r} 54081 \\ \underline{49} \end{array}$$

Name 3 numbers which, when rounded to the nearer hundred, would equal the given number.

20. 6200 6219, 6249, 6185 and many others

21. 400 ______

22. 1500 ______

23. 2300 ______

24. 3800 ______

Find each product.

25. $0 \times 1 =$ ________

26. $2 \times 1 =$ ________

27. $7 \times 0 =$ ________

28. $1 \times 12 =$ ________

Assignment 6

For each problem
- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. Sue had 15 quarters. After she spent some of them, she had 9 quarters left. How many quarters did Sue spend?

Placeholder: Let q be the number of quarters Sue spent.

Equation: $15 - q = 9$

Computation

$$\begin{array}{r}15\\9\\\hline 6\end{array}$$

Answer: $q = 6$

Statement: Sue spent six quarters.

2. Ann bought some postage stamps. She has used 14 of them. Now she has 18 stamps left. How many stamps did Ann buy?

Placeholder: Let s be ________

____

Equation ________ Computation

Answer: ________

Statement: ________

____

3. Bill earned some money. He spent $48 for hiking boots. Then he had $23 left in his bank account. How much money did he have at first?

Placeholder: ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

4. In Winnipeg, 100,875 cars were tested for safety. The total number of cars was 115,642. How many cars were not tested yet?

Placeholder: ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

5. One week Mr. Thoms travelled 1476 kilometres. The following week he travelled 1829 kilometres. How many kilometres in all did Mr. Thoms travel?

Placeholder: ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

Key to the Self-correcting Exercises in Lesson 2

Exercise 1, page 8

1. 1170 (since 39 × 3 = 117)
2. 1200 (since 15 × 8 = 120)
3. 53,600 (since 67 × 8 = 536)
4. 2400 (since 24 × 1 = 24)
5. 245,000 (since 49 × 5 = 245)
6. 2,208,000 (since 368 × 6 = 2208)
7. 3,500,000 (since 50 × 7 = 350)
8. 6,220,000 (since 311 × 2 = 622)
9. 154,400 (since 4 × 386 = 1544)
10. 11,100,000 (since 111 × 1 = 111)

Exercise 2, page 10

1.

$$\begin{array}{r l} 1137 & \\ \underline{96} & \\ 6822 & \longleftarrow 1137 \times 6 \\ \underline{102330} & \longleftarrow 1137 \times 90 \\ 109{,}152 & \longleftarrow 1137 \times 96 \end{array}$$

2.

$$\begin{array}{r l} 4631 & \\ \underline{274} & \\ 18524 & \longleftarrow 4631 \times 4 \\ 324170 & \longleftarrow 4631 \times 70 \\ \underline{926200} & \longleftarrow 4631 \times 200 \\ 1{,}268{,}894 & \longleftarrow 4631 \times 274 \end{array}$$

Exercise 3, page 12

1. 0
2. 0
3. 21
4. 13
5.

$$\begin{array}{r} 1706 \\ \underline{21} \\ 1706 \\ \underline{34120} \\ 35{,}826 \end{array}$$

6.

$$\begin{array}{r} 9102 \\ \underline{307} \\ 63714 \\ \underline{2730600} \\ 2{,}794{,}314 \end{array}$$

Exercise 4, page 14

1. 30 (7 is more than half of 10.)
2. 340 (5 is half of 10.)
3. 160 (4 is less than half of 10.)
4. 1270 (5 is half of 10.)
5. 240 (3 is less than half of 10.)
6. 700 (8 is more than half of 10.)
7. 205, 206, 207, 208, 209, 210, 211, 212, 213, 214
8. 325, 326, 327, 328, 329, 330, 331, 332, 333, 334
9. 695, 696, 697, 698, 699, 700, 701, 702, 703, 704
10. 485, 486, 487, 488, 489, 490, 491, 492, 493, 494

End of Lesson 2

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ______

Time Spent on Lesson ______

(If label is missing
or incorrect)

File Number ______

Lesson Number ______

Student's Questions and Comments

Apply Lesson Label Here

Name ______

Address ______

______

______

Postal Code ______

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned
Teacher: ______

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: ______

Graded by: ______

Assignment Code: ______

Date Lesson Received:

______

Lesson Recorded ______

Teacher's Comments:

______
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

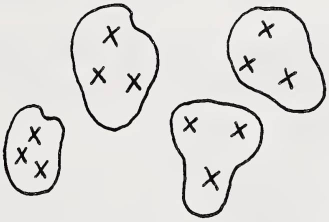
Division

Topic One: Division

DIVISION is the operation we use to separate one large group into a number of smaller groups. The process of division can be thought of as repeated subtraction.

e.g. $12 \div 3 = \square$ We can subtract groups of 3 from the total number of objects to find the number of smaller groups that we have.

One large group of
12 objects



A group of 12 objects contains
4 smaller groups of 3 objects
with no objects remaining.

If we divide the number of objects in the larger group by the number of objects in each smaller group, we obtain the number of smaller groups.

We have a special name for each part of a division problem. The number being divided (the large group) is called the DIVIDEND. The number that you divide by (the number of objects in each smaller group) is called the DIVISOR. The result that is obtained (the number of groups) is called the QUOTIENT, and the number that is left if the division is not exact is called the REMAINDER.

$$\underset{\text{DIVIDEND}}{\underset{\uparrow}{32}} \div \underset{\text{DIVISOR}}{\underset{\nwarrow}{5}} = \underset{\text{QUOTIENT}}{\underset{\nwarrow}{6}} \quad \text{REMAINDER } 2$$

$$\begin{array}{rrl} & 6 & \leftarrow \text{QUOTIENT} \\ 5 & \overline{)\,32} & \leftarrow \text{DIVIDEND} \\ & \underline{30} & \\ & 2 & \leftarrow \text{REMAINDER} \\ \uparrow & & \\ \text{DIVISOR} & & \end{array}$$

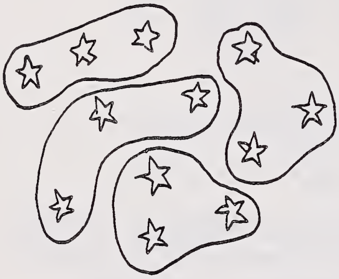
Assignment 1

1. In each picture below, a large group is separated into a number of smaller groups. You are to show mathematically how you would find the number of groups. One is done for you.

The large group separated into smaller groups.	The number of smaller groups

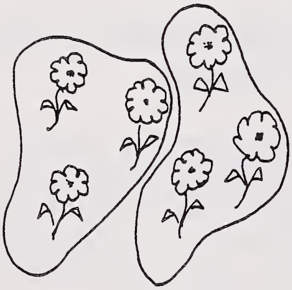
Number of objects in large group ÷ Number of objects in each smaller group = The number of groups

a.



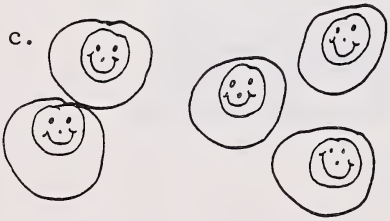
12 ÷ 3 = 4

b.



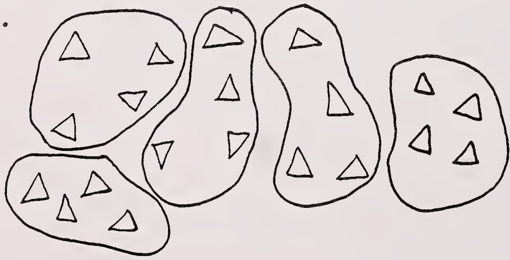
_____ ÷ _____ = _____

c.



_____ ÷ _____ = _____

d.



_____ ÷ _____ = _____

2. Consider each division problem given below and fill in the blanks with the appropriate words or numbers. One is done for you.

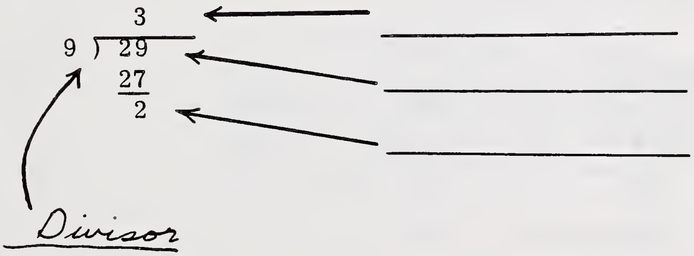
   a. In the division problem 15 ÷ 3 = 5, the dividend is 15, the divisor is 3 and the quotient is 5. What is the remainder? 0.

   b. In the division problem 21 ÷ 7 = 3, the number 21 is the ____________, 3 is the ____________, 7 is the divisor and ____________ is the remainder.

   c. Write the division problem that uses 8 as a dividend, 2 as a divisor and 4 as a quotient. Draw a picture to illustrate the problem.

   ________ ÷ ________ = ________

   d.



3. Find the quotient.

a.	10 ÷ 2 = 5	e.	56 ÷ 7 = ________
b.	16 ÷ 8 = ________	f.	54 ÷ 9 = ________
c.	36 ÷ 6 = ________	g.	48 ÷ 6 = ________
d.	24 ÷ 3 = ________	h.	28 ÷ 4 = ________

Topic Two: Two methods of division

To divide we must use two operations that you already know about — multiplication and subtraction. We must multiply to see how much of the large group has been used up. Then we must subtract to see how much of the large group remains. We want the amount that remains to be less than the divisor (the number of objects in the smaller group). If it is larger, we know that at least one more group could be formed.

To perform the division there are a variety of methods; you may have seen or used several different ones, but chances are you normally use either the subtractive method of division of the shortened method.

e.g. $33345 \div 247 = \square$

The subtractive method:

$$\begin{array}{r|l} 247\,\overline{)\,33345} & 100 \\ \underline{24700} & \\ 8645 & 30 \\ \underline{7410} & \\ 1235 & 5 \\ \underline{1235} & \\ 0 & \overline{135} \longleftarrow \text{QUOTIENT} \end{array}$$

$247 \times 100 = 24700$

$247 \times 30 = 7410$

$247 \times 5 = 1235$

$100 + 30 + 5 = 135$

The shortened method:

$$\begin{array}{rl} 135 & \longleftarrow \text{QUOTIENT} \\ 247\,\overline{)\,33345} & \\ \underline{247} & \longleftarrow 1 \times 247 \\ 864 & \\ \underline{741} & \longleftarrow 3 \times 247 \\ 1235 & \\ \underline{1235} & \longleftarrow 5 \times 247 \\ 0 & \end{array}$$

Both methods of division may be used, although we encourage you to use the shortened method because it involves less writing and is faster to do.

Topic Three: One-digit divisors

EXAMPLES: 325 ÷ 9 = ☐

Solution

$$\begin{array}{r@{\,}l} & 3 \\ \enclose{circle}{9} & \overline{)\,\enclose{circle}{32}5} \end{array}$$

9 will not divide into 3, so we begin by dividing 9 into 32. 9 divides into 32 about 3 times. Place this partial quotient above the last digit in 32.

$$\begin{array}{r@{\,}l} & \enclose{circle}{3} \\ \enclose{circle}{9} & \overline{)\,325} \\ & \underline{\enclose{circle}{27}} \\ & 5 \end{array}$$

Multiply 9 by 3 and place the product 27 below 32. Subtract 27 from 32 and obtain a remainder of 5. Check to be sure that the remainder is less than the divisor.

$$\begin{array}{r@{\,}l} & 3 \\ 9 & \overline{)\,32\enclose{circle}{5}} \\ & \underline{27}^{\times} \\ & 5\enclose{circle}{5} \end{array}$$

Bring down the next digit in the dividend and place it to the right of the remainder, 5. (The small × under the 5 indicates that it has been brought down.)

$$\begin{array}{r@{\,}l} & 3\enclose{circle}{6} \\ \enclose{circle}{9} & \overline{)\,325} \\ & \underline{27}^{\times} \\ & \enclose{circle}{55} \end{array}$$

Divide 9 into 55. It goes about 6 times. Place this partial quotient, 6, beside the first partial quotient, 3. (i.e. The quotient 36 appears above the 25 in the dividend.)

$$\begin{array}{r@{\,}l} & 3\enclose{circle}{6} \\ \enclose{circle}{9} & \overline{)\,325} \\ & \underline{27}^{\times} \\ & 55 \\ & \underline{54} \\ & 1 \end{array}$$

Multiply 9 by 6 and place the product 54 below 55. Subtract 54 from 55 and obtain a remainder of 1. Again, check to make sure that your remainder is less than the the divisor.

The division is complete since there are no more digits to bring down. (If there were, you would bring down the digits one at a time, and continue in the same manner.)

325 ÷ 9 = $\underline{36}$ remainder $\underline{\ 1\ }$

Self-correcting Exercise 1

Divide. Find the quotients.

1. $5\overline{)475}$ 2. $4\overline{)744}$ 3. $3\overline{)936}$

475 ÷ 5 = ____, r ____ 744 ÷ 4 = ____, r ____ 936 ÷ 3 = ____, r ____

Corrections are on page 20.

Assignment 2

Divide to find the quotients.

1. $\begin{array}{r} 21 \\ 6\overline{)131} \\ \underline{12}^{\times} \\ 11 \\ \underline{6} \\ 5 \end{array}$ 2. $9\overline{)146}$ 3. $8\overline{)328}$

131 ÷ 6 = 21, r 5 146 ÷ 9 = ____, r ____ 328 ÷ 8 = ____, r ____

4. $6\overline{)256}$ 5. $7\overline{)360}$ 6. $3\overline{)7965}$

256 ÷ 6 = ____, r ____ 360 ÷ 7 = ____, r ____ 7965 ÷ 3 = ____, r ____

Topic Four: Division by larger numbers

Students often find division difficult. The reasons for this may be that division calls upon many other skills and a mistake can be made at any step along the way. To divide by a larger number, you must know how to find the correct quotient; know where to place the first quotient figure; be able to multiply and subtract correctly and be able to use zero as a placeholder in the quotient. Of course to find the correct quotient it is necessary to know the basic division facts first of all.

EXAMPLE: Divide 1472 by 32.

Solution

$$\textcircled{32}\,\overline{)\,\textcircled{147}2}$$

32 will not divide into 1 or 14, so we begin by dividing 32 into 147. How many times will 32 divide into 147? To estimate what our quotient will be, we can round the divisor to 30 and the partial dividend to 140. We use our basic division facts to tell us that 30 will divide into 140 about 4 times (Think of 14 ÷ 3 = $\boxed{}$). This tells us that we can expect 32 to go into 147 about 4 times.

$$\begin{array}{r} \textcircled{4} \\ 32\,\overline{)\,1472} \\ \underline{128}^{\times} \\ 192 \end{array}$$

We place this partial quotient above the 7 in the dividend. This is very important. Now we can multiply 32 by 4, and subtract the product 128 from 147. Bring down the next digit.

$$\begin{array}{r} 4\textcircled{6} \\ \textcircled{32}\,\overline{)\,1472} \\ \underline{128}^{\times} \\ \textcircled{192} \\ \underline{192} \\ 0 \end{array}$$

Now we divide 32 into 192. This time we can round the partial dividend to 190. We know that 30 will go into 190 about 6 times. So we can expect 32 to divide into 192 about 6 times. Place this figure above the 2 in the dividend. We complete the division problem by multiplying 32 by 6 and subtracting the product from 192. Our remainder is 0.

1472 ÷ 32 = $\underline{46}$ remainder $\underline{\ 0\ }$

Estimating the quotient figure is perhaps the hardest step in a division problem. By rounding the divisor to a greater number and the dividend to a lesser number, there is less chance of using a quotient that is too large. However even when rounding the dividend and divisor, we do not always obtain the correct estimate for the quotient right away. If your estimated quotient is too large, you will not be able to subtract the product obtained in the multiplication step from the dividend. If the estimated quotient figure is too large, make it less one number at a time until the true quotient figure is found. If your estimated quotient is too small, you will obtain a remainder that is larger than the divisor. Increase the estimated quotient one number at a time until the true quotient figure is found.

EXAMPLE: Divide 8123 by 17.
Solution

$$\textcircled{17}\,\overline{)\,\textcircled{81}23}$$

Think: 80 ÷ 20 = 4
17 will go into 81 about 4 times.

$$\begin{array}{r}
4\\
\textcircled{17}\,\overline{)\,8123}\\
68^{\times}\\
\hline
\textcircled{132}
\end{array}$$

Think: 130 ÷ 20 = 6
If we use 6 as the next figure in our quotient we will find the product 17 × 6 = 102. When we subtract, our remainder will be 30. Since the remainder is too large, we know that our estimated quotient is too small. Try the next larger figure, 7.

$$\begin{array}{r}
47\\
17\,\overline{)\,8123}\\
68^{\times\times}\\
\hline
132\\
119\\
\hline
133
\end{array}$$

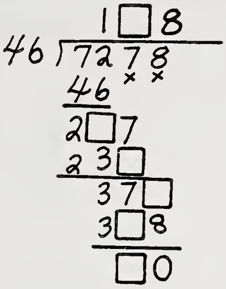
Now we are ready to divide 17 into 133. From the step above we know that it will go about 7 times.

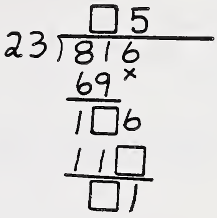
$$\begin{array}{r}
477\\
\textcircled{17}\,\overline{)\,8123}\\
68^{\times\times}\\
\hline
132\\
119\\
\hline
\textcircled{133}\\
119\\
\hline
14
\end{array}$$

8123 ÷ 17 = 477 remainder 14

Self-correcting Exercise 2

1. Work through each division problem below, filling in the boxes as you go along.

a. 

b. 

7278 ÷ 46 = ____ remainder ____

816 ÷ 23 = ____ remainder ____

2. Find the quotients.

a. 23) 534

b. 26) 1880

534 ÷ 23 = ______, r ________

1880 ÷ 26 = ________, r ________

c. Divide 6250 by 18.

6250 ÷ 18 = ________, r ________

Corrections are on pages 20 and 21.

Assignment 3

Divide.

1.
$$\begin{array}{r} 24 \\ 24\,\overline{)\,578} \\ \underline{48} \\ 98 \\ \underline{96} \\ 2 \end{array}$$

2. $22\,\overline{)\,946}$

3. $34\,\overline{)\,6494}$

4. $63\,\overline{)\,5796}$

5. $67\,\overline{)\,7683}$

6. $48\,\overline{)\,1638}$

7. $78\,\overline{)\,4436}$

8. $19\,\overline{)\,1672}$

Topic Five: Zeros In division

Zeros are important as placeholders in the quotient. Study the following example carefully.

EXAMPLE: Divide 329,982 by 47.
Solution

```
       7
47 ) 329,982
     329
       0
```

There is no remainder from the partial product 47 × 7.

```
       7
47 ) 329,982
     329  x
         9
```

Bring down the next digit, 9, from the dividend.

```
       7 0
47 ) 329,982
     329  x
         9
         0
         9
```

47 will not divide into 9 so we write a 0 above the 9 in the dividend. Then multiply 0 × 47 = 0 and write the partial product 0 below the 9. Subtract 9 - 0 = 9.

```
       7 0
47 ) 329,982
     329  xx
         9
         0
         98
```

Bring down the next digit, 8, from the dividend. Place it along side the 9 remainder.

```
       7 02
47 ) 329,982
     329  xx
         9
         0
         98
```

Divide 47 into 98. It goes about twice. Place this partial quotient, 2, beside the placeholder 0. (The 2 should be directly above the 8 in the dividend.)

```
       7 02
47 ) 329,982
     329  xxx
         9
         0
         98
         94
          42
```

Multiply 47 by 2 and place the product 94 below 98. Subtract 94 from 98 and obtain the remainder 4.

Bring down the next digit, 2, from the dividend.

...Solution is continued on new page.

... Solution continued

$$\begin{array}{r}
7\,02\textcircled{0}\\
\textcircled{47}\;\overline{)\,329,982}\\
\underline{329}\\
9\\
\underline{0}\\
98\\
\underline{94}\\
42\\
\underline{0}\\
\textcircled{42}
\end{array}$$

47 will not divide into 42. To show this, use the placeholder 0 directly above the 2 in the dividend. You can continue and multiply 47 × 0 = 0 and place this partial product below the 42. If you subtract 42 - 0 = 42 the remainder is still 42. All of the digits from the dividend have been brought down, so the division is complete.

329, 982 ÷ 47 = 7020 remainder 42

Self-correcting Exercise 3

1. Divide 79338 by 319.

79338 ÷ 319 = ______ remainder ______

2. $835\,\overline{)\,61530}$

61530 ÷ 835 = ______ remainder ______.

Corrections are on page 21.

Assignment 4

Find the quotients in each of the following.

1. $18 \overline{)\,7243}$

402
72××
4
0
43
36
7

2. $15 \overline{)\,7514}$

3. $48 \overline{)\,9648}$

4. $17 \overline{)\,9532}$

5. $68 \overline{)\,7344}$

6. $73 \overline{)\,45361}$

7. $88 \overline{)\,8262}$

8. $642 \overline{)\,37,236}$

Topic Six: Rounding quotients

Often you are required to express quotients to the nearer whole number. This is the same as being asked to round the quotient to the nearer one. To round the quotient to the nearer one, you must consider the remainder.

Remember these rules:

1. If the remainder is zero, the quotient does not change.
2. If the remainder is less than half of the divisor, the quotient does not change.
3. If the remainder is half or more than half of the divisor, the quotient is increased by 1. To find half of the divisor, divide it by 2.

EXAMPLE: Find the quotient of 5694 ÷ 74 = ☐ and round to the nearer one.

Solution

$$\begin{array}{r} 76\\ 74\,\overline{)\,569\underset{\times}{4}}\\ \underline{518}\\ 514\\ \underline{444}\\ \enclose{circle}{70} \end{array}$$

Half of the divisor is 74 ÷ 2 = 37. Since 70 is greater than 37, the remainder is more than half of the divisor. Thus the quotient should be increased by 1.

5694 ÷ 74 = 77, to the nearer one.

EXAMPLE: Find the quotient of 298 ÷ 46 = ☐ and round to the nearer one.

Solution

$$\begin{array}{r} 6\\ 46\,\overline{)\,298}\\ \underline{276}\\ 22 \end{array}$$

Half of the divisor is 46 ÷ 2 = 23. Since 22 is less than 23, the remainder is less than half the divisor. Thus the quotient should be left unchanged.

298 ÷ 46 = 6, to the nearer one.

Self-correcting Exercise 4

Divide. Round each of the quotients to the nearer one.

1. 18) 424

2. 35) 14212

424 ÷ 18 = ________, to the nearer one

14212 ÷ 35 = ________, to the nearer one

Corrections are on page 21.

Assignment 5

Divide. Round each of the quotients to the nearer one.

1.
```
         376
   34 ) 12816
        102xx
        ----
         261
         238      32 is more
         ---
          236     than half of
          204
          ---  <- the divisor.
           32
```

2. 76) 17784

12816 ÷ 34 = ________, rounded to the nearer one

17784 ÷ 76 = ________, rounded to the nearer one

3. $81 \overline{)\,14336}$

14336 ÷ 81 = ________, rounded to the nearer one

4. $519 \overline{)\,108373}$

108373 ÷ 519 = ________, rounded to the nearer one

5. $278 \overline{)\,18335}$

18335 ÷ 278 = ________, rounded to the nearer one

6. $27 \overline{)\,9269}$

9269 ÷ 27 = ________, rounded to the nearer one

Topic Seven: The short method of division

On page 4 you were shown two methods of division; the subtractive method and the shortened method. There is also another method of division that is a very short way of dividing by one digit. This method is very handy when you have a limited amount of time in which to do your work. The quotient is written above the division symbol as in the shortened method, but the products and differences are not written below each other.

EXAMPLE: Divide 72 by 6.
Solution

$$\begin{array}{r} 1 \\ 6\,\overline{)\,7_{1}2} \end{array}$$

Think of 7 TENS being divided into groups of 6. 7 TENS ÷ 6 = 1 TEN remainder 1 TEN. Write 1 above 7. Then write a small 1 down and to the left of 2. This small 1 can now be thought of as 1 TEN changed to 10 ONES. There were 2 ONES. Now there is a total of 12 ONES.

$$\begin{array}{r} 12 \\ 6\,\overline{)\,7_{1}2} \end{array}$$

Divide 12 ONES by 6. 12 ONES ÷ 6 = 2 ONES. Write 2 above 2 in 72.

$72 \div 6 = \underline{12}$

Now let's do a division question that has a remainder.

EXAMPLE: Divide 683 by 8.
Solution

$$8\,\overline{)\,683}$$

Since 6 HUNDREDS cannot be divided into groups of 8, consider the first two digits of the dividend. Think of 6 HUNDREDS 8 TENS as 68 TENS.

$$\begin{array}{r} 8 \\ 8\,\overline{)\,68_{4}3} \end{array}$$

68 TENS ÷ 8 = 8 TENS remainder 4 TENS. 4 TENS 3 ONES can be thought of as 43 ONES.

$$\begin{array}{r} 85 \text{ remainder } 3 \\ 8\,\overline{)\,68_{4}3}\phantom{\text{ remainder } 3} \end{array}$$

43 ONES ÷ 8 = 5 ONES remainder 3 ONES. The remainder is written following the quotient.

$683 \div 8 = \underline{85}$ remainder $\underline{3}$

Assignment 6

Use the short method to find each quotient.

1. $\begin{array}{r} 103 \\ 5\overline{)518} \end{array}$ remainder 3

2. $4\overline{)1400}$

3. $3\overline{)2049}$

4. $7\overline{)2385}$

5. $9\overline{)308}$

6. $8\overline{)400}$

Assignment 7

For each problem:

- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. Jim paid 18¢ for each package of notebook paper. He bought 3 packages. How many cents in all did Jim pay for the 3 packages of paper.

Placeholder: Let n be the cost of 3 packages of notebook paper

Equation: $3 \times 18 = n$

Computation

$$\begin{array}{r} \overset{2}{1}8 \\ 3 \\ \hline 54 \end{array}$$

Answer: $n = 54$

Statement: Jim paid 54¢ for 3 packages of notebook paper.

2. Most of the known planets in our solar system have one or more satellites or moons. Jupiter has 6 times as many satellites as Mars. Mars has 2 satellites. How many satellites does Jupiter have?

Placeholder: Let z be ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

3. There were 16 times as many motor vehicles in Canada in 1935 as there were in 1914. In 1914, there were 74,246 motor vehicles. How many motor vehicles were there in Canada in 1935?

Placeholder: Let m be ______________________________

Equation: Computation

Answer: ______________________________

Statement: ______________________________

4. The heat energy in food is measured in calories. There are 4 times as many calories in a piece of mince pie as in a glass of milk. There are 170 calories in a glass of milk. How many calories are there in a piece of mince pie?

Placeholder: Let c be ______________________________

Equation: Computation

Answer: ______________________________

Statement: ______________________________

5. The population density of a land area is the average number of persons per square mile (one square mile). The population density of Canada is approximately 6 persons per square mile. The land area of Canada is 3,560,238 square miles. Approximately how many people live in Canada?

Placeholder: Let p be ________________________________

Equation: ________________________ Computation

Answer: ________________________

Statement: ________________________

____

Key to Self-correcting Exercises in Lesson 3

Exercise 1, page 6

1.
$$\begin{array}{r} 95 \\ 5\,\overline{)\,475} \\ 45^{x} \\ \hline 25 \\ 25 \\ \hline 0 \end{array}$$

2.
$$\begin{array}{r} 186 \\ 4\,\overline{)\,744} \\ 4^{xx} \\ \hline 34 \\ 32 \\ \hline 24 \\ 24 \\ \hline 0 \end{array}$$

3.
$$\begin{array}{r} 312 \\ 3\,\overline{)\,936} \\ 9^{xx} \\ \hline 3 \\ 3 \\ \hline 6 \\ 6 \\ \hline 0 \end{array}$$

475 ÷ 5 = 95, r 0

744 ÷ 4 = 186, r 0

936 ÷ 3 = 312, r 0

Exercise 2, page 9

1. a.
$$\begin{array}{r} 158 \\ 46\,\overline{)\,7278} \\ 46^{xx} \\ \hline 267 \\ 230 \\ \hline 378 \\ 368 \\ \hline 10 \end{array}$$

b.
$$\begin{array}{r} 35 \\ 23\,\overline{)\,816} \\ 69^{x} \\ \hline 126 \\ 115 \\ \hline 11 \end{array}$$

7278 ÷ 46 = 158, remainder 10.

816 ÷ 23 = 35, remainder 11.

Exercise 2, page 9 continued

2. a.
$$\begin{array}{r}
23 \\
23\,\overline{)\,534} \\
\underline{46^{x}} \\
74 \\
\underline{69} \\
5
\end{array}$$

$534 \div 23 = \underline{23}$, r $\underline{5}$

b.
$$\begin{array}{r}
72 \\
26\,\overline{)\,1880} \\
\underline{182^{x}} \\
60 \\
\underline{52} \\
8
\end{array}$$

$1880 \div 26 = \underline{72}$, r $\underline{8}$

c.
$$\begin{array}{r}
347 \\
18\,\overline{)\,6250} \\
\underline{54^{xx}} \\
85 \\
\underline{72} \\
130 \\
\underline{126} \\
4
\end{array}$$

$6250 \div 18 = \underline{347}$, r $\underline{4}$

Exercise 3, page 12

1.
$$\begin{array}{r}
248 \\
319\,\overline{)\,79338} \\
\underline{638^{xx}} \\
1553 \\
\underline{1276} \\
2778 \\
\underline{2552} \\
226
\end{array}$$

$79338 \div 319 = \underline{248}$, r $\underline{226}$

2.
$$\begin{array}{r}
73 \\
835\,\overline{)\,61530} \\
\underline{5845^{x}} \\
3080 \\
\underline{2505} \\
575
\end{array}$$

$61530 \div 835 = \underline{73}$, r $\underline{575}$

Exercise 4, page 15

1.
$$\begin{array}{r}
23 \\
18\,\overline{)\,424} \\
\underline{36^{x}} \\
64 \\
\underline{54} \\
\bigcirc\!\!\!\!\!\!10
\end{array}$$

10 is more than half of the divisor.

$424 \div 18 = \underline{24}$, to the nearer one

2.
$$\begin{array}{r}
406 \\
35\,\overline{)\,14212} \\
\underline{140^{xx}} \\
21 \\
\underline{0} \\
212 \\
\underline{210} \\
\boxed{2}
\end{array}$$

2 is less than half of the divisor.

$14212 \div 35 = \underline{406}$, to the nearer one

End of Lesson 3

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned Teacher: __________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: ____________

Graded by: __________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Numbers

Topic One: Factors

We used the word "factor" in Lesson 1. In this lesson we will try to improve your understanding of factors. The word "factor" is used when dealing with products.

e.g. $\underset{\text{factor}}{\overset{2}{\uparrow}} \times \underset{\text{factor}}{\overset{3}{\uparrow}} = \underset{\text{product}}{\overset{6}{\uparrow}}$

2 is a factor of 6.
3 is a factor of 6.

Some numbers have many factors. To name the factors of a number, think of all the different products you can write for the number. The numbers in these products are all factors of the given number.

EXAMPLE: Name the factors of 12.

Solution

Think of all the products you can write for 12.
1 × 12 = 12 or 12 × 1 = 12
2 × 6 = 12 or 6 × 2 = 12
3 × 4 = 12 or 4 × 3 = 12
3 × 2 × 2 = 12 or 2 × 3 × 2 = 12 or 2 × 2 × 3 = 12

Circle all the different numbers in the products.
① × ⑫ = 12 or 12 × 1 = 12
② × ⑥ = 12 or 6 × 2 = 12
③ × ④ = 12 or 4 × 3 = 12
3 × 2 × 2 = 12 or 2 × 3 × 2 = 12 or 2 × 2 × 3 = 12

All the circled numbers are factors of 12.
The factors of 12 are 1, 2, 3, 4, 6, 12.

Sometimes a factor in a product will be used more than once.

e.g. 12 = 2 × 2 × 3 2 is used as a factor more than once.

In such cases, we can make use of exponential notation (see Lesson 1). Remember that this notation is a short way of writing a product that contains a factor which is used more than once.

e.g. 2 × 2 × 3 = 12, using exponential notation, is $2^2 \times 3^1 = 12$.

Numbers, other than 1, always have at least two factors. These factors are 1 and the number itself.

e.g. 1 × 12 = 12 1 and 12 (the number itself) are factors of 12.

Some numbers can be written as a product of more than two factors.

e.g. 30 = 2 × 3 × 5

2 is a factor of 30.
3 is a factor of 30.
5 is a factor of 30.

Self-correcting Exercise 1

Write each of the numbers as products of two factors. (Do not use 1 or the number itself.)

1. 8 ____________________

2. 14 ____________________

3. 35 ____________________

4. 77 ____________________

Write each of the numbers as products of more than 2 factors. (Do not use 1 or the number itself.)

5. 30 ____________________

6. 70 ____________________

7. 24 ____________________

8. 100 ____________________

Name 3 factors of each of the numbers.

9. 9 ____________________

10. 16 ____________________

11. 24 ____________________

12. 32 ____________________

13. Which of 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12 are not factors of 64?

____

Write each power as a product. Then give the standard name of each product.

14. 10^3 ____

15. 4^6 ____

16. 6^4 ____

17. 3^2 ____

Corrections are on page 20.

You probably realized from Lesson 2 and Lesson 3 that multiplication and division are related.

e.g. If $2 \times 3 = 6$ then $6 \div 3 = 2$
or if $3 \times 2 = 6$ then $6 \div 2 = 3$.

We can apply the words "factor" and "product" to both multiplication and division questions.

e.g. Two factors are multiplied to give a product.

$$\underset{\text{factor}}{\underset{\uparrow}{2}} \times \underset{\text{factor}}{\underset{\uparrow}{3}} = \underset{\text{product}}{\underset{\uparrow}{6}}$$

When the product is divided by one of the factors, the other factor is obtained.

$$\underset{\text{product}}{\underset{\uparrow}{6}} \div \underset{\text{factor}}{\underset{\uparrow}{3}} = \underset{\text{factor}}{\underset{\uparrow}{2}}$$

We can determine if one number is a factor of another number by dividing the first into the second. If the division process yields no remainder, then the first number is a factor of the second.

EXAMPLE: Is 32 a factor of 256?

Solution
Divide 256 by 32.

$$\begin{array}{r} 8 \\ 32\,\overline{)\,256} \\ \underline{256} \\ 0 \end{array}$$

32 is a factor of 256 because 32 divides evenly into 256 (the remainder is 0).
Since $8 \times 32 = 256$, 8 is also a factor of 256.

It is helpful to know about factor pairs when you are attempting to find all of the factors of a number. Each factor of a number has another factor of that number that "it goes with". This is possible because factor × factor = product.

e.g. $2 \times 3 = 6$ 2 and 3 are a factor pair of 6.
$1 \times 6 = 6$ 1 and 6 are a factor pair of 6.

Discovering factor pairs will help you to name all of the factors of a number. If you name one factor, be sure that you name the other factor of that particular factor pair. To name all of the factors you must test each of the natural numbers. (The natural numbers are 1, 2, 3, 4 and so on.) Ask yourself the following questions:

1. Is 1 a factor of the number? (Yes, 1 is a factor of every number.)
2. Is 2 a factor of the number? (Divide the number by 2. If 2 divides evenly into the number, 2 is a factor.)
3. Is 3 a factor of the number? (Divide the number by 3. If 3 divides evenly into the number, 3 is a factor.)

Carry on with the next numbers in the same way. When the factor pairs begin to repeat, there is no need to test any more numbers.

EXAMPLE: Find all of the factors of 36.

1. $\begin{array}{r} 36 \\ 1\overline{)36} \end{array}$ 1 and 36 are factors of 36.
2. $\begin{array}{r} 18 \\ 2\overline{)36} \end{array}$ 2 and 18 are factors of 36.
3. $\begin{array}{r} 12 \\ 3\overline{)36} \end{array}$ 3 and 12 are factors of 36.
4. $\begin{array}{r} 9 \\ 4\overline{)36} \end{array}$ 4 and 9 are factors of 36.
5. $\begin{array}{l} 7 \text{ remainder } 1 \\ 5\overline{)36} \end{array}$ 5 is not a factor of 36.
6. $\begin{array}{r} 6 \\ 6\overline{)36} \end{array}$ 6 is a factor of 36.
7. $\begin{array}{l} 5 \text{ remainder } 1 \\ 7\overline{)36} \end{array}$ 7 is not a factor of 36.
8. $\begin{array}{l} 4 \text{ remainder } 4 \\ 8\overline{)36} \end{array}$ 8 is not a factor of 36.
9. $\begin{array}{r} 4 \\ 9\overline{)36} \end{array}$ 9 and 4 are factors of 36.

The factor pairs are beginning to repeat. Therefore we have found all the factors of 36.

The factors of 36 are 1, 2, 3, 4, 6, 9, 12, 18, 36.

The curved lines join factor pairs. 6 does not have a curved line because 6 × 6 = 36. The factor pair is 6 and 6. We do not write the same factor twice.

Self-correcting Exercise 2

72 ÷ 24 = 3

1. Is 24 a factor of 72? ______________ Explain your answer.

____

2. Is 3 a factor of 72? ______________ Explain your answer.

____

3. Is 72 a factor of 24? ______________ Explain your answer.

____

83 ÷ 8 = 10 remainder 3

4. Is 8 a factor of 83? ____________ Explain your answer.

____

5. Is 10 a factor of 83? ____________ Explain your answer.

____

6. Is 3 a factor of 83? ____________ Explain your answer.

____

7. Is 27 a factor of 81? ____________ Explain your answer.

____

____

8. Are 7 and 3 a factor pair of 42? ____________
Explain your answer.

____

____

9. The following are factors of 24. Show the factor pairs by curved lines.

3, 6, 1, 12, 24, 2, 8, 4

10. Show how you would find all the factors of 48.

Corrections are on pages 20 and 21.

Assignment 1

Write two division statements for each product. The division statements must be related to the given multiplication statement.

1. 6 × 7 = 42 42 ÷ 7 = 6 42 ÷ 6 = 7
2. 3 × 1 = 3
3. 9 × 10 = 90
4. 4 × 8 = 32
5. 8 × 9 = 72

n is used as a placeholder in each of the following statements. In each case n is holding the place of what number?

6. n × 8 = 88 n = 11
7. 7 × n = 49 n =
8. 3 × 21 = n n =
9. n ÷ 6 = 17 n =
10. 120 ÷ n = 30 n =
11. 128 ÷ 4 = n n =

Which of these numbers 1, 2, 3, 4, 7, 8, 9, 11, 12, 13, 14 are factors of each of the following numbers?

12. 20 1, 2, 4

13. 32 ____

14. 45 ____

15. 54 ____

16. 36 ____

Name 3 factors of each of the following.

17. 125 1, 5, 25

18. 99 ____

19. 84 ____

20. 300 ____

21. 105 ____

Name the factors of each number. Use curved lines to show the factor pairs in each group of factors.

22. 28

$$\begin{tikzcd}[column sep=small] 1 \arrow[rrrrr, leftrightarrow, bend right] & 2 \arrow[rrr, leftrightarrow, bend right] & 4 \arrow[r, leftrightarrow, bend right] & 7 & 14 & 28 \end{tikzcd}$$

23. 37 ____

24. 56 ____

25. 68 ____

26. 72 ____

27. Is 25 a factor of 875? ______________ Explain your answer.

____

____

28. Is 11 a factor of 111? ______________ Explain your answer.

____

____

29. Are both 20 and 25 factors of 250? ______________
Explain your answer.

____

____

____

Topic Two: Even numbers

The set of whole numbers contains the numbers 0, 1, 2, 3 and so on.

EVEN NUMBERS are those numbers that have 2 as a factor. In other words, even numbers are evenly divisible by 2.

EXAMPLE: Is 126 an even number?

Solution

2 is a factor of even numbers. Divide 126 by 2 to see whether it is a factor of 126.

$$2\overset{63}{\overline{)126}} \qquad 2 \times 63 = 126$$

2 is a factor of 126. Therefore 126 is an even number.

Some even numbers are 6, 10, 18, 24, 32, 54 and 60. You will notice that the digit in the ones' place is either 0, 2, 4, 6 or 8.

Two methods of deciding whether or not a number is even are:

1. Divide the number by 2. If 2 divides evenly into the number, then 2 is a factor of the number and the number is even.
2. Look at the digit in the ones' place. If it is 0, 2, 4, 6 or 8, the number is an even number.

EXAMPLE: Is 137 an even number?

Solution

You may use either method.

1. $$\begin{array}{l} \;\;68 \text{ remainder } 1 \\ 2\overline{)137} \end{array}$$ 137 is not evenly divisible by 2, therefore 2 is not a factor of 137. 137 is not an even number.

2. In 137, the digit in the ones' place is 7. Since the digit in the ones' place is not 0, 2, 4, 6 or 8, 137 is not an even number.

Topic Three: Odd numbers

ODD NUMBERS do not have 2 as a factor. In other words, odd numbers are not evenly divisible by 2.

When dividing by 2, the only possible remainders are 0 and 1. If the remainder is 2, 3, 4 or any larger number, this means that you have not completed the division question. At least one more group of 2 objects could be formed. If the remainder is 0, then the number is evenly divisible by 2 and it is an even number. If the remainder is 1, then the number is not evenly divisible by 2 and is an odd number.

EXAMPLE: Is 137 an odd number?

Solution

2 is not a factor of an odd number. Divide 137 by 2 to see whether or not there is a remainder of 1.

$$\begin{array}{l} \;\;68 \text{ remainder } 1 \\ 2\overline{)137} \end{array}$$

2 is not a factor of 137. Therefore 137 is an odd number.

Some odd numbers are 7, 11, 19, 25, 33, 55 and 61. You will notice that the digit in the ones' place is either 1, 3, 5, 7 or 9.

Two methods of deciding whether or not a number is odd are:

1. Divide the number by 2. If there is a remainder of 1, then 2 is not a factor of the number and the number is an odd number.
2. Look at the digit in the ones' place. If it is 1, 3, 5, 7 or 9, the number is an odd number.

EXAMPLE: Is 126 an odd number?

Solution

You may use either method.

1. $\begin{array}{r}63\\2\overline{)126}\end{array}$ 126 is evenly divisible by 2. The remainder is 0; therefore 2 is a factor of 126. 126 is not an odd number.

2. In 126, the digit in the ones' place is 6. Since the digit in the ones' place is not 1, 3, 5, 7 or 9, 126 is not an odd number.

Assignment 2

Using division decide whether or not 2 is a factor of each of the following numbers. Then decide whether each number is odd or even.

		Division	Odd or even
1.	221	$\begin{array}{r}110 \text{ remainder } 1\\2\overline{)221}\end{array}$	odd
2.	186		
3.	322		
4.	70		
5.	84		
6.	97		

By looking at the digit in the ones' place decide whether each of the following is odd or even.

7. 113 odd

8. 48 __________

9. 50 __________

10. 655 __________

11. 101 __________

12. 29 __________

Topic Four: Composite numbers

A COMPOSITE NUMBER is a natural number (1, 2, 3 and so on) that has more than two factors. One of the factors of a composite number is 1. Another factor of the composite number is the number itself. Besides these two factors, a composite number must have at least one more factor.

EXAMPLE: Is 27 a composite number?

Solution

Find at least one factor of 27 other than 1 and 27.

$\begin{array}{r} 27 \\ 1\overline{)27} \end{array}$ 1 and 27 are factors of 27.

$\begin{array}{r} 13 \text{ remainder } 1 \\ 2\overline{)27} \end{array}$ 2 is not a factor of 27.

$\begin{array}{r} 9 \\ 3\overline{)27} \end{array}$ 3 and 9 are factors of 27.

We have found 4 factors of 27. They are 1, 3, 9, 27. Since 27 has more than 2 factors, it is a composite number.

To tell whether a number is composite, we need to find only one factor of the number other than 1 and the number itself. We can use some basic rules for finding another factor of the number.

1. Every number with 0, 2, 4, 6 or 8 in the ones' place is an even number. It has 2 as a factor. Therefore every even number is a composite number.
   e.g. 18 { 2 is a factor of 18 since 18 is an even number. Three factors of 18 and 1, 2, 18. Therefore 18 is a composite number.

2. If the sum of the digits in a number is 3, 6 or 9, the number is evenly divisible by 3. It has 3 as a factor. Therefore every number, whose digits add up to 3, 6 or 9, is a composite number.
   e.g. 729 { The sum of 7, 2 and 9 is 18. The sum of 1 and 8 is 9. 9 is evenly divisible by 3. 3 is a factor of 729. Three factors of 729 are 1, 3, 729. Therefore 729 is a composite number.

3. If the digit in the ones' place is 5 or 0, the number is evenly divisible by 5. 5 is a factor of the number. Therefore every number, that has 5 or 0 in the ones' place, is a composite number.
   e.g. 55 { The digit in the ones' place is 5. 5 is a factor of 55. Three factors of 55 are 1, 5, 55. Therefore 55 is a composite number.

Self-correcting Exercise 3

Without dividing, tell whether 2, 3 or 5 are factors of each number. In each case explain how you know which number or numbers are factors.

1. 21 ____________________

   Explanation ____

2. 25 ____________________

   Explanation ____

3. 26 ____________________

   Explanation ____

4. 18 ____________________

   Explanation ____

   ____

5. 20 ____________________

   Explanation ____

   ____

6. 45 ____________________

Explanation ____________________

____

____

7. 30 ____________________

Explanation ____________________

____

____

Corrections are on page 21.

Assignment 3

Name all of the factors of each of the numbers. Then in the column below "Composite" write "yes" if the number is a composite number; write "no" if the number is not a composite number.

		Factors	Composite
1.	10	1,2,5,10	yes
2.	13		
3.	14		
4.	25		
5.	29		
6.	47		
7.	51		
8.	54		
9.	72		
10.	100		

Name 3 factors of each of the following composite numbers. Try to use the rules given on pages 11-12.

11. 325 1,5,325

12. 177 ______________

13. 441 ______________

14. 387 ______________

15. 312 ______________

Topic Five: Prime numbers

A PRIME NUMBER is a natural number that has only 2 factors. One of the factors of a prime number is 1, the only other factor of the prime number is the number itself.

EXAMPLE: Is 7 a prime number?

Solution

1 and 7 are factors of 7. $\begin{array}{r} 7 \\ 1\overline{)7} \end{array}$

2, 3, 4, 5 and 6 are not factors of 7. None of these numbers divide evenly into 7.

Since the only factors of 7 are 1 and itself, 7 is a prime number.

The smallest factor of a number, other than 1, is 2. The quotient of the number and 2 is the largest factor of the number, other than the number itself.

e.g. $\begin{array}{r} 46 \\ 1\overline{)46} \end{array}$ 1 and 46 are factors of 46. 1 is the smallest factor of 46. 46 is the largest factor of 46.

$\begin{array}{r} 23 \\ 2\overline{)46} \end{array}$ 2 and 23 are factors of 46. 2 is the smallest factor of 46, other than 1. 23 is the largest factor of 46, other than 46.

Therefore when trying to find factors of a number, test numbers that are smaller than or equal to one-half of the number. There is no value in testing numbers that are larger than one-half of the given number because we know that they are not factors of the given number.

e.g. There is no value in testing 31 to see whether it is a factor of 46 because it is more than one-half of 46.

We have used the idea of factor pairs to show this fact.

e.g.. 1, 2, 23, 46

Self-correcting Exercise 4

Divide each number by 2 to find the largest number to be tested as a factor.

1. 11 ____________________
2. 12 ____________________
3. 17 ____________________
4. 21 ____________________
5. 28 ____________________
6. 33 ____________________
7. 43 ____________________
8. 57 ____________________

Name all of the factors of each number. Tell whether the given number is a prime number or a composite number.

		Factors	Prime or composite
9.	2		
10.	15		
11.	16		
12.	41		

13. Is 1 a prime or composite number? Explain your answer.

____

____

Corrections are on page 22.

When asked to find all of the prime numbers between two numbers, it is convenient to use a sieve. Write down all of the numbers. Then cross out each number that has factors other than 1 and itself. The remaining numbers will be prime numbers.

EXAMPLE: Name all of the prime numbers between 1 and 20.

Solution

Name all of the numbers.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20

All of the even numbers have 2 as a factor. Therefore cross out all even numbers except 2. 2 is a prime number because its only factors are 1 and 2.

1	2	3	~~4~~	5
~~6~~	7	~~8~~	9	~~10~~
11	~~12~~	13	~~14~~	15
~~16~~	17	~~18~~	19	~~20~~

Cross out all of the numbers that have 3 as a factor except 3. 3 is a prime number because its only factors are 1 and 3.

1	2	3	~~4~~	5
~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~
~~16~~	17	~~18~~	19	~~20~~

The remaining numbers, except 1, have only 1 and themselves as factors. They are prime numbers.

The prime numbers between 1 and 20 are 2, 3, 5, 7, 11, 13, 17, 19.

Assignment 4

Tell whether each of the numbers is prime or composite. Name 3 factors of each composite number. Name 2 factors of each prime number.

		Prime or composite	Factors
1.	50	composite	1,5,50
2.	49		
3.	97		
4.	105		
5.	19		
6.	86		
7.	64		
8.	5		
9.	39		
10.	88		

11. Is every even number prime? ____________
Explain your answer.

____

____

12. What is the smallest prime number? ____________

13. What is the smallest composite number? ____________

14. What is the smallest composite odd number? ____________

15. Use a sieve to name all of the prime numbers between 1 and 50. Cross out all the numbers that have 2 as a factor, then all of the numbers that have 3 as a factor, then those that have 5 as a factor and those that have 7 as a factor.

1	2	3	4	5
6	7	8	9	10
11	12	13	14	15
16	17	18	19	20
21	22	23	24	25
26	27	28	29	30
31	32	33	34	35
36	37	38	39	40
41	42	43	44	45
46	47	48	49	50

The prime numbers are: ______________________________

Assignment 5

For each problem:

- tell what the placeholder represents
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. In an election for class secretary, Janet received 39 votes and Sue received 13 votes. Janet received how many times as many votes as Sue?

Placeholder: Let t be the number of times as many votes Janet received as compared to those Sue received.

Equation: $39 \div 13 = t$

Answer: $3 = t$

Statement: Janet received 3 times as many votes as Sue received.

Computation

$$\begin{array}{r} 3 \\ 13\overline{)39} \\ 39 \\ \hline 0 \end{array}$$

2. The population density of Australia is about 3 persons. About 8,925,000 persons live in Australia. The land area of Australia is about how many square miles?

Placeholder: Let x be ____________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

3. An office building that was 76 feet high above the ground was demolished. Another office building that was 228 ft. high above the ground was built in its place. The height of the new building was how many times the height of the old building?

Placeholder: Let h be ____________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

4. In 1956, the population density of Saskatchewan was 4 persons. About 880,000 persons lived in Saskatchewan in 1956. The area of Saskatchewan is about how many square miles?

Placeholder: Let a be ____________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

5. Lois had 95 coins from England and 19 coins from France in her collection of foreign coins. She has how many times as many English coins as French coins?

Placeholder: Let c be ________________________________

____

Equation: ______________________ Computation

Answer: ______________________

Statement: ______________________

____

Key to the Self-correcting Exercises in Lesson 4

Exercise 1, page 2

1. $2 \times 4 = 8$ or $4 \times 2 = 8$
2. $2 \times 7 = 14$ or $7 \times 2 = 14$
3. $5 \times 7 = 35$ or $7 \times 5 = 35$
4. $7 \times 11 = 77$ or $11 \times 7 = 77$

These factors may be written in any order.

5. $30 = 2 \times 3 \times 5$
6. $70 = 2 \times 5 \times 7$
7. $24 = 2 \times 3 \times 4$
8. $100 = 2 \times 5 \times 10$

9. 1, 3, 9
10. any 3 of 1, 2, 4, 8, 16
11. any 3 of 1, 2, 3, 4, 6, 8, 12, 24
12. any 3 of 1, 2, 4, 8, 16, 32

13. 3, 5, 6, 7, 9, 10, 11, 12

14. $10^3 = 10 \times 10 \times 10 = 1000$
15. $4^6 = 4 \times 4 \times 4 \times 4 \times 4 \times 4 = 4096$
16. $6^4 = 6 \times 6 \times 6 \times 6 = 1296$
17. $3^2 = 3 \times 3 = 9$

Exercise 2, page 4

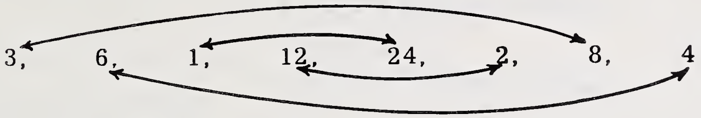
1. Yes, because 72 is evenly divisible by 24.
2. Yes, because 72 is evenly divisible by 3.
3. No, because 24 is not evenly divisible by 72.

4. No, because 83 is not evenly divisible by 8.
5. No, because 83 is not evenly divisible by 10.
6. No, because 83 is not evenly divisible by 3.

7. Yes, because 3 × 27 = 81 and 81 is evenly divisible by 27.

8. No, because 7 × 3 does not equal 42.

9.



10. $\begin{array}{r} 48 \\ 1\overline{)48} \end{array}$ 1 and 48 are factors of 48.

$\begin{array}{r} 24 \\ 2\overline{)48} \end{array}$ 2 and 24 are factors of 48.

$\begin{array}{r} 16 \\ 3\overline{)48} \end{array}$ 3 and 16 are factors of 48.

$\begin{array}{r} 12 \\ 4\overline{)48} \end{array}$ 4 and 12 are factors of 48.

$\begin{array}{l} \;\;\;9 \text{ remainder } 3 \\ 5\overline{)48} \end{array}$ 5 is not a factor of 48.

$\begin{array}{r} 8 \\ 6\overline{)48} \end{array}$ 6 and 8 are factors of 48.

$\begin{array}{l} \;\;\;6 \text{ remainder } 6 \\ 7\overline{)48} \end{array}$ 7 is not a factor of 48.

$\begin{array}{r} 6 \\ 8\overline{)48} \end{array}$ 8 and 6 are factors of 48.

Factors of 48 are 1, 2, 3, 4, 6, 8, 12, 16, 24 and 48.

Exercise 3, page 12

1. 3; because 2 + 1 = 3 and 3 is evenly divisible by 3.
2. 5; because the digit in the ones' place is 5.
3. 2; because the digit in the ones' place is even (6).
4. 2, because the digit in the ones' place is even (8).
   3; because 1 + 8 = 9 and 9 is evenly divisible by 3.
5. 2; because the digit in the ones' place is even (0).
   5; because the digit in the ones' place is 0.
6. 3; because 4 + 5 = 9 and 9 is evenly divisible by 3.
   5; because the digit in the ones' place is 5.
7. 2; because the digit in the ones' place is even (0).
   3; because 3 + 0 = 3 and 3 is evenly divisible by 3.
   5; because the digit in the ones' place is 0.

Exercise 4, page 15

1.	$\begin{array}{r} 5 \text{ remainder } 1 \\ 2\overline{)11} \end{array}$	Largest number is 5.
2.	$\begin{array}{r} 6 \\ 2\overline{)12} \end{array}$	Largest number is 6.
3.	$\begin{array}{r} 8 \text{ remainder } 1 \\ 2\overline{)17} \end{array}$	Largest number is 8.
4.	$\begin{array}{r} 10 \text{ remainder } 1 \\ 2\overline{)21} \end{array}$	Largest number is 10.
5.	$\begin{array}{r} 14 \\ 2\overline{)28} \end{array}$	Largest number is 14.
6.	$\begin{array}{r} 16 \text{ remainder } 1 \\ 2\overline{)33} \end{array}$	Largest number is 16.
7.	$\begin{array}{r} 21 \text{ remainder } 1 \\ 2\overline{)43} \end{array}$	Largest number is 21.
8.	$\begin{array}{r} 28 \text{ remainder } 1 \\ 2\overline{)57} \end{array}$	Largest number is 28.
9.	1, 2	Prime
10.	1, 3, 5, 15	Composite
11.	1, 2, 4, 8, 16	Composite
12.	1, 41	Prime

13. 1 is neither prime nor composite because it has only one factor, 1.

End of Lesson 4

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received:

____

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Greatest Common Factor

Topic One: Factors

Let's review what we know about factors.

1. A factor times a factor equals a product.
   e.g. $3 \times 4 = 12$ 3 and 4 are factors of 12.

2. A product divided by a factor equals a factor.
   e.g. $12 \div 3 = 4$ 3 and 4 are factors of 12.

3. To be a factor, a number must divide evenly into another number.
   e.g. $12 \div 5 = 2$ remainder 2 5 is not a factor of 12 because it does not divide evenly into 12.

4. The number itself is a factor of the number.
   e.g. 12 is a factor of 12 because $1 \times 12 = 12$.

5. 1 is a factor of every number.
   e.g. 1 is a factor of 12 because $1 \times 12 = 12$.

6. 2 is a factor of every even number.
   e.g. 12 is an even number. 2 is a factor of 12 because $2 \times 6 = 12$.

7. A product may contain more than two factors.
   e.g. $24 = 2 \times 3 \times 4$ 24 can be written as a product of 3 factors.

8. The same factor may be used more than once in a product.
   e.g. $12 = 2 \times 2 \times 3$ The factor $\underline{2}$ is used 2 times.

9. We can use exponential notation when a factor is used more than once.
   e.g. $12 = 2 \times 2 \times 3$ can be written as $12 = 2^2 \times 3^1$.

10. All numbers larger than 1 have at least two factors.

    a. Numbers which have only two factors are prime numbers.
       e.g. Since the only factors of 7 are 1 and 7, 7 is a prime number.

    b. Numbers which have more than two factors are composite numbers.
       e.g. Since 9 has three factors (1, 3, 9), 9 is a composite number.

Self-correcting Exercise 1

$1 \times 16 = 16$

$12 \div 2 = 6$

$20 \div 1 = 20$

$20 = 2 \times 2 \times 5$

$12 = 2 \times 2 \times 3$

$19 \div 19 = 1$

$2 \times 8 = 16$

$12 \div 4 = 3$

$16 \div 4 = 4$

$12 \times 1 = 12$

$20 \div 5 = 4$

$2 \times 10 = 20$

Using the above statements, name the factors of each number.

1. 16 ______________________
2. 12 ______________________
3. 20 ______________________
4. 19 ______________________

Name all the factors of each number. Then state whether the number is a prime or composite number.

	Factors	Prime or composite
5. 10		
6. 4		
7. 29		
8. 13		
9. 26		

Use exponential notation to write each product.

10. $125 = 5 \times 5 \times 5$ ______________________

11. $100 = 2 \times 5 \times 2 \times 5$ ______________________

12. $42 = 2 \times 3 \times 7$ ______________________

13. $60 = 2 \times 2 \times 3 \times 5$ ______________________

Corrections are on page 18.

Topic Two: Complete Factorization

In writing numbers as products of factors we have used both prime and composite numbers as factors.

e.g. 8 may be written as the product of the prime number 2 and the composite number 4. $8 = 2 \times 4$

Now, suppose we wished to write 8 as the product of prime numbers only. We would have to rewrite the composite number 4 as the product of the two prime numbers 2 and 2.

$$8 = 2 \times 2 \times 2$$

When a number is written as a product of primes, it is called the **complete factorization** of the number. To find the complete factorization of a number we first write the number as a product of two of its factors. If these factors are not prime numbers then we rewrite each composite number as a product of primes.

Example: Find the complete factorization of 24.

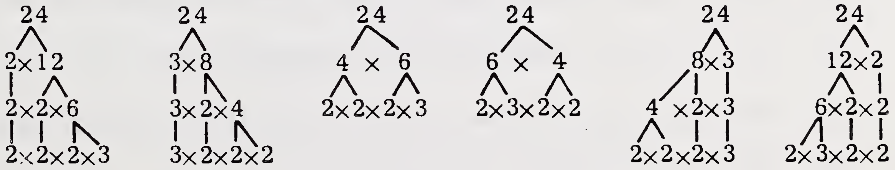
Solution

Write 24 as a product of two of its factors.
$24 = 4 \times 6$
4 and 6 are composite numbers.
Rewrite 4 as a product of two of its factors. Rewrite 6 as a product of two of its factors.
Since $4 = 2 \times 2$ and $6 = 3 \times 2$; $24 = 2 \times 2 \times 2 \times 3$.
2 and 3 are prime numbers.

The complete factorization of 24 is $\underline{24 = 2 \times 2 \times 2 \times 3}$.

Though there are many ways of arriving at the final answer, any number has only one complete factorization.

e.g.



In each case, $24 = 2 \times 2 \times 2 \times 3$ or $24 = 2^3 \times 3^1$ using exponential notation.

A prime number is considered the complete factorization of itself.

e.g. The complete factorization of 7 is 7.

Assignment 1

Name all of the factors of each number.

1. 14 1, 2, 7, 14
2. 36
3. 30
4. 47
5. 72

6. Name all of the prime numbers between 1 and 30.

1	2	3	~~4~~	5	~~6~~
7	~~8~~	~~9~~	~~10~~	11	12
13	14	15	16	17	18
19	20	21	22	23	24
25	26	27	28	29	30

The prime numbers are

Name all the prime factors of each number.

7. 36 2, 3
8. 14
9. 30
10. 47
11. 72

Write the complete factorization for each number.

12. 36 $36 = 2 \times 2 \times 3 \times 3$
13. 14
14. 30
15. 47
16. 72

Topic Three: Common factors

When a number is a factor of each of two numbers, it is a COMMON FACTOR of the two numbers.

As you learned earlier, 2 is a factor of every even number. We can say that 2 is a common factor of all even numbers. In other words, every even number is evenly divisible by two.

We can find a common factor of two numbers by first naming all of the factors of each number and then picking out a factor that is the same for both numbers.

EXAMPLE: Name the common factors of 12 and 20.

Solution

Name all the factors of each number.
The factors of 12 are ①, ②, 3, ④, 6 and 12.
The factors of 20 are ①, ②, ④, 5, 10 and 20.

Circle all the factors that are the same for both numbers.
The common factors of 12 and 20 are 1, 2 and 4.

Self-correcting Exercise 2

1. Name the common factors of 8 and 20.

   a. The factors of 8 are ______________________________

   b. The factors of 20 are ______________________________

   c. Circle the common factors of 8 and 20.

   The common factors of 8 and 20 are __________________

2. Name the common factors of 12 and 30.

   a. The factors of 12 are ______________________________

   b. The factors of 30 are ______________________________

   c. Circle the common factors of 12 and 30.

   The common factors of 12 and 30 are __________________

3. Name the common factors of 3 and 7.

a. The factors of 3 are ____________________

b. The factors of 7 are ____________________

c. Circle the common factors of 3 and 7.

The common factors of 3 and 7 are ____________________

4. The only common factor of any two prime numbers is __________

Corrections are on page 18.

Assignment 2

Name the common factors of each group of numbers.

1. 6, 8

The factors of 6 are ①,②,3,6.

The factors of 8 are ①,②,4,8.

Circle the common factors of 6 and 8.

The common factors of 6 and 8 are 1,2.

2. 12, 16

The factors of 12 are ____________________

The factors of 16 are ____________________

Circle the common factors of 12 and 16.

The common factors of 12 and 16 are ____________________

3. 15, 30

The factors of 15 are ____________________

The factors of 30 are ____________________

Circle the common factors of 15 and 30.

The common factors of 15 and 30 are ____________________

4. 15, 27

The factors of 15 are ______________________________

The factors of 27 are ______________________________

Circle the common factors of 15 and 27.

The common factors of 15 and 27 are ____________________

5. 30, 72

The factors of 30 are ______________________________

The factors of 72 are ______________________________

Circle the common factors of 30 and 72.

The common factors of 30 and 72 are ____________________

6. 36, 27, 18

The factors of 36 are ______________________________

The factors of 27 are ______________________________

The factors of 18 are ______________________________

Circle the common factors of 36, 27 and 18.

The common factors of 36, 27 and 18 are ________________

Topic Four: Greatest common factor

The GREATEST COMMON FACTOR of two numbers is the largest number that is a factor of both numbers.

There are two methods of finding the greatest common factor.
1. List all of the factors of each number. Choose the largest number that is a factor of both numbers.
2. Use the complete factorization of each number. The greatest common factor is the product of all the factors which are common to both numbers.

Let's use the first method.

EXAMPLE: Name the greatest common factor of 12 and 16.

Solution

Name all of the factors of 12 and 16.
The factors of 12 are ①, ②, 3, ④, 6 and 12.
The factors of 16 are ①, ②, ④, 8 and 16.

Circle the common factors of 12 and 16.
The common factors of 12 and 16 are 1, 2 and 4.

Name the largest number in the set of common factors.
$\underline{4}$ is the greatest common factor of 12 and 16.

Let's use the second method.

EXAMPLE: Name the greatest common factor of 12 and 16.

Solution

Find the complete factorization of each number.
$12 = 2 \times 6$ (6 is not a prime number. Write 6 as a product.)
$12 = 2 \times 2 \times 3$
$16 = 4 \times 4$ (4 is not a prime number. Write 4 as a product.)
$16 = 2 \times 2 \times 2 \times 2$

The greatest common factor is the product of all the factors which both numbers have in common. If we look at the complete factorization of 12 and 16 we can see they both have 2 factors of 2 in common (shown by circles).

$$12 = \boxed{2 \times 2} \times 3$$
$$16 = \boxed{2 \times 2} \times 2 \times 2$$

Therefore the greatest common factor of 12 and 16 is 2×2 which is $\underline{4}$.

Usually it is convenient to use the first method to find the greatest common factor of smaller numbers.
e.g. Use it for numbers like 8, 24, 36 and 50.

When numbers are large and have many factors it is more convenient to use the second method.
e.g. Use it for numbers like 100, 168, 210 and 350.

Self-correcting Exercise 3

1. The factors of 12 are ______________________

   The factors of 30 are ______________________

   The common factors of 12 and 30 are ______________________

   The greatest common factor of 12 and 30 is ______________________

2. The factors of 18 are ______________________

   The factors of 24 are ______________________

   The common factors of 18 and 24 are ______________________

   The greatest common factor of 18 and 24 is ______________________

3. 70 = 2 × 5 × 7
   20 = 2 × 2 × 5
   The greatest common factor of 20 and 70 is ______________________

4. 66 = 2 × 11 × 3
   165 = 11 × 3 × 5
   The greatest common factor of 66 and 165 is ______________________

Corrections are on page 18.

Assignment 3

Name all the factors of each number.

1. 24 ______________________

2. 60 ______________________

3. 25 ______________________

4. 90 ______________________

Using the above information, name all of the common factors of each group of numbers.

5. 24 and 60 ______________________

6. 24 and 25 ______________________

7. 24 and 90 ______________________

8. 60 and 25 ____________________

9. 60 and 90 ____________________

10. 25 and 90 ____________________

11. 60, 25 and 90 ____________________

Using the information on page 9, name the greatest common factor of each group of numbers.

12. 24 and 60 ____________________

13. 24 and 25 ____________________

14. 24 and 90 ____________________

15. 60 and 25 ____________________

16. 60 and 90 ____________________

17. 25 and 90 ____________________

18. 60, 25 and 90 ____________________

Find the complete factorization for each number.

19. 108 ____________________

20. 125 ____________________

21. 234 ____________________

22. 306 ____________________

Using the above information, name the greatest common factor of each group of numbers.

23. 108 and 125 ____________________

24. 108 and 234 ____________________

25. 108 and 306 ____________________

26. 125 and 234 ____________________

27. 125 and 306 ____________________

28. 234 and 306 ____________________

Marks

Review

Carefully restudy Lessons 1, 2, 3, 4 and the first part of 5. Notice where you made errors. Try to eliminate the errors (similar to those you made earlier) from this review.

Consider the number 16, 925, 109, 587.

1 1. This number, written in words, is ____ ____________

____

____________________________ ____________

10 2. The digit in the hundreds' place is __________,

in the hundred millions' place is __________,

in the hundred thousands' place is __________,

in the millions' place is __________,

in the ones' place is __________,

in the tens' place is __________,

in the ten millions' place is __________,

in the ten thousands' place is __________,

in the thousands' place is __________ and

in the trillions' place is __________.

Write each number in expanded form.

2 3. 352 = $(3 \times 100) +$ ______________________

4 4. 7, 830 = ______________________

7 5. 9, 473. 432 = ______________________

Marks

Round 9,603,239 to the

6. nearer hundred 9,603,200

1 7. nearer million ____________

1 8. nearer hundred thousand ____________

1 9. nearer ten ____________

1 10. nearer ten thousand ____________

1 11. nearer thousand ____________

Give the standard name of each number.

12. $10^4 =$ $10 \times 10 \times 10 \times 10 = 10{,}000$

1 13. $16^2 =$ ____________

1 14. $3 \times 10^2 =$ ____________

1 15. $2^4 =$ ____________

1 16. $5^3 =$ ____________

Convert each number to a base ten number.

3 17. $324_6 = (3 \times 6^2) + (2 \times 6) + (4 \times 1)$

$= (3 \times 36) + \quad +$

$= 108 \quad + \quad +$

$=$ ____________

4 18. $42_8 =$ ____________

____

____

____

Marks

Use scientific notation to write each number.

19. 6000 = 6×10^3

1 20. 800 = ____________

1 21. 1 = ____________

1 22. 300,000 = ____________

1 23. 40,000 = ____________

Write the standard name for each number.

1 24. 273 + 10 = ____________

1 25. 4261 - 10 = ____________

1 26. 8327 × 100 = ____________

1 27. 49,896 + 10 = ____________

1 28. 83012 - 10 = ____________

1 29. 6271 × 20 = ____________

Find the following products.

6 30.
$$\begin{array}{r} 892 \\ \underline{83} \end{array}$$

31.
$$\begin{array}{r} 41087 \\ \underline{507} \end{array}$$

7 32.
$$\begin{array}{r} 892 \\ \underline{18} \end{array}$$

33.
$$\begin{array}{r} 1764 \\ \underline{137} \end{array}$$

Marks

Find the following quotients. Then round your answer to the nearer one.

3 34. $25\overline{)86487}$

Rounded quotient is __________.

3 35. $327\overline{)94360}$

Rounded quotient is __________.

3 36. $85\overline{)62989}$

Rounded quotient is __________.

3 37. $57\overline{)60913}$

Rounded quotient is __________.

Marks

Use the short method of division to find each quotient.

7 38. $7\overline{)1357}$ 39. $3\overline{)18373}$

7 40. $6\overline{)31416}$ 41. $9\overline{)4641}$

Classify each number as being even or odd and then decide if it is prime or composite.

			Even or odd	Prime or composite
2	42.	17	________	________
2	43.	22	________	________
2	44.	26	________	________
2	45.	29	________	________
2	46.	39	________	________
2	47.	43	________	________
2	48.	57	________	________
2	49.	63	________	________
2	50.	71	________	________
2	51.	89	________	________

Name all of the factors of each number.

2 52. 33 ________________________________

2 53. 42 ________________________________

2 54. 56 ________________________________

2 55. The greatest common factor of 33, 42 and 56 is ____________

Marks

Find the complete factorization of each number.

2 56. 24 = ________________________

2 57. 56 = ________________________

2 58. 60 = ________________________

2 59. The greatest common factor of 56, 60 and 24 is ____________

For each problem:

- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

4 60. When Margaret went to camp, she weighed 98 pounds. She lost 3 pounds while she was at camp. How many pounds did Margaret weigh when she came home from camp?

Placeholder: Let n be the number of pounds that Margaret weighed when she came home from camp.

Equation: ________________________ Computation

Answer: ________________________

Statement: ________________________

____

5 61. Sam scored 6 points during the first quarter of a basketball game. By the end of the second quarter, he scored a total of 14 points. How many points did Sam score during the second quarter?

Placeholder: ________________________

____

Equation: ________________________ Computation

Answer: ________________________

Statement: ________________________

____

Marks

5 62. Persons who have checking accounts are usually charged a small amount for each check that they write. This amount is called a service charge. If a bank charges 8¢ per check, what is the service charge for 12 checks?

Placeholder: ______________________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

5 63. What is the cost of one can of soup if 3 cans of this soup cost 54¢?

Placeholder: ______________________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

144 marks
TOTAL

Key to Self-correcting Exercises in Lesson 5

Exercise 1, page 2

1. 1, 16, 2, 8, 4
2. 2, 6, 3, 4, 1, 12
3. 20, 1, 2, 5, 10, 4
4. 1, 19

5. 1, 2, 5, 10 Composite
6. 1, 2, 4 Composite
7. 1, 29 Prime
8. 1, 13 Prime
9. 1, 2, 13, 26 Composite

10. $125 = 5 \times 5 \times 5 = 5^3$
11. $100 = 2 \times 5 \times 2 \times 5 = 2^2 \times 5^2$
12. $42 = 2 \times 3 \times 7 = 2^1 \times 3^1 \times 7^1$
13. $60 = 2 \times 2 \times 3 \times 5 = 2^2 \times 3^1 \times 5^1$

Exercise 2, page 5

1. a. ①, ②, ④, 8
   b. ①, ②, ④, 5, 10, 20
   c. 1, 2, 4

2. a. ①, ②, ③, 4, ⑥, 12
   b. ①, ②, ③, 5, ⑥, 10, 15, 30
   c. 1, 2, 3, 6

3. a. ①, 3
   b. ①, 7
   c. 1

4. 1

Exercise 3, page 9

1. 1, 2, 3, 4, 6, 12
   1, 2, 3, 5, 6, 10, 15, 30
   1, 2, 3, 6
   6

2. 1, 2, 3, 6, 9, 18
   1, 2, 3, 4, 6, 8, 12, 24
   1, 2, 3, 6
   6

3. $2 \times 5 = 10$

4. $11 \times 3 = 33$

End of Lesson 5

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing or incorrect)

File Number ____________________

Lesson Number __________

FOR SCHOOL USE ONLY

Assigned Teacher: __________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Least Common Multiples

Topic One: Multiples

Any number is a multiple of its factors.

e.g. $8 = 2 \times 2 \times 2$ 2 is a factor of 8.
8 is a multiple of 2.
$8 = 2 \times 4$ 4 is a factor of 8.
8 is a multiple of 4.
$8 = 1 \times 8$ 1 and 8 are factors of 8.
8 is a multiple of 1 and of 8.

Each number has many multiples. To find some multiples, multiply the number by 1, then by 2, then by 3 and so on.

EXAMPLE: Name four multiples of 3.

Solution

$3 \times 1 = 3$
$3 \times 2 = 6$
$3 \times 3 = 9$
$3 \times 4 = 12$

Four multiples of 3 are 3, 6, 9 and 12. Of course, 3 has many more multiples. You might have named 48 because $3 \times 16 = 48$ or
39 because $3 \times 13 = 39$ or
231 because $3 \times 77 = 231$.

You will notice that the small numbers are factors of the large numbers; the large numbers are multiples of the small numbers.

In division, the dividend is a multiple of the divisor if the divisor divides evenly into the dividend.

e.g. $\underset{\substack{\uparrow \\ \text{dividend}}}{12} \div \underset{\substack{\uparrow \\ \text{divisor}}}{6} = 2$ Since 6 divides evenly into 12, 12 is a multiple of 6.

Self-correcting Exercise 1

Fill in the blanks in the sentences.

1. The first eight multiples of 4 are ____, 8, ____, ____, ____, ____, 28 and ____.
2. The factors of 4 are ____, ____ and ____.
3. The first eight multiples of 6 are 6, ____, ____, ____, ____, 36, ____ and ____.
4. The factors of 6 are ____, ____, ____ and ____.
5. Any even number is a multiple of ____.
6. ____ is a factor of every even number.
7. 15, 20, 25 and 30 are all multiples of ____.
8. ____ is a common factor of 15, 20, 25 and 30.
9. 21 is a multiple of ____ and also of ____.
10. ____ and ____ are factors of 21.

Corrections are on page 15.

Topic Two: Common multiples

When a number is a multiple of each of two numbers, it is a COMMON MULTIPLE of the two numbers.

We can find a common multiple of two numbers by naming many multiples of each number. (Since each number has countless multiples we cannot name all of them.) Then we can pick out multiples that are common to both numbers.

EXAMPLE: Find 2 common multiples of 4 and 6.

Solution
Some multiples of 4 are 4, 8, ⑫, 16, 20, ㉔, 28.
Some multiples of 6 are 6, ⑫, 18, ㉔, 30.
The numbers that are encircled are multiples of 4 as well as multiples of 6. Two common multiples of 4 and 6 are 12 and 24.

Self-correcting Exercise 2

Name three common multiples for each group of numbers.

1. 5, 10
   The first six multiples of 5 are ____________________

   The first six multiples of 10 are ____________________

   Three common multiples of 5 and 10 are ________________

2. 6, 9
   The first nine multiples of 6 are ____________________

   The first nine multiples of 9 are ____________________

   Three common multiples of 6 and 9 are ________________

3. 15, 25
   The first ten multiples of 15 are ____________________

   The first ten multiples of 25 are ____________________

   Two common multiples of 15 and 25 are ________________

4. 22, 33
   The first six multiples of 22 are ____________________

   The first six multiples of 33 are ____________________

   Two common multiples of 22 and 33 are ________________

Corrections are on page 15.

Assignment 1

1. The numbers 4, 8, 12, 16 and 20 are all ____________ of 4.
2. Five multiples of 3 are ________________________
3. How many multiples does 12 have? ____________________
4. How many factors does 12 have? ____________________
5. Circle the numbers below that are multiples of 18.

   36 6 9 18 54 24

6. Fill in this multiplication table.
e.g. Look across from 7 until you are down from 6.
7 × 6 = 42
Write 42 in the square that is across from 7 and below 6.

	×	1	2	3	4	5	6	7	8	9	10
a.	1										
b.	2										
c.	3										
d.	4										
e.	5										
f.	6										
g.	7						42				
h.	8										
i.	9										
j.	10										

k. Each number in Row a is a multiple of 1.

l. Each number in Row b is a multiple of ______.

m. Each number in Row g is a multiple of ______.

n. Each number in Row j is a multiple of ______.

o. Using the above chart we can say that:

42 is a common multiple of 6 and 7,

32 is a common multiple of _____ and _____,

90 is a common multiple of _____ and _____,

27 is a common multiple of _____ and _____ and

6 is a common multiple of _____ and _____.

p. Using the above chart we can say that:

a common multiple of 6 and 7 is 42,

a common multiple of 3 and 8 is _____,

a common multiple of 10 and 1 is _____,

a common multiple of 2 and 5 is _____ and

a common multiple of 4 and 9 is _____.

Topic Three: Least common multiple

The LEAST COMMON MULTIPLE of two numbers is the smallest number that is a multiple of both numbers. There are three common methods of finding the least common multiple.

A. Use of common multiples
This method makes use of what you learned about common multiples in the earlier part of this lesson. To make use of this method you must:

1. List many multiples of each number. To do this multiply each number by 1, then by 2, then 3 and so on. Continue in this order. Do not omit any multiples.

2. List the numbers that are multiples of both the given numbers. These are the common multiples.

3. From the common multiples choose the smallest number. This number is the least common multiple of the given numbers.

Assignment 2

Name the least common multiple for each group of numbers.

1. 8, 10
The first ten multiples of 8 are 8,16,24,32,40,48,56,64,72,80.

The first ten multiples of 10 are 10,20,30,40,50,60,70,80,90,100.

The common multiples of 8 and 10 are 40,80.

The least common multiple of 8 and 10 is 40.

2. 9, 12

The first eight multiples of 9 are ______________________

The first eight multiples of 12 are ______________________

The common multiples of 9 and 12 are ______________________

The least common multiple of 9 and 12 is ______________________

3. 4, 5

The first ten multiples of 4 are ______________________

The first ten multiples of 5 are ______________________

The common multiples of 4 and 5 are ______________________

The least common multiple of 4 and 5 is ______________________

4. 2, 3, 4

The first ten multiples of 2 are ______________________

The first ten multiples of 3 are ______________________

The first ten multiples of 4 are ______________________

The common multiples of 2, 3 and 4 are ______________________

The least common multiple of 2, 3 and 4 is ______________________

B. Using multiples of the larger number

To use this method you must.

1. List many multiples of the greater number. That is multiply the greater number by 1, then by 2, then 3 and so on.
2. Decide which of the multiples of the greater number is also a multiple of the lesser number. To do this, divide by the lesser number. If it divides evenly into the multiple of the greater number, then that multiple is a common multiple of the two numbers. The first number you arrive at by this method is the least common multiple of the two numbers.

EXAMPLE: Find the least common multiple of 5 and 12.

Solution

Some multiples of 12 are 12, 24, 36, 48, 60, 72, 84, 96, 108, 120. Now start dividing these multiples by 5. The first number that you arrive at that 5 divides into evenly is the least common multiple.

12 ÷ 5 = 2 remainder 2	12 is not a multiple of 5.
24 ÷ 5 = 4 remainder 4	24 is not a multiple of 5.
36 ÷ 5 = 7 remainder 1	36 is not a multiple of 5.
48 ÷ 5 = 9 remainder 3	48 is not a multiple of 5.
60 ÷ 5 = 12	60 is a multiple of 5.

Therefore the least common multiple of 5 and 12 is 60.

Assignment 3

Find the least common multiple of each group of numbers. Use the above method.

1. 4 and 20
The first five multiples of 20 are 20, 40, 60, 80, 100.

20 ÷ 4 = 5 (We can stop at the first multiple, 20, because 4 divides evenly into 20.)

The least common multiple of 4 and 20 is 20.

2. 8, 30
The first five multiples of 30 are __________

The least common multiple of 8 and 30 is __________

3. 7, 9
The first ten multiples of 9 are ______________________________ .

The least common multiple of 7 and 9 is ______________________ .

4. 12, 16
The first five multiples of 16 are ____________________________ .

The least common multiple of 12 and 16 is ___________________ .

5. 9. 12, 18
The first four multiples of 18 are ____________________________ .

The least common multiple of 9, 12 and 18 is _________________ .

C. Use of complete factorization

To use this method:

1. Write the complete factorization of each number.
2. Remember that the complete factorization of a number can be written using exponential notation. Rewrite the complete factorization using exponential notation.
3. Write down all prime factors used in the complete factorization of the first number. Also write down any different factors that were used in the complete factorization of the second, third and fourth numbers.
4. Using an exponent show the greatest number of times each factor was used in the complete factorization of the numbers.
5. Find the product of the factors.

EXAMPLE: Find the least common multiple of 12 and 45.

Solution

The complete factorization of 12 is $2 \times 2 \times 3$.
The complete factorization of 45 is $3 \times 3 \times 5$.

Using exponential notation, the complete factorization of 12 is $2^2 \times 3^1$.
Using exponential notation, the complete factorization of 45 is $3^2 \times 5^1$.

The prime factors in $2^2 \times 3^1$ and $3^2 \times 5^1$ are 2, 3, 5.

The greatest number of times 2 is used in $2^2 \times 3^1$ is ② times.
The greatest number of times 3 is used in $3^2 \times 5^1$ is ② times.
The greatest number of times 5 is used in $3^2 \times 5^1$ is ① time.

Therefore using exponential notation we can write the least common multiple as $2^2 \times 3^2 \times 5^1$ which equals 180.
The least common multiple of 12 and 45 is 180.

Self-correcting Exercise 3

Find the complete factorization of each number using exponential notation.

1. 15 = ________________________________
2. 24 = ________________________________
3. 45 = ________________________________

Using the above information, find the least common multiple for each group of numbers.

4. 15 and 24 ________________________________
5. 24 and 45 ________________________________

6. 15 and 45 ________________

7. 15, 24 and 45 ________________

Corrections are on pages 15 and 16.

Assignment 4

Name four multiples of each number.

1. 7 7, 14, 21, 28

2. 20 ________________

3. 16 ________________

4. 2 ________________

5. Is 5 a multiple of 20? ________ Explain your answer.

____

____

6 Is 50 a multiple of 20? ________ Explain your answer.

____

____

7. Is 20 a multiple of 20? ________ Explain your answer.

____

____

Find the least common multiple of each group of numbers using the complete factorization of the numbers. (Use exponential notation to write the complete factorizations.)

8. 5 and 15

The complete factorization of 5 is 5^1.

The complete factorization of 15 is $3^1 \times 5^1$.

The least common multiple of 5 and 15 is $3^1 \times 5^1 = 15$

9. 30 and 45
The complete factorization of 30 is ____________________

The complete factorization of 45 is ____________________

The least common multiple of 30 and 45 is ______________

10. 32 and 48
The complete factorization of 32 is ____________________

The complete factorization of 48 is ____________________

The least common multiple of 32 and 48 is ______________

11. 24 and 60
The complete factorization of 24 is ____________________

The complete factorization of 60 is ____________________

The least common multiple of 24 and 60 is ______________

Topic Four: Problems involving least common multiples

The idea of least common multiple can be used to find the least amount that is needed, with no waste, in different situations.

EXAMPLE: John wants to build a bookcase. He is not sure whether it should contain 2, 3 or 4 shelves. What is the shortest length of board he could buy so that there would be no waste when he decided upon the number of shelves?

Solution

John is not concerned with the length of each shelf, only with the number of shelves. Let's find the smallest number that 2, 3 and 4 will divide into evenly.
Give the complete factorization of each number.
$2 = 2^1$
$3 = 3^1$
$4 = 2^2$
The least common multiple (L.C.M.) of 2, 3 and 4 is $2^2 \times 3^1$ or 12.
The shortest board John could buy would be 12 feet long (and still be able to put in 2, 3 or 4 shelves with no waste).

Assignment 5

Show your work and give an answer for each problem.

1. What is the shortest length that I can measure evenly with a 6-inch rod, or with an 8-inch rod, or with a 9-inch rod?

$6 = 2^1 \times 3^1$

$8 = 2^3$

$9 = 3^2$

L.C.M. of 6, 8 and 9 is $2^3 \times 3^2 = 8 \times 9 = 72$.

The shortest length I can measure evenly with a 6-inch rod, an 8-inch rod or a 9-inch rod is 72 inches.

2. What is the smallest sum of money which I can pay completely in nickels, in dimes or in quarters?

$5 = 5^1$

$10 = 2^1 \times 5^1$

$25 =$

3. What is the least number of men that can be divided evenly into 4-man teams, 5-man teams or 6-man teams for a rope-pulling contest?

4. A grocer wishes to divide bulk flour into 5 lb. bags, 10 lb. bags or 7 lb. bags. What is the least amount of flour that he can have in order to divide it evenly into any one of these amounts?

Using any of the three methods you learned, find the least common multiple of each group of numbers.

5. 10, 20, 25

6. 36, 48, 72

7. 15, 40, 80

Key to the Self-correcting Exercises in Lesson 6

Exercise 1, page 2

1. 4, 8, 12, 16, 20, 24, 28, 32
2. 1, 2, 4
3. 6, 12, 18, 24, 30, 36, 42, 48
4. 1, 2, 3, 6
5. 2
6. 2
7. 5
8. 5
9. 3, 7, 21 or 1
10. 1, 3, 7 or 21

Exercise 2, page 3

1. 5, 10, 15, 20, 25, 30
   10, 20, 30, 40, 50, 60
   10, 20, 30

2. 6, 12, 18, 24, 30, 36, 42, 48, 54
   9, 18, 27, 36, 45, 54, 63, 72, 81
   18, 36, 54

3. 15, 30, 45, 60, 75, 90, 105, 120, 135, 150
   25, 50, 75, 100, 125, 150, 175, 200, 225, 250
   75, 150

4. 22, 44, 66, 88, 110, 132
   33, 66, 99, 132, 165, 198
   66, 132

Exercise 3, page 9

1. $15 = 3 \times 5 = 3^1 \times 5^1$
2. $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3^1$
3. $45 = 3 \times 3 \times 5 = 3^2 \times 5^1$

4. $15 = 3^1 \times 5^1$ *The greatest number of times 2 is used is 3 times;*
   $24 = 2^3 \times 3^1$ *3 is used 1 time; 5 is used 1 time.*
   L.C.M. of 15 and 24 is $2^{\textcircled{3}} \times 3^{\textcircled{1}} \times 5^{\textcircled{1}} = \underline{120}$.

5. $24 = 2^3 \times 3^1$ *The greatest number of times 2 is used is 3 times;*
   $45 = 3^2 \times 5^1$ *3 is used 2 times; 5 is used 1 time.*
   L.C.M. of 24 and 45 is $2^{\textcircled{3}} \times 3^{\textcircled{2}} \times 5^{\textcircled{1}} = \underline{360}$.

The greatest number of times 3 is used is 2 times; 5 is used 1 time.

6. $15 = 3^1 \times 5^1$
$45 = 3^2 \times 5^1$
L.C.M. of 15 and 45 is $3^{\textcircled{2}} \times 5^{\textcircled{1}} = \underline{45}$.

The greatest number of times 2 is used is 3 times; 3 is used 2 times; 5 is used 1 time.

7. $15 = 3^1 \times 5^1$
$24 = 2^3 \times 3^1$
$45 = 3^2 \times 5^1$
L.C.M. of 15, 24 and 45 is $2^{\textcircled{3}} \times 3^{\textcircled{2}} \times 5^{\textcircled{1}} = \underline{360}$.

End of Lesson 6

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: __________

Graded by: ________

Assignment Code: _____

Date Lesson Received:

____

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Angles and Polygons

Topic One: Basic geometric figures

A geometric figure is a set of points.

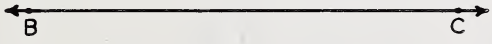
We use a dot to represent a point in space. Actually a point does not have size or shape but since we must use a point to develop other geometric figures we represent it by a dot. To name a point we use a single capital letter.

e.g.

•A This is "point A".

A line is made up of infinitely many points. By the term "line" we mean straight line. A line is endless. It goes on and on in opposite directions. Although a line does not have thickness, we use a long dash to represent it. An arrow is put on each end of the dash to indicate that it goes on and on. A line can be named by any two points in it. The symbol for the word "line" is "$\longleftrightarrow$".

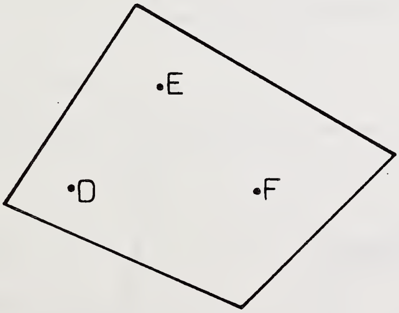
e.g.



This is line BC or $\overleftrightarrow{BC}$.

A plane is made up of infinitely many points. It is endless. It has no edges or boundaries. Usually a four-sided figure is used to represent a plane. This page represents a plane. Every time you move this page (as long as you don't bend it) it represents a different plane. A plane is named by any three points which are not in a line.

e.g.



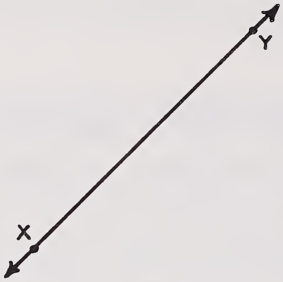
This is plane DEF or
plane DFE or
plane EDF or
plane EFD or
plane FDE or
plane FED.

Any one of these names is acceptable.

Assignment 1

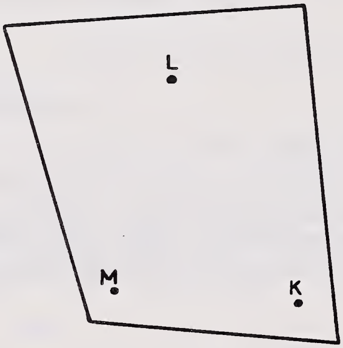
Name the geometric figures represented by each diagram.

1.

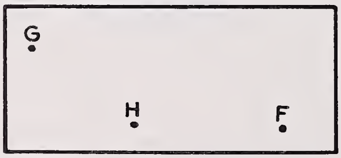


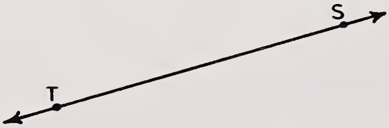
line XY or $\overleftrightarrow{XY}$

2.

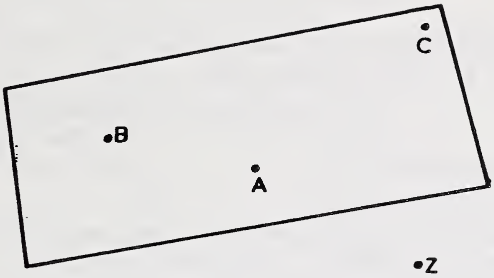


3.

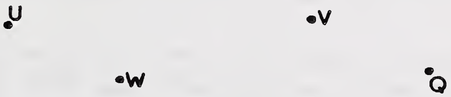




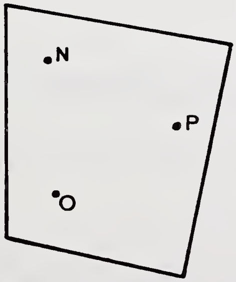
4.

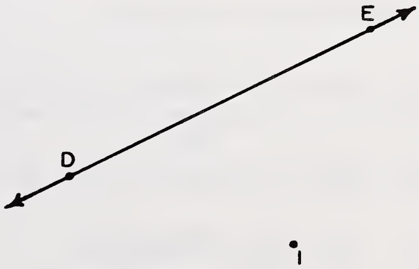


5.



6.

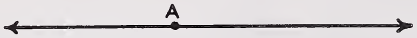




Topic Two: Angles

Let's think about a half-line.

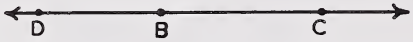
e.g.



To the left of A is one half-line. To the right of A is another half-line.

If we think about a half-line and the point that separates the line into half-lines, then we have a RAY. A ray has direction. It is endless in only one direction. To name a ray, we name the point that separates the line into half-lines and another point which is to the right or the left of this point. The symbol for the word "ray" is "•⟶".

e.g.

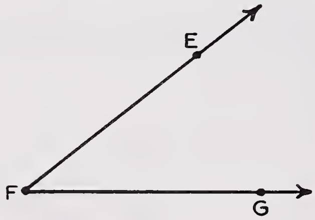


If B separates $\overleftrightarrow{DC}$ into two half-lines, then the ray to the left is $\overrightarrow{BD}$ and the ray to the right is $\overrightarrow{BC}$.

Since we can draw a line on this sheet of paper (which represents a plane) we can think of a line as part of a plane.

If we consider a plane and two rays, we can draw an angle. When we draw two rays on this page, we can see that they are part of the plane represented by this page. We must draw the rays in a special way. The two rays must have the same endpoint.

e.g.



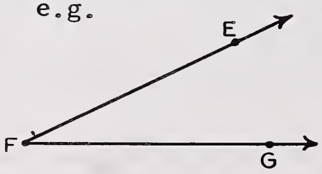
$\overrightarrow{FE}$ and $\overrightarrow{FG}$ have the same endpoint (F).

The common endpoint of the two rays is the VERTEX of the angle. Each ray is a side of the angle.

An angle is named in one of two ways:

A. By its vertex alone

e.g.

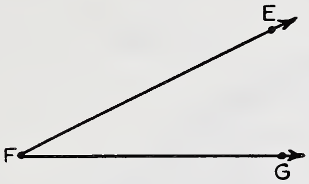


The vertex of this angle is point F. The symbol for the word "angle" is "∠". Therefore the name of this angle can be ∠F(angle F).

B. By its vertex and a point in each ray

These points must be named in a special order. The first point named is in one ray, the second point is always the vertex and the third point is in the other ray.

e.g.



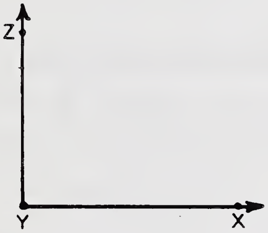
E is a point in $\overrightarrow{FE}$.
F is the vertex.
G is a point in the other ray, $\overrightarrow{FG}$.
Therefore the name of this angle can be ∠EFG or ∠GFE.

Vertex is the middle letter.

The sides of an angle go on and on. They do not end. One side of an angle is not longer or shorter than the other side of the angle because of this fact.

Self-correcting Exercise 1

1.



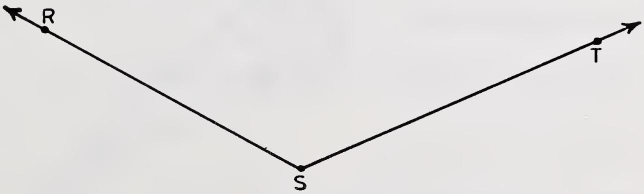
a. The name of the angle using the vertex only is ______________.

b. Two names of the angle using three points are ____________ and __________.

c. The vertex is _______________.

d. The sides of the angle are ____________ and _________________.

2.



a. The name of the angle using the vertex only is __________.

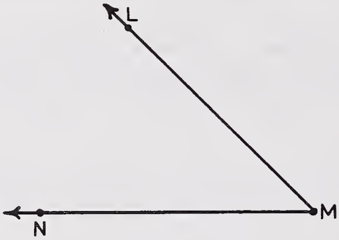
b. Two names of the angle using three points are ______ and

________.

c. The vertex is __________.

d. The sides of the angle are ____________ and ____________.

3.



a. The name of the angle using the vertex only is __________.

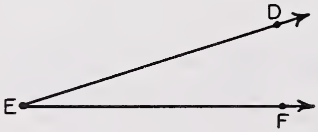
b. Two names of the angle using three points are ________ and

__________.

c. The vertex is __________.

d. The sides of the angle are ____________ and ____________.

4.



a. The name of the angle using the vertex only is _________.

b. Two names of the angle using three points are ____ and ____.

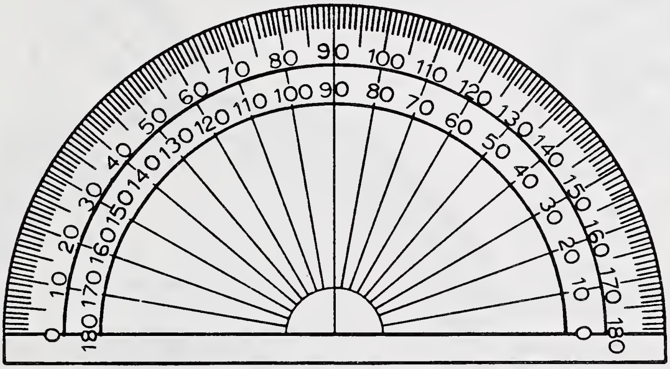
c. The vertex is _________.

d. The sides of the angle are ______________ and _____________.

Corrections are on page 23.

Topic Three: Measuring angles

As you can see from the angles that were drawn earlier, angles are not all the same shape. To indicate how angles are different, we use an instrument called a PROTRACTOR to measure the distance from one side of the angle to the other side of the angle.

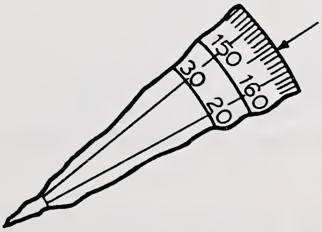


The protractor has two different scales. The one that begins at 0 on the right and ends at 180 on the left is the counter-clockwise scale. It can also be called the inner scale because it is to the inside of the protractor. The other scale which begins at 0 on the left and ends at 180 on the right is the clockwise scale. It can also be called the outer scale because it is near the edge of the protractor.

Each long line from the center of the protractor represents a multiple of 10. Each short line represents a whole number or divides the scale into ones. The line from the edge of the protractor to the outer scale divides the scale into fives.

Each line on the protractor represents two different numbers. One number is read from the counter-clockwise (inner) scale. The other number is read from the clockwise (outer) scale.

e.g.

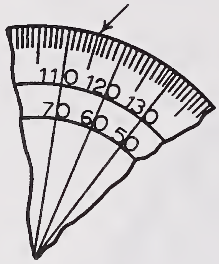


The point indicated by the arrowhead is 24 on the inner scale and 156 on the outer scale.

Self-correcting Exercise 2

Give the clockwise (outer) and counter-clockwise (inner) readings indicated by the arrow in each diagram.

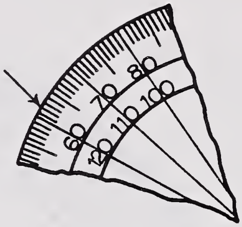
1.



clockwise ___________

counter-clockwise ___________

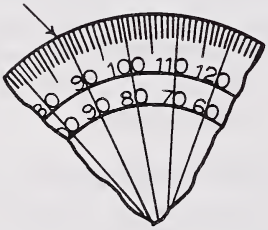
2.



clockwise ___________

counter-clockwise ___________

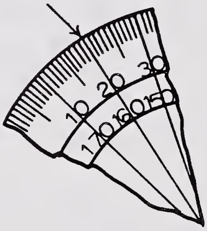
3.



clockwise ___________

counter-clockwise ___________

4.



clockwise ___________

counter-clockwise ___________

Corrections are on page 23.

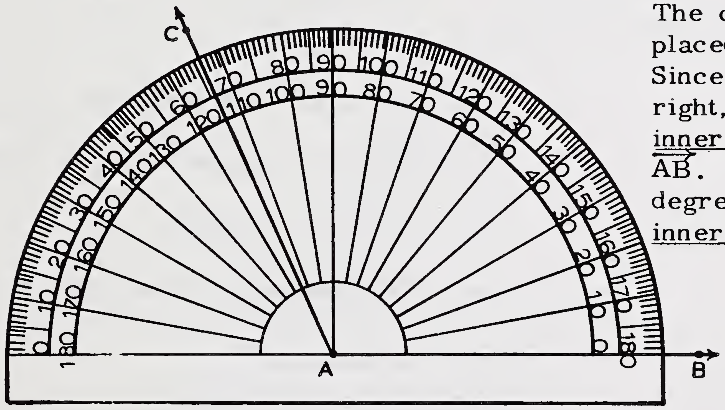
The unit of measurement for an angle is the degree. On the protractor, from 0 to 1 (on either scale) is 1 degree. The symbol for degrees is a small circle that is placed above and to the right of the number.

e.g. 15° (fifteen degrees)

To measure an angle we must place the protractor very carefully. On your protractor there is a point which is directly below the 90 mark and on the line between 0 on the left and 0 on the right. This central point must be placed on the vertex of the angle. If the angle opens to the left, make sure that the 0 mark on the left is on the lower side of the angle. If the angle opens to the right, make sure that the 0 mark on the right is on the lower side of the angle. Using the appropriate scale, read the number on the protractor above the other side of the angle.

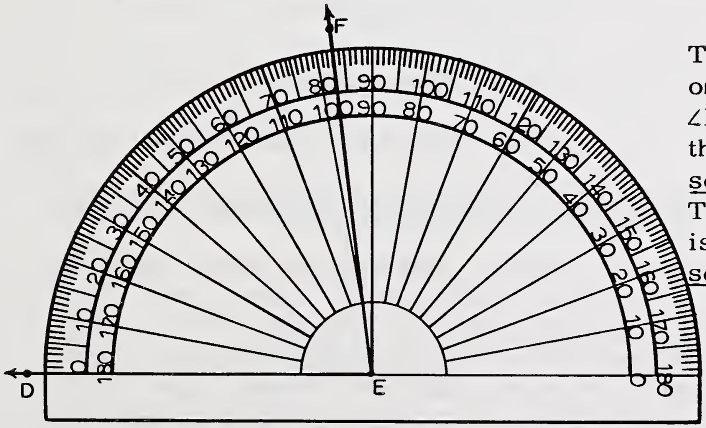
To write the measurement of an angle we would write $\angle XAB^\circ = 35$. This is read "the measure in degrees (°) of angle XAB ($\angle XAB$) is (=) 35.

e.g.



The central point is placed on the vertex, A. Since $\angle CAB$ opens to the right, the 0 mark on the inner scale is placed over $\overrightarrow{AB}$. The measure in degrees is shown on the inner scale by $\overrightarrow{AC}$.

$\angle BAC^\circ = 114$



The central point is placed on the vertex, E. Since $\angle FED$ opens to the left, the 0 mark on the outer scale is placed over $\overleftrightarrow{ED}$. The measure in degrees is shown on the outer scale by $\overrightarrow{EF}$.

$\angle DEF^\circ = 83$

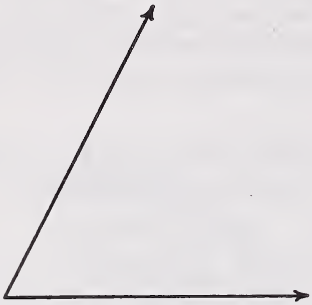
If the sides of the angle are not long enough to pass under the scale on the protractor, extend them.

Here is another way of remembering which scale to use.

1. If the bottom side of the angle is under the 0 from the inner scale, then the inner scale is used to read the number of degrees.
2. If the bottom side of the angle is under the 0 from the outer scale, then the outer scale is used to read the number of degrees.

Assignment 2

1.



a. The angle opens to the right.

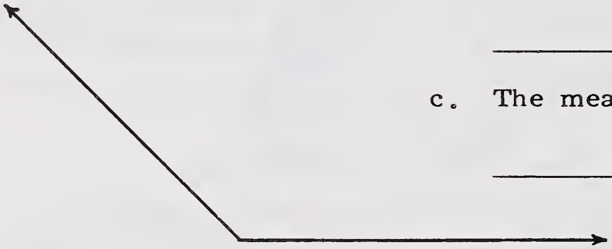
b. Read the measurement on the

inner scale.

c. The measure of the angle is

63°.

2.



a. The angle opens to the ______

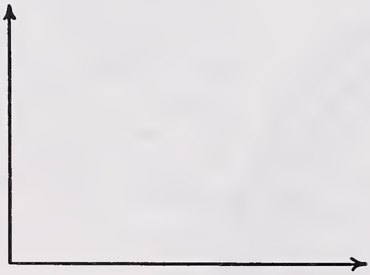
b. Read the measurement on the

______ scale.

c. The measure of the angle is

______

3.



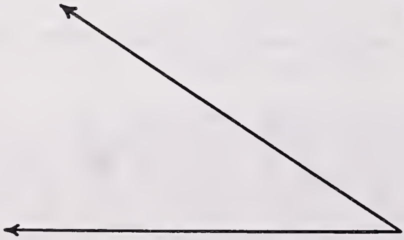
a. The angle opens to the ______

b. Read the measurement on the

______ scale.

c. The measure of the angle is ______

4.



a. The angle opens to the ______

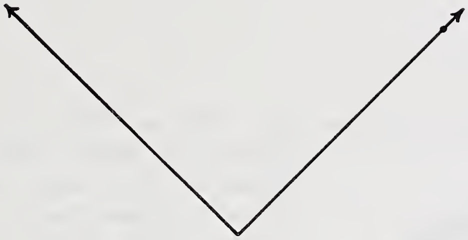
b. Read the measurement on the

______ scale.

c. The measure of the angle is

______

5.



a. The angle opens to the

____

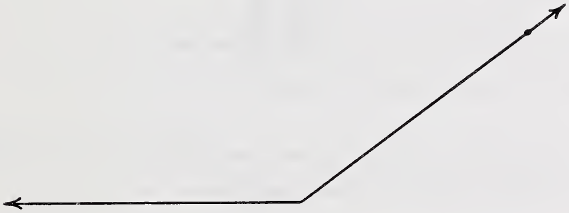
b. Read the measurement

on the _________ scale.

c. The measure of the

angle is _________

6.



a. The angle opens to the

____

b. Read the measurement

on the ________ scale.

c. The measure of the

angle is _________

Topic Four: Using a protractor to make an angle

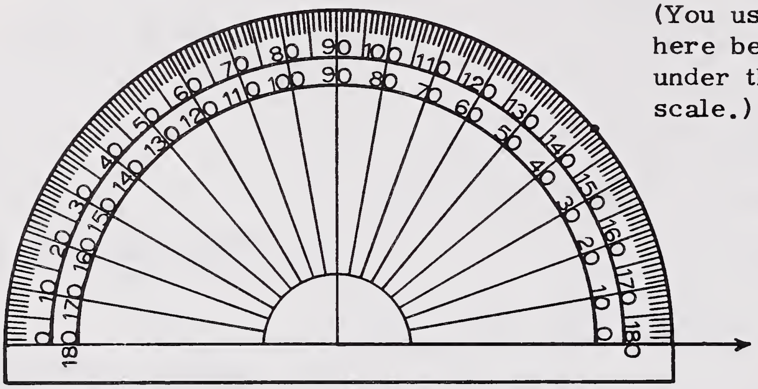
Sometimes you are required to draw an angle of a certain size. To do this, draw a ray. The endpoint of this ray will be the vertex of the angle. Place the point of the protractor (that is the point below 90 and on the line between 0 on the left and 0 on the right) on the endpoint of the ray. Make sure that one of the 0 marks lies along the ray as well. Find the required measure of the angle on the protractor. Make a dot at this measure. Using the bottom edge of the protractor, join the dot to the endpoint of the ray.

EXAMPLE: Draw a 40° angle.

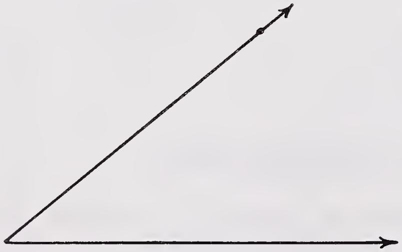
Solution

Draw a ray.

Place the protractor on the endpoint of the ray. Put a dot at 40 using the inner scale. (You use the inner scale here because the ray is under the 0 on the inner scale.)



Remove the protractor and draw a line from the dot to the vertex. Put arrows at the ends of the arms of the angle.



This is a 40° angle.

Assignment 3

In each case make an angle of the given size.

1. 105°

2. 65°

3. 127°

4. 85°

5. 89°

Topic Five: Kinds of angles

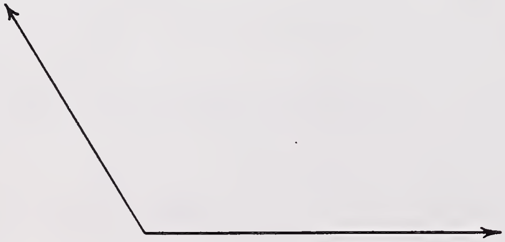
You need to know about three kinds of angles.

1. Right angles which have a measurement of exactly 90°.
2. Acute angles which have a measurement of less than 90°.
3. Obtuse angles which have a measurement of more than 90°.

Assignment 4

Measure each of the angles with a protractor. Then tell whether it is a right angle, an acute angle or an obtuse angle.

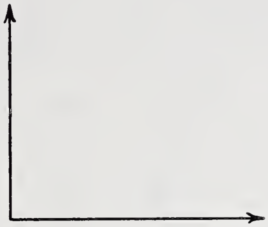
1.



121°

obtuse angle

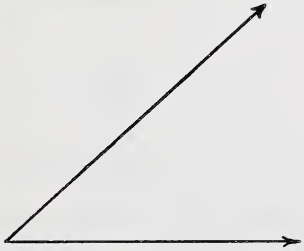
2.



____

____

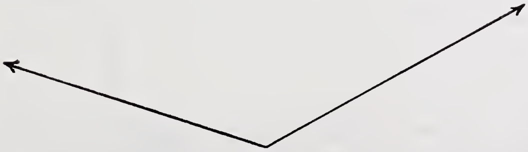
3.



____

____

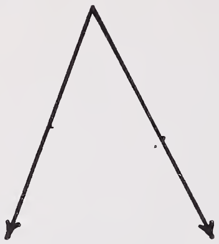
4.



____

____

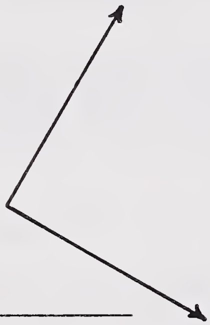
5.



____

____

6.



____

____

7. Can you tell, just by looking, , whether an angle is a right angle, an acute angle or an obtuse angle?

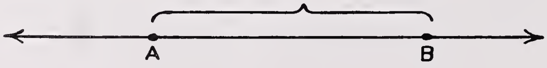
____

Topic Six: Polygons

Let's think about a line again. If we designate two points in the line and think about these two points and all the points between them, we are thinking about a SEGMENT. To name a segment, we use the two endpoints. The symbol for the word "segment" is "$\overline{}$".

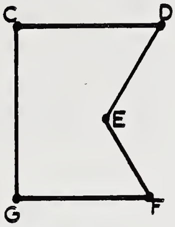
e.g.

This is segment AB or $\overline{AB}$.



If we carefully arrange a group of segments, we can make a polygon. The endpoint of each segment must also be the endpoint of another segment that is part of the polygon.

e.g.

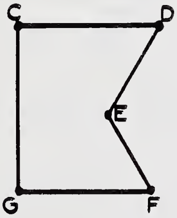


The endpoints of $\overline{CD}$ are C and D.
C is also the endpoint of $\overline{CG}$.
D is also the endpoint of $\overline{DE}$.

The segments must be joined so that the letters are in alphabetical order in either a clockwise or counter-clockwise direction.

To name a polygon we list the endpoints of the segments in a clockwise or counter-clockwise order. The clockwise method is most commonly used.

e.g.



This is polygon CDEFG
polygon DEFGC
polygon EFGCD
polygon FGCDE
polygon GCDEF

Any one of these names is correct. Polygon CDEFG would be the most commonly used name since the letters appear in alphabetical order.

One segment does not form a polygon.

e.g.

Two segments do not form a polygon.

e.g.

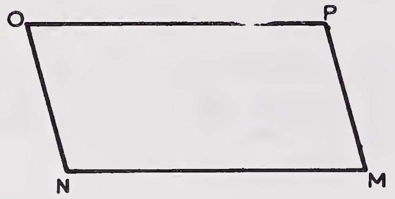
Three segments or more do form polygons. Polygons can be classified into special groups by the number of sides they have.

A three-sided polygon is a triangle (tri meaning three).
A four-sided polygon is a quadrilateral (quadri meaning four).
A five-sided polygon is a pentagon (penta meaning five).
A six-sided polygon is a hexagon (hexa meaning six).
A seven-sided polygon is a heptagon (hepta meaning seven).
An eight-sided polygon is an octagon (octa meaning eight).

Assignment 5

Name each polygon and classify each into the group to which it belongs.

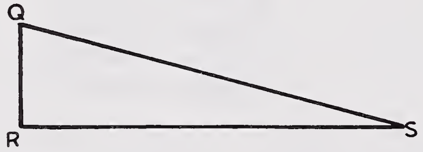
1.



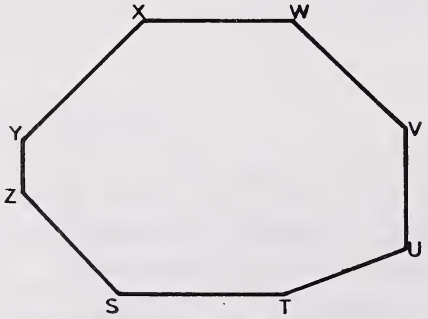
polygon MNOP

quadrilateral

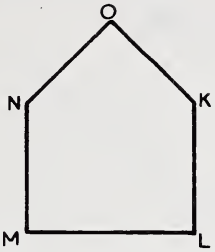
2.



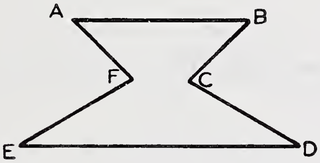
3.



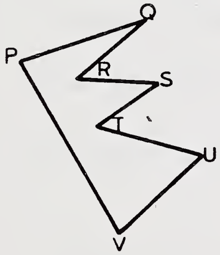
4.



5.

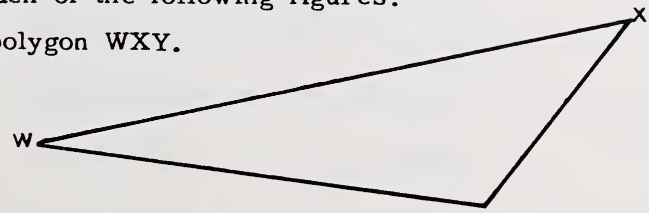


6.



Draw each of the following figures:

7. A polygon WXY.



This is a *triangle.*

8. A polygon with sides $\overline{LM}$, $\overline{MN}$, $\overline{NO}$, $\overline{OP}$ and $\overline{PL}$.

This is a ____________________.

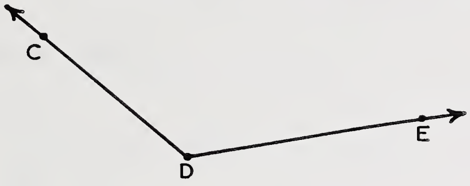
9. A polygon that has 4 sides. Name the endpoints of each segment. Use the letters "A, B, C, D".

This is a ____________________.

The name of this polygon is ____________________.

10. A heptagon.

11.

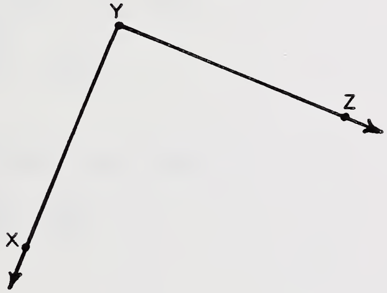


a. A name of this angle is ∠EDC.

b. The measure of this angle is 132°.

c. It is a(n) obtuse angle.

12.

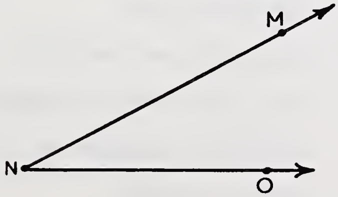


a. A name of this angle is __________.

b. The measure of this angle is __________.

c. It is a(n) __________ angle.

13.



a. A name of this angle is __________.

b. The measure of this angle is __________.

c. It is a(n) __________ angle.

14. Draw an acute angle. Measure it.

____

15. Draw an obtuse angle. Measure it.

____

16. Match Column B with Column A.

	A		B
f	hexagon	a.	has two endpoints
_____	ray	b.	has five sides
_____	inner scale	c.	has only one endpoint
_____	vertex	d.	two rays with a common endpoint
_____	triangle	e.	common endpoint of two segments
_____	pentagon	f.	has six sides
_____	polygon	g.	can determine three angles
_____	segment	h.	has segments for sides
_____	angle	i.	found on a protractor
_____	octagon	j.	has eight sides

Key to Self-correcting Exercise in Lesson 7

Exercise 1, page 5

1. a. $\angle Y$
   b. $\angle ZYX$ and $\angle XYZ$
   c. Y
   d. $\overrightarrow{YZ}$, $\overrightarrow{YX}$

2. a. $\angle S$
   b. $\angle RST$ and $\angle TSR$
   c. S
   d. $\overrightarrow{SR}$, $\overrightarrow{ST}$

3. a. $\angle M$
   b. $\angle LMN$ and $\angle NML$
   c. M
   d. $\overrightarrow{ML}$, $\overrightarrow{MN}$

4. a. $\angle E$
   b. $\angle DEF$ and $\angle FED$
   c. E
   d. $\overrightarrow{ED}$, $\overrightarrow{EF}$

Exercise 2, page 8

1. 115
   65

2. 62
   118

3. 89
   91

4. 20
   160

End of Lesson 7

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: __________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ______________

Graded by: ____________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________
Address ____________________
____
____
Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

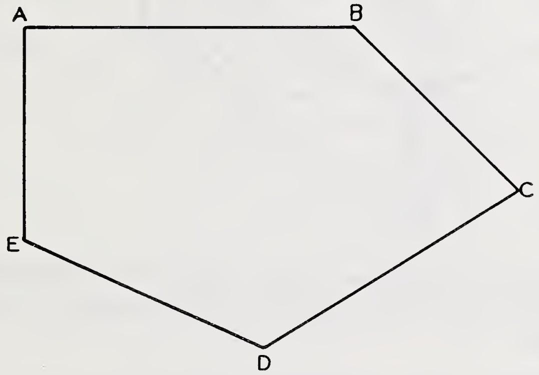
When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Triangles and Quadrilaterals

Topic One: Measuring the sides and angles of a polygon

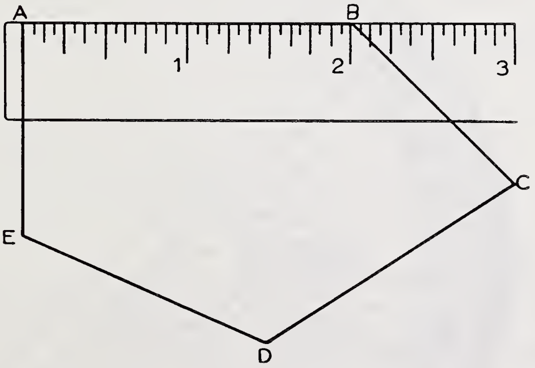
We use a ruler to measure the sides of a polygon. By carefully placing the ruler so that the zero mark is below one vertex, we can read the measure of the segment by looking at the marking on the ruler below the other vertex.

EXAMPLE: Measure $\overline{AB}$ and $\overline{DC}$ in polygon ABCDE.



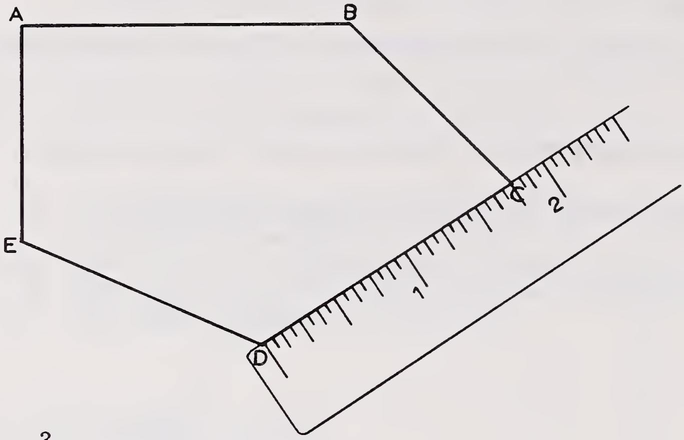
Solution

To measure $\overline{AB}$: place the ruler below $\overline{AB}$. Be sure that 0 is below point A. Read the measure of $\overline{AB}$ by looking below point B.



m($\overline{AB}$)= 2 in. (This is read "the measure of segment AB is equal to 2 inches".)

To measure $\overline{DC}$: place the ruler below $\overline{DC}$. Be sure that 0 is below point D. Read the measure of $\overline{DC}$ by looking below point C.



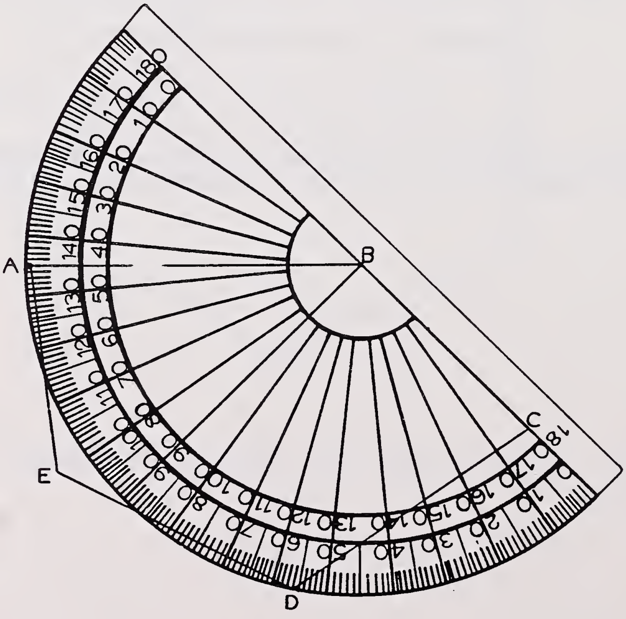
$m(\overline{DC}) = 1\frac{3}{4}$ in.

We use a protractor to measure the angles of a polygon. Restudy Lesson 7, pages 8 and 9 for notes on how to use a protractor for measuring angles.

EXAMPLE: Measure ∠B and ∠E of polygon ABCDE (see above).

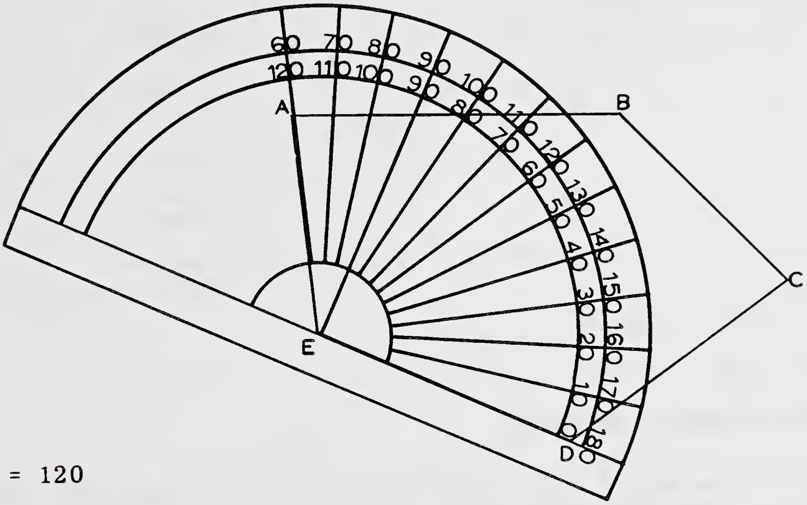
Solution

To measure ∠B: place the central point at the bottom of the protractor on B.



∠B° = 135

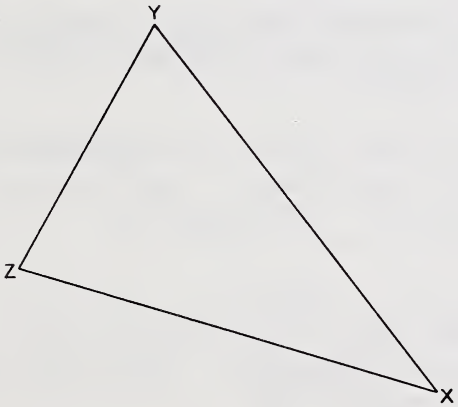
To measure ∠E: place the central point at the bottom of the protractor on E.



∠E° = 120

Assignment 1

1.



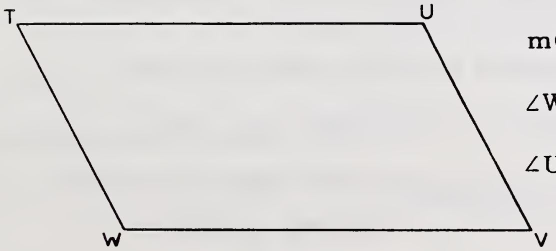
m($\overline{XY}$) = ______________

m($\overline{YZ}$) = ______________

∠XYZ° = ______________

∠YXZ° = ______________

2.



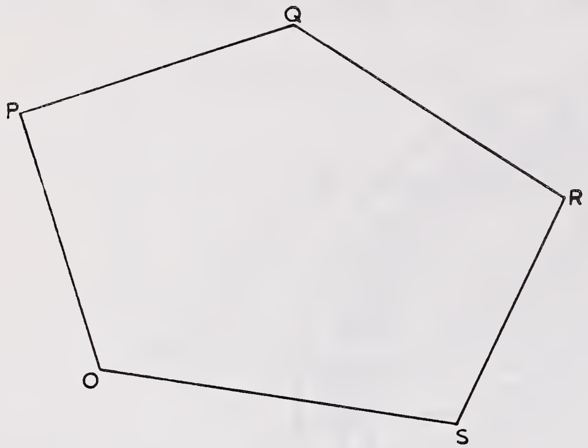
m($\overline{TU}$) = ______________

m($\overline{WV}$) = ______________

∠WTU° = ______________

∠UVW° = ______________

3.



$m(\overline{PO})$ = ________

$m(\overline{QR})$ = ________

$m(\overline{RS})$ = ________

$\angle RSO^\circ$ = ________

$\angle OPQ^\circ$ = ________

$\angle QRS^\circ$ = ________

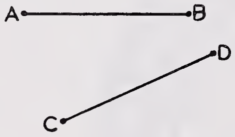
Topic Two: Kinds of triangles

You learned in Lesson 7 that triangles are a kind of polygon. Now you will learn about different kinds of triangles. Triangles can be classified according to the lengths of their sides or the size of their angles.

To use the length of sides to classify triangles, we must understand what the term "congruent" means. Two geometric figures are congruent if they are the same size and shape.

In triangles we are concerned with the sides, which are segments. To be congruent, segments must be the same length. They are the same shape because they are both segments. The symbol for "is congruent to" is " ≅ ".

e.g.

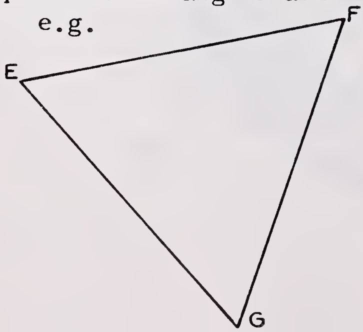


$m(\overline{AB}) = 1$ in. $m(\overline{CD}) = 1$ in. Therefore $\overline{AB} \cong \overline{CD}$. This is read "segment AB is congruent to segment CD". This means $\overline{AB}$ is the same length as $\overline{CD}$.

Using the idea of congruent segments, we can divide all triangles into three groups according to the lengths of their sides.

1. Equilateral triangles are triangles which have all three sides congruent.

e.g.

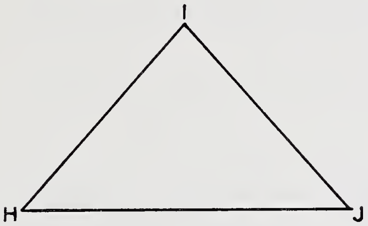


Triangle EFG is an equilateral triangle since each of its sides measures 2 inches.

$\overline{EF} \cong \overline{EG}$
$\overline{EF} \cong \overline{FG}$
$\overline{FG} \cong \overline{EG}$

2. Isosceles triangles are triangles which have any two sides congruent.

e.g.

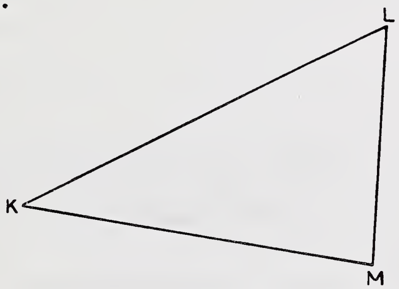


Triangle HIJ is an isosceles triangle since two of its sides measure $1\frac{1}{2}$ inches.

$\overline{HI} \cong \overline{IJ}$

3. Scalene triangles are triangles which have no sides that are congruent.

e.g.

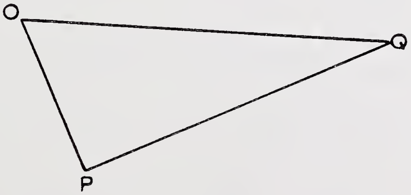


Triangle KLM is scalene. Each side of this triangle has a different measure.

The symbol for the word "triangle" is "$\triangle$". Thus, triangle ABC may be written, "$\triangle$ABC".

Assignment 2

1.



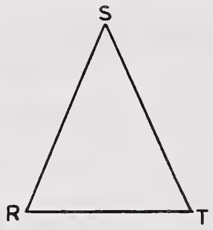
a. The name of this triangle is $\triangle$ OPQ.

b. The three sides of the triangle are $\overline{OP}$, $\overline{OQ}$ and $\overline{PQ}$.

c. No sides of the triangle are congruent.

d. This triangle is scalene.

2.



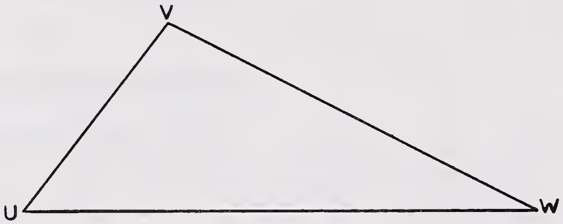
a. The name of this triangle is ______________________

b. The three sides of the triangle are ________________

c. ______________ sides of the triangle are congruent.

d. This triangle is __________________________________

3.



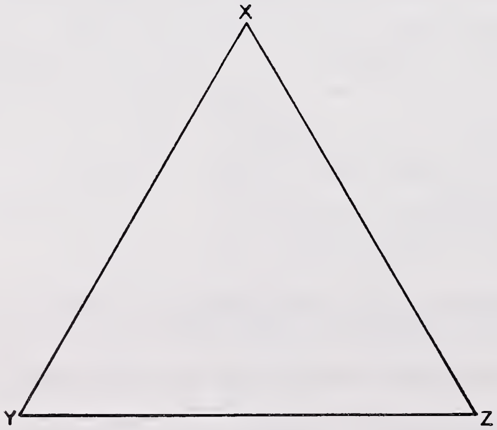
a. The name of this triangle is ______________________

b. The three sides of the triangle are ________________

c. ______________ sides of the triangle are congruent.

d. This triangle is __________________________________

4.



a. The name of this triangle is ______________________

b. The three sides of the triangle are ________________

c. ______________ sides of the triangle are congruent.

d. The triangle is __________________________________

5. Could an equilateral triangle also be called an isosceles triangle? Explain your answer.

____

____

____

6. Could an isosceles triangle also be called an equilateral triangle? Explain your answer.

____

____

____

7. $\triangle PQR$ is an equilateral triangle. Sketch it. Use the names of the sides and the congruent symbol to write statements which show that it is an equilateral triangle.

____

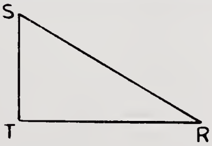
____

____

The other way in which we can classify triangles is by the types of angles determined by the segments.

You will remember (from Lesson 7) that an angle is made up of two rays that have a common endpoint. A triangle is made up of 3 segments. Any 2 of these segments have a common endpoint.

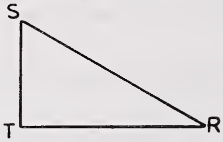
e.g.



In $\triangle STR$,
S is the common endpoint of $\overline{ST}$ and $\overline{SR}$,
R is the common endpoint of $\overline{RS}$ and $\overline{RT}$ and
T is the common endpoint of $\overline{TS}$ and $\overline{TR}$.

The endpoints of the segments of a triangle are called the vertices (plural for vertex) of the triangle just as the common endpoint of the rays of an angle is the vertex of the angle.

e.g.



The vertices of △STR are S, T and R.

Although segments are not endless in length as rays are, we say that any two segments of a triangle determine an angle.

Self-correcting Exercise 1

1. A right angle contains ____________°.

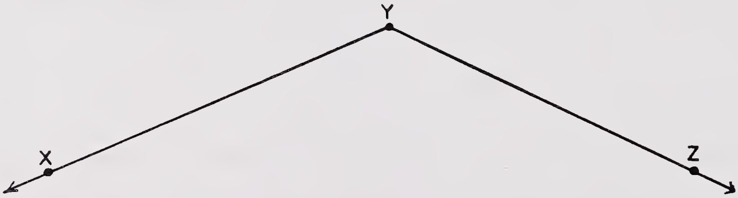
2. An obtuse angle is ____________ than 90°.

3. An acute angle is ____________ than 90°.

4. An obtuse angle is ____________ than a right angle.

5. An acute angle is ____________ than a right angle.

6.



This is ____________________ angle.

The name of this angle is ____________________.

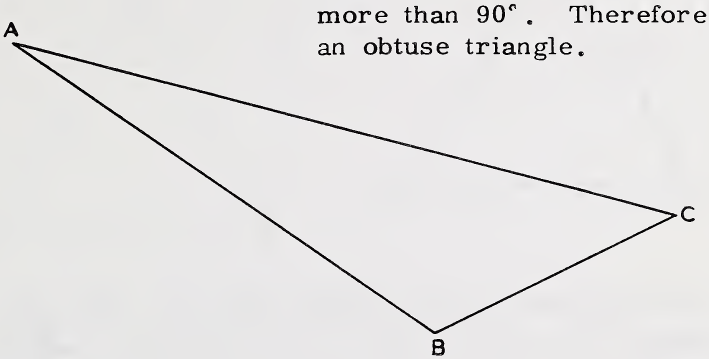
The vertex of the angle is ____________________.

Corrections are on page 29.

We can divide all triangles into three groups according to the measure of their angles.

1. If a triangle has one obtuse angle, it is called an obtuse triangle.

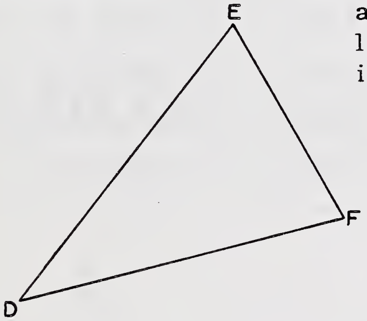
e.g.



∠ABC is obtuse since it measures more than 90°. Therefore △ABC is an obtuse triangle.

2. If a triangle has three acute angles, it is called an acute triangle.

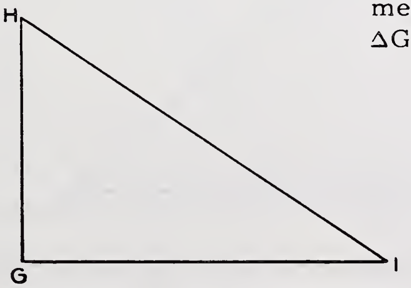
e.g.



∠DEF, ∠DFE and ∠EDF are all acute angles since they measure less than 90°. Therefore △DEF is an acute triangle.

3. If a triangle has one right angle, it is called a right triangle.

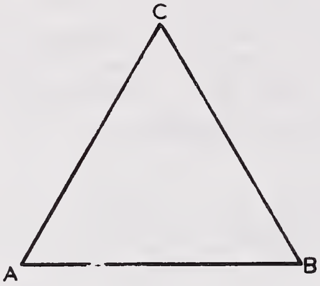
e.g.



∠HGI is a right angle since it measures 90°. Therefore △GHI is a right triangle.

Assignment 3

1.



a. The vertices of this triangle are A , B and C

b. Name any obtuse angles in this triangle. There are none.

c. Name any acute angles in this triangle. $\angle ABC, \angle BCA, \angle CAB$

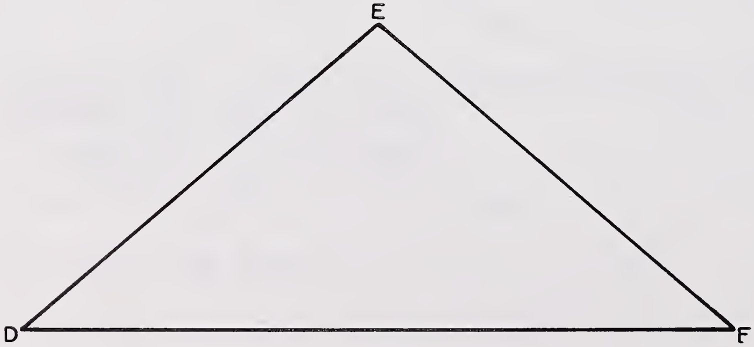
d. Name any right angles in this triangle. There are none.

e. Classify the triangle according to the size of its angles.
acute triangle

f. List any sides that are congruent. $\overline{AC} \cong \overline{AB} \cong \overline{CB}$

g. Classify the triangle according to the lengths of its sides.
equilateral triangle

2.



a. The vertices of this triangle are ____, ____ and ____.

b. Name any obtuse angles in this triangle. ____________

c. Name any acute angles in this triangle. ____________

d. Name any right angles in this triangle. ____________

e. Classify the triangle according to the size of its angles.

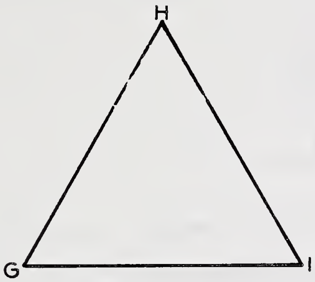
____

f. List any sides that are congruent. ____________________

g. Classify the triangle according to the lengths of its sides.

____

3.



a. Name any obtuse angles in this triangle. ______________

b. Name any acute angles in this triangle. ______________

c. Name any right angles in this triangle. ______________

d. Classify the triangle according to the size of its angles.

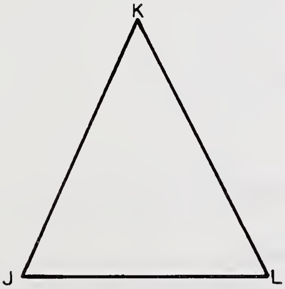
____

e. List any sides that are congruent. ____________________

f. Classify the triangle according to the lengths of its sides.

____

4.



a. Name any obtuse angles in this triangle. ______________

b. Name any acute angles in this triangle. ______________

c. Name any right angles in this triangle. ______________

d. Classify the triangle according to the size of its angles.

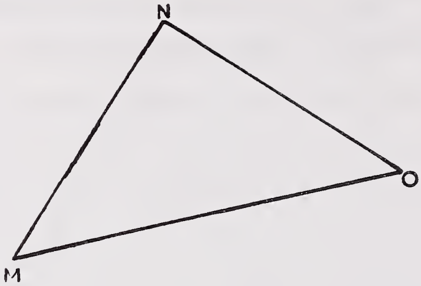
____

e. List any sides that are congruent. ____________________

f. Classify the triangle according to the lengths of its sides.

____

5.



a. Name any obtuse angles in this triangle. ______________

b. Name any acute angles in this triangle. ______________

c. Name any right angles in this triangle. ______________

d. Classify the triangle according to the size of its angles.

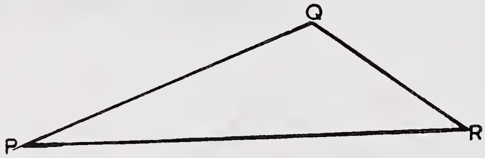
____

e. List any sides that are congruent. ______________

f. Classify the triangle according to the length of its sides.

____

6.



a. Name any obtuse angles in this triangle. ______________

b. Name any acute angles in this triangle. ______________

c. Name any right angles in this triangle. ______________

d. Classify the triangle according to the size of its angles.

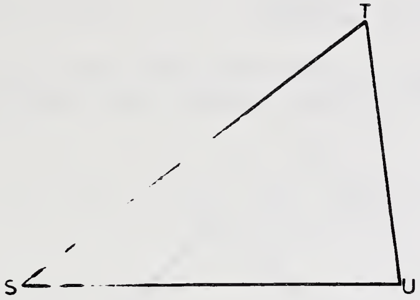
____

e. List any sides that are congruent. ______________

f. Classify the triangle according to the length of its sides.

____

7.



a. Name any obtuse angles in this triangle. ____________

b. Name any acute angles in the triangle. ____________

c. Name any right angles in the triangle. ____________

d. Classify the triangle according to the size of its angles.

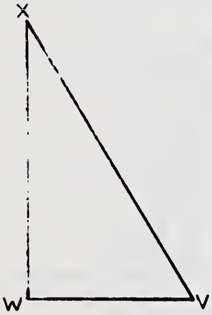
____

e. List any sides that are congruent. ____________

f. Classify the triangle according to the lengths of its sides.

____

8.



a. Name any obtuse angles in this triangle. ____________

b. Name any acute angles in this triangle. ____________

c. Name any right angles in this triangle. ____________

d. Classify the triangle according to the size of its angles.

____

e. List any sides that are congruent. ____________

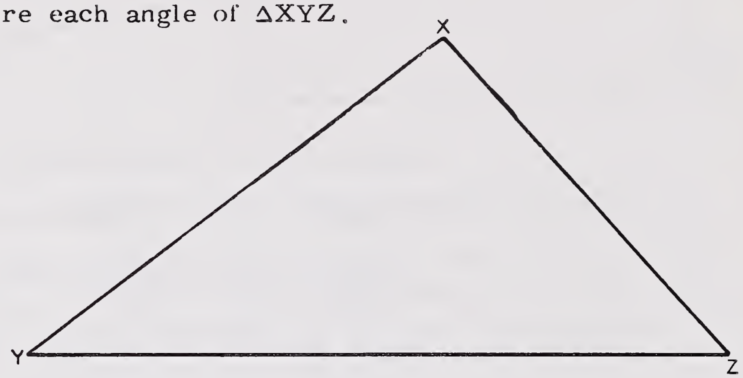
f. Classify the triangle according to the lengths of its sides.

____

Topic Three: Sum of the angles of a triangle

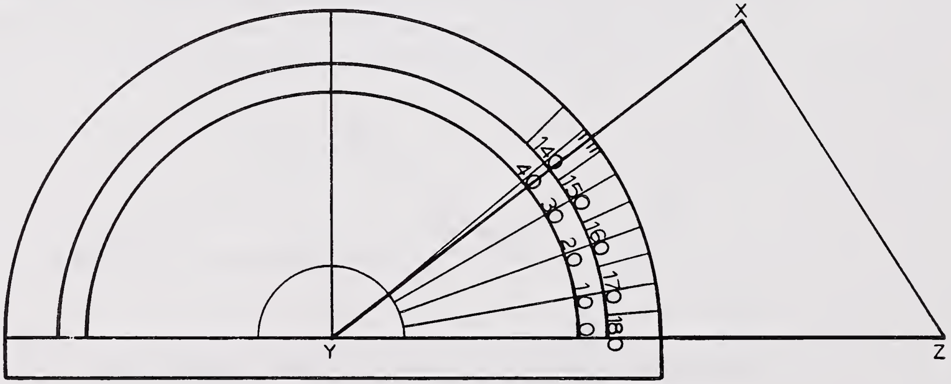
You measure the angles of a triangle in the same way that you measure an angle. Since a triangle determines three angles, you must measure each angle separately.

EXAMPLE: Measure each angle of ΔXYZ.



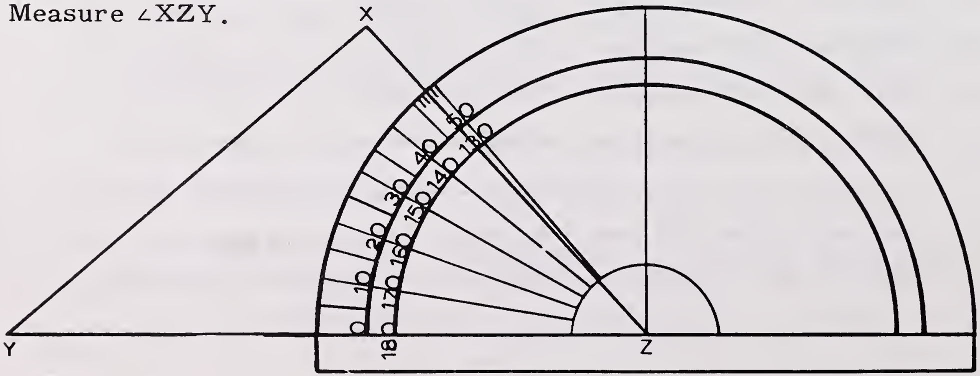
Solution

Measure ∠XYZ.



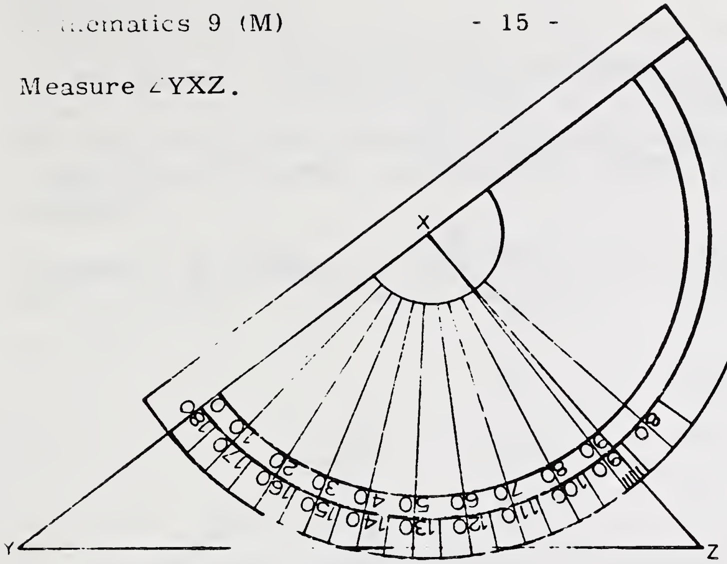
∠XYZ° = 38

Measure ∠XZY.



∠XZY° = 48

Measure ∠YXZ.



∠YXZ° = 94

If we add together the measures of the angles of a triangle the sum is 180° (if you measured carefully).

Assignment 4

1. Complete this chart. Find the measure of the missing angle. Then classify the triangle by its angles.

Triangle	First angle	Second angle	Third angle	Triangle (angles)
ΔABC	75°	25	80°	acute
ΔDEF	45°	48		
ΔGHI		27°	53°	
ΔJKL		45°	45°	
ΔMNO	36°		44°	
ΔPQR	72		18°	

$$\begin{array}{r} 75 \\ 25 \\ \hline 100 \end{array} \qquad \begin{array}{r} 180 \\ -100 \\ \hline 80 \end{array}$$

2. In each of the following cases, is it possible for the three given angles to belong to a triangle? If it is possible, classify the triangle by its angles.

First angle	Second angle	Third angle	Yes or No	Triangle (angles)
30°	70°	80° 30 70 80 180	Yes	acute
90°	12°	98°		
86°	90°	4°		
110°	30°	40°		
10°	8°	112°		

3. What is the greatest number of obtuse angles a triangle can have? Why?

4. What is the greatest number of right angles a triangle can have? Why?

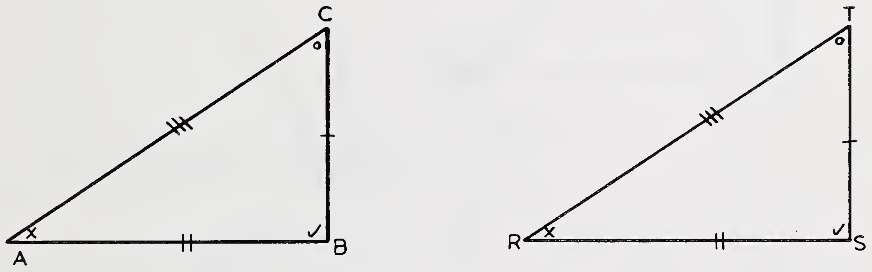
5. We say that an acute triangle has three acute angles. Why must we say three acute angles, why not one or two acute angles?

Topic Four: Congruent triangles

Triangles are congruent if they have the same size and shape. We must compare the segments of one triangle with the segments of another triangle to determine if the triangles are congruent. The three segments, which form the sides of one triangle, must be congruent to (or be the same length as) the three segments which form the sides of the other triangle. Also, the angles determined by one triangle have the same measure as the angles determined by the other triangle.

There are certain markings we can use on diagrams to indicate that particular sides or angles are congruent.

e.g.



In $\triangle ABC$ and $\triangle RST$ above, note that:

1. A single stroke has been placed on $\overline{CB}$ and $\overline{TS}$ to indicate that these two segments are congruent.

2. Double strokes have been placed on $\overline{AB}$ and $\overline{RS}$ to indicate that they are congruent.

3. Triple strokes have been placed on $\overline{AC}$ and $\overline{RT}$ to indicate that they are congruent.

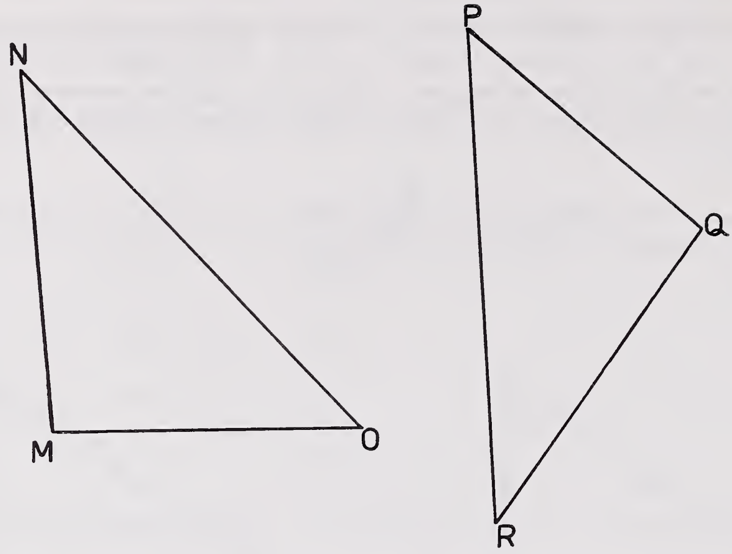
4. A small "x" has been placed on $\angle CAB$ and on $\angle TRS$ to show that they are congruent.

5. A small "o" has been placed on $\angle BCA$ and on $\angle STR$ to show that they are congruent.

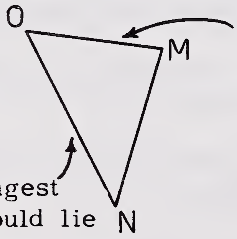
6. A small "✓" has been placed on $\angle ABC$ and $\angle RST$ to show that they are congruent.

In some cases, when two triangles are not lying in the same position, it is more difficult to find the corresponding parts. In such cases, you should draw a sketch of one of the triangles so it lies in the same position as the other.

e.g.



ΔNMO and ΔPQR are congruent, but it is difficult to find the corresponding parts since they are not lying in the same position. Move ΔMNO mentally so that it lies in the same position as ΔPQR and then draw a sketch of it in this position.



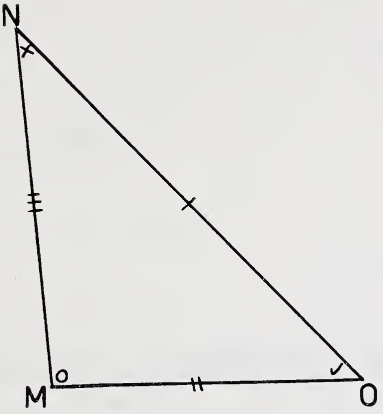
$\overline{MO}$ is the shortest side. (It should lie in the same position as $\overline{PQ}$, the shortest side of ΔPQR.)

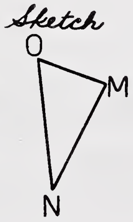
$\overline{NO}$ is the longest side. (It should lie in the same position as $\overline{PR}$, the longest side of ΔPQR.)

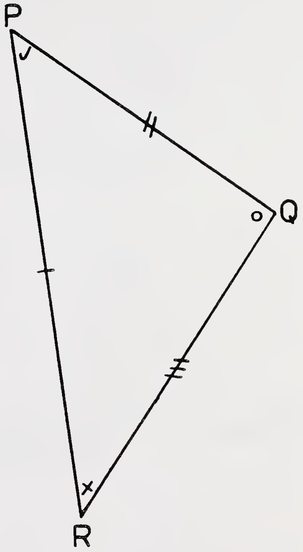
Now, we can name the congruent sides more easily.

$\overline{NO} \cong \overline{PR}$ $\angle NOM \cong \angle RPQ$
$\overline{MO} \cong \overline{PQ}$ $\angle OMN \cong \angle PQR$
$\overline{MN} \cong \overline{QR}$ $\angle MNO \cong \angle QRP$

Since we know which sides are congruent and which angles are congruent we can show this on the diagram by using I , II , III for the sides and x, o, ✓ for the angles.

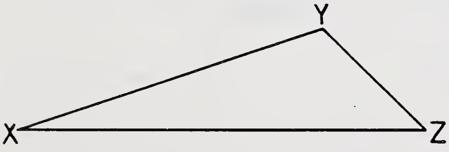


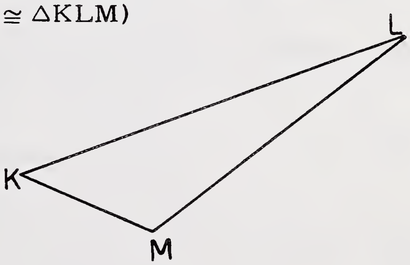




Assignment 5

1. Mark the triangles showing congruent sides and angles. Use \ , \\ , \\\ for the sides. Use x, o, ✓ for the angles. Then complete the chart by referring to the marked sides and angles of △XYZ and △KLM. (△XYZ ≅ △KLM)

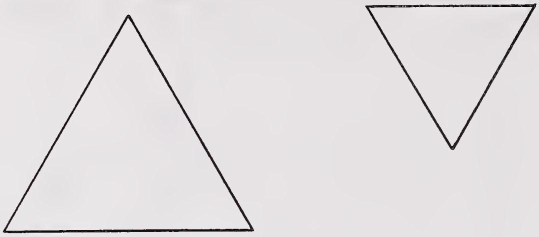




△KLM	≅	△XYZ
$\overline{LM}$	≅	$\overline{XY}$
_____	≅	$\overline{YZ}$
∠KLM	≅	_____
_____	≅	∠YZX
$\overline{KL}$	≅	_____
_____	≅	∠XYZ

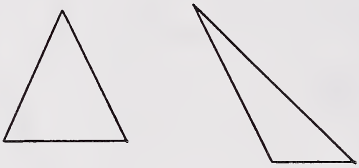
2. By looking at the following pairs of triangles, tell whether or not they are congruent.

a.



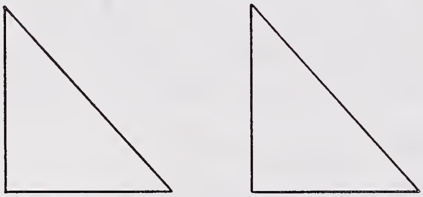
no ____________

b.



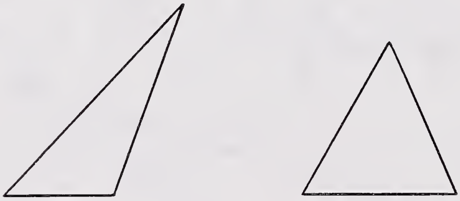
____

c.



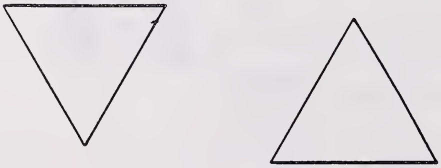
____

d.



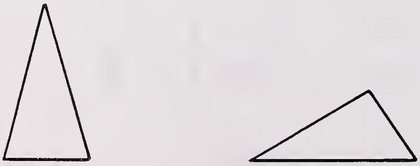
____

e.



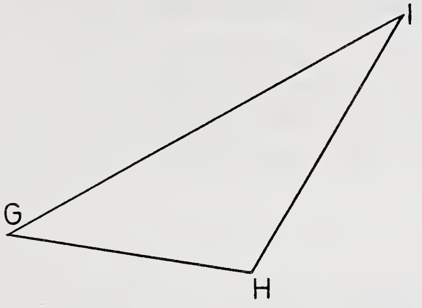
____

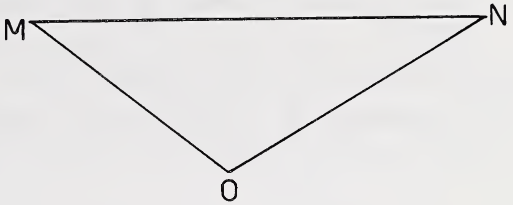
f.



____

3. $\triangle GHI$ and $\triangle MNO$ are congruent. Use / , // , /// to show which side of $\triangle GHI$ is congruent to a side of $\triangle MNO$. Use x, o, ✓ to show which angle of $\triangle GHI$ is congruent to an angle of $\triangle MNO$.





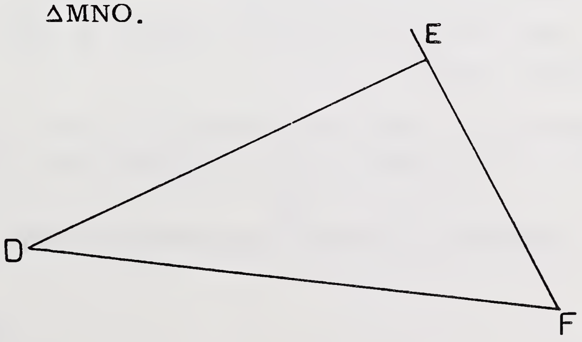
Topic Five: Constructing congruent triangles

It is interesting to learn how to construct a triangle that is congruent to a given triangle. There are two ways of doing this.

A. Using a ruler and a protractor

You can use your ruler and protractor to copy one side and the two angles, that have this side as a common side, from the given triangle. Draw a segment equal in length to any side of the given triangle. Measure the two angles of the given triangle that have the endpoints of this segment as their vertices. Using the segment that you drew earlier, construct angles congruent to the ones you measured. Extend the sides till they meet. They will form the third vertex.

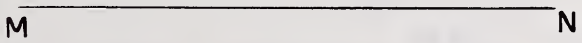
EXAMPLE: Construct a triangle congruent to $\triangle DEF$. Name it $\triangle MNO$.



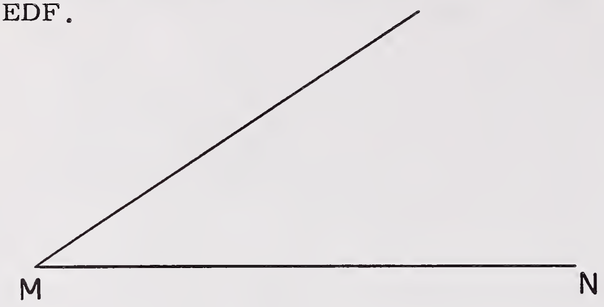
Solution

Draw a segment equal in length to $\overline{DF}$.

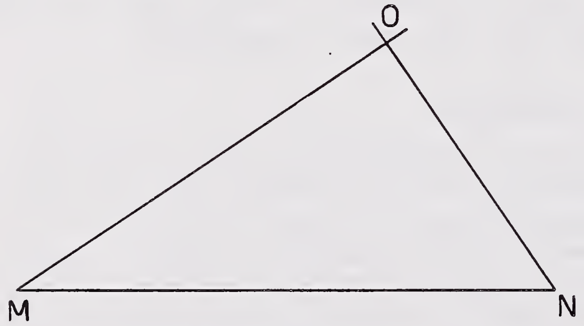
$\overline{DF}$ is $3\frac{1}{4}$ inches long.



Measure the angles whose vertices are the endpoints of this segment. The angles are ∠EDF and ∠EFD. ∠EDF° = 33 ∠EFD° = 56 (Notice that $\overline{EF}$ had to be extended so that ∠EFD could be measured.) Using point M as the vertex, we can draw an angle congruent to ∠EDF.



Using point N as the vertex, we can draw an angle congruent to ∠EFD.



∠OMN and ∠ONM meet at point O. Point O is the third vertex of the triangle.

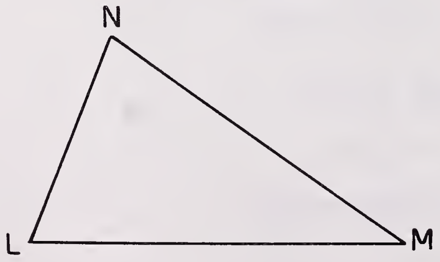
This triangle is congruent to ΔDEF.

B. Using a straight edge and a compass

You can use your straight edge and compass to copy the three sides of a given triangle.

A straight edge is the same as a ruler but it has no measurements indicated on it. It is used to draw straight lines; not to measure lines.

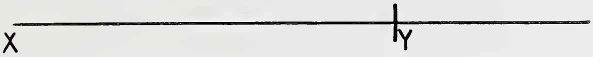
EXAMPLE: Construct a triangle congruent to ΔLMN. Name it ΔXYZ.



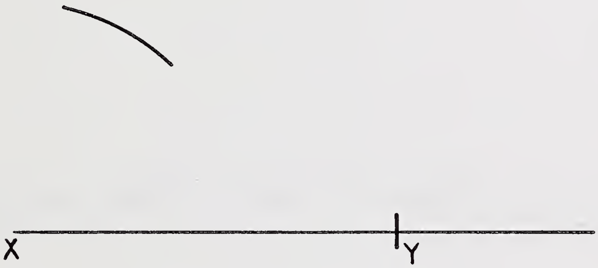
Solution

Draw a line segment.

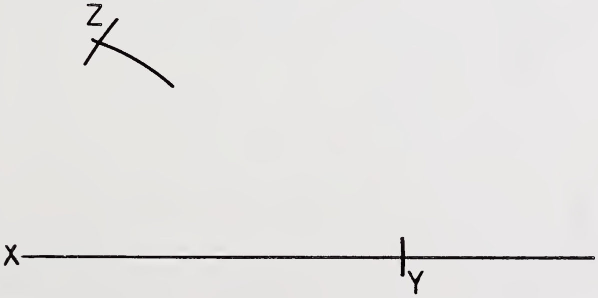
Measure $\overline{LM}$ with a compass. (Place the tip of the compass on L and the pencil on M.) Then, keeping this measured distance between the legs of the compass, place the tip on one end of the line segment and draw an arc on the line with the pencil point.



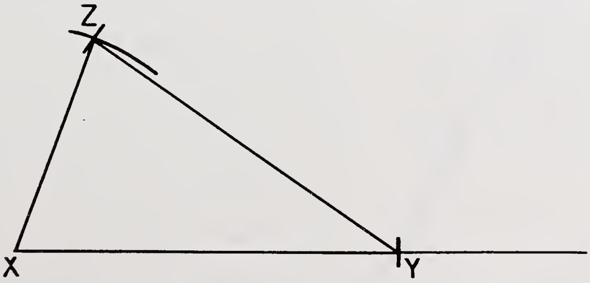
Move the compass back to the triangle. Place the tip on L and the pencil point on N. Move the compass back to the line. Place the tip on X and draw an arc above the line.



Move the compass back to the triangle. Place the tip on M and the pencil point on N. Move the compass back to the above drawing. Place the tip on Y and draw an arc so that it crosses the other arc above the line.



Use your ruler to join Z and X and to join Z and Y.

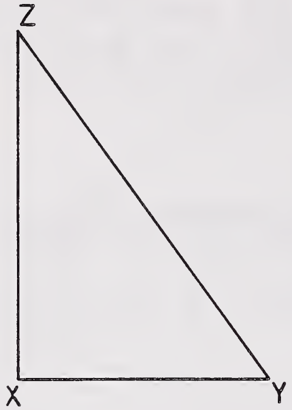


This triangle is congruent to ΔLMN.

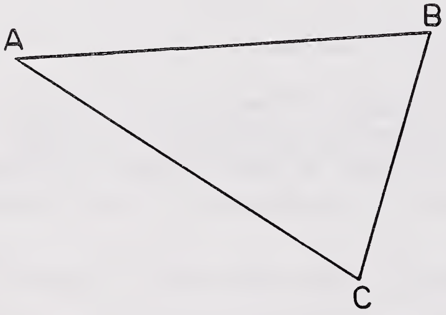
Assignment 6

Construct a triangle that is congruent to each of the given triangles. (Use either method.)

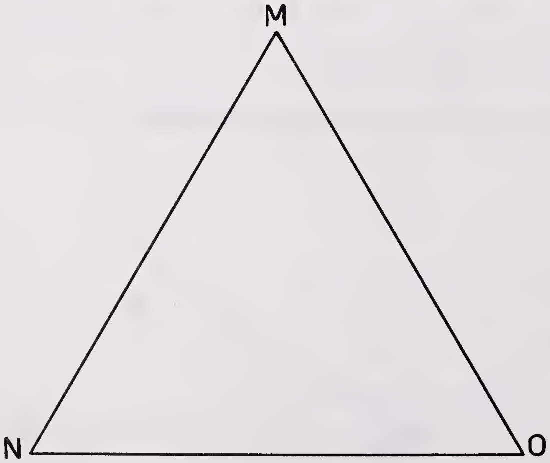
1.



2.



3.



4. Construct $\triangle ABC$. $\overline{AB}$ is $1\frac{1}{2}$ inches long. $\overline{BC}$ is 2 inches long. $\overline{AC}$ is $2\frac{1}{2}$ inches long. (To find these measurements for your compass place the compass tip on the zero mark on the ruler. Extend the compass until the pencil tip is above the required measurement.)

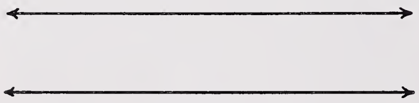
5. Construct $\triangle GHI$. $\angle GHI = 60$ $\overline{GH}$ is $1\frac{3}{4}$ inches long. $\overline{HI}$ is 2 inches long.

6. Construct $\triangle NOP$. $\angle NOP^\circ = 45$ and $\angle NPO^\circ = 55$. $\overline{OP}$ is 2 inches long.

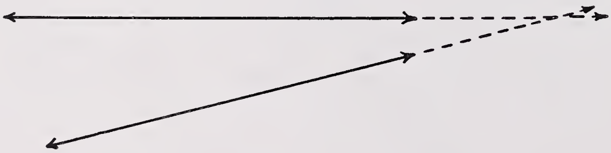
Topic Six: Quadrilaterals

There are five kinds of quadrilaterals. To compare quadrilaterals, we need to know about parallel lines. Lines are parallel if they never meet.

e.g.



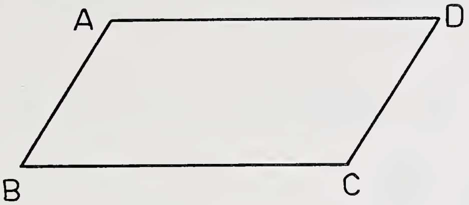
These lines are parallel.
They never meet each other.



These lines are not parallel. They meet (see the extended drawing).

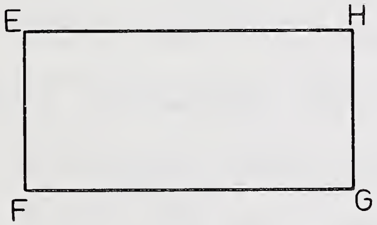
The kinds of quadrilaterals are:

1. Parallelogram
   The opposite sides are congruent and parallel.



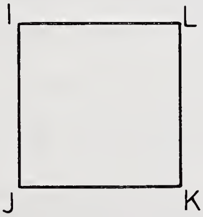
In parallelogram ABCD,
$\overline{AD} \cong \overline{BC}$ and $\overline{AB} \cong \overline{CD}$.
$\overline{AD}$ is parallel to $\overline{BC}$.
$\overline{AB}$ is parallel to $\overline{CD}$.

2. Rectangle
   The opposite sides are congruent and parallel. Each angle determined by the sides is a right angle.



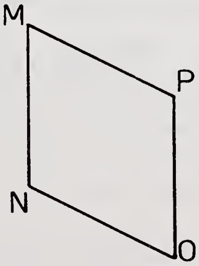
In rectangle EFGH.
$\overline{EH} \cong \overline{FG}$ and $\overline{EF} \cong \overline{HG}$.
$\overline{EH}$ is parallel to $\overline{FG}$.
$\overline{EF}$ is parallel to $\overline{HG}$.
$\angle E$, $\angle F$, $\angle G$ and $\angle H$ are right angles.

3. Square
   Each side is congruent to each of the other sides. The opposite sides are parallel. Each angle determined by the sides is a right angle.



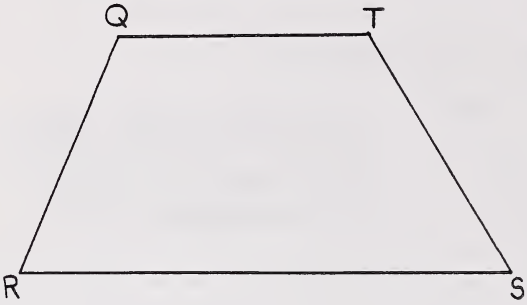
In square IJKL,
$\overline{IJ}$, $\overline{JK}$, $\overline{KL}$ and $\overline{LI}$ are congruent.
$\overline{IL}$ is parallel to $\overline{JK}$.
$\overline{IJ}$ is parallel to $\overline{LK}$.
$\angle I$, $\angle J$, $\angle K$ and $\angle L$ are right angles.

4. Rhombus
   Each side is congruent to each of the other sides. The opposite sides are parallel.



In rhombus MNOP,
$\overline{MN}$, $\overline{NO}$, $\overline{OP}$ and $\overline{PM}$ are congruent.
$\overline{MP}$ is parallel to $\overline{NO}$.
$\overline{MN}$ is parallel to $\overline{OP}$.

5. Trapezoid
Only two sides are parallel.



In trapezoid QRST, $\overline{QT}$ is parallel to $\overline{RS}$. $\overline{QR}$ is not parallel to $\overline{TS}$.

Assignment 7

1. Can all squares be classified as rectangles? Why?

Yes. A rectangle has opposite sides which are congruent and parallel. The opposite sides of a square are parallel and congruent. (A square is a special rectangle with four equal sides.)

2. Can all trapezoids be classified as quadrilaterals? Why?

3. Can all squares be classified as rhombi? Why?

4. Can all parallelograms be classified as rectangles? Why?

5. Can all rhombi be classified as parallelograms? Why?

____

____

____

6. Match Column B with Column A.

A	B
_g_ acute angle	a. has two congruent sides
___ rectangle	b. name for all 4-sided figures
___ rhombus	c. sides are congruent and parallel
___ isosceles triangle	d. each angle is 90°
___ trapezoid	e. never meet
___ square	f. sides are congruent and parallel, and each angle is 90°
___ parallel lines	g. measures less than 90°
___ quadrilateral	h. is a three-sided figure
___ triangle	i. two parallel sides only
___ equilateral triangle	j. 3 sides are congruent

Key to Self-correcting Exercises in Lesson 8

Exercise 1, page 8

1. 90
2. greater
3. less
4. greater
5. less
6. an obtuse
   $\angle XYZ$ or $\angle ZYX$ or $\angle Y$
   Y

End of Lesson 8

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted

____

Time Spent on Lesson

____

(If label is missing
or incorrect)

File Number

____

Lesson Number ________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received:

____

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____

Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Graphs

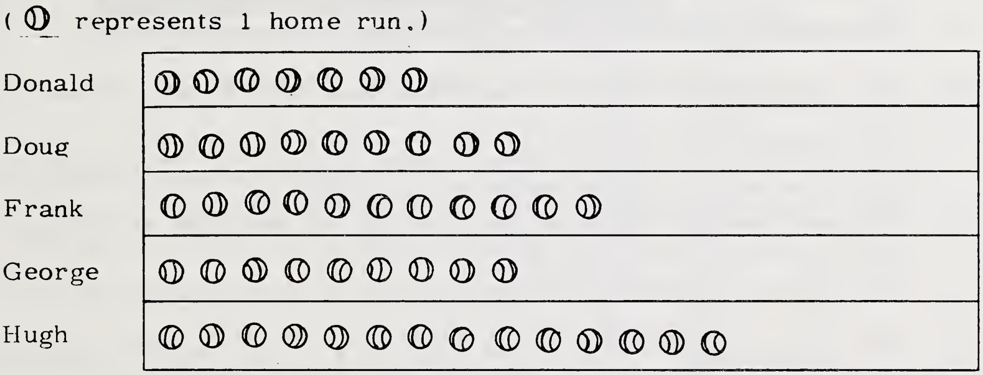
Topic One: Pictographs

Graphs are used to show numerical facts. It is easier to compare facts when they are represented on a graph than when they are included in a wordy paragraph.

A pictograph is a special type of graph. It represents information by using picture symbols. Each picture symbol has a value.

Self-correcting Exercise 1

Number of Home Runs in a Season by Five Ballplayers



1. The title of the pictograph is ______________________________

____

2. ________________ made the most home runs.

3. ________________ made the least home runs.

4. ____________ and ______________ made the same number of runs.

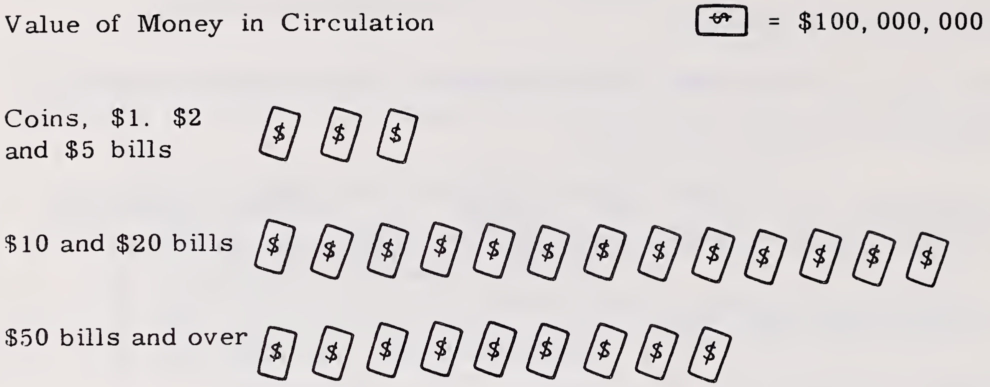
5. Frank made ______________ more home runs than George did.

6. Frank made fewer home runs than ____________________

7. Doug made fewer home runs than ___________ and ___________

Corrections are on page 19.

Assignment 1



1. Each symbol on the graph represents ________________

2. The approximate value of the coins and bills of $1, $2 and $5 in circulation is ________________

3. The approximate value of the bills of $10 and $20 in circulation is ________________

4. The approximate value of the bills of $50 and over in circulation is ________________

5. The approximate value of all the bills in circulation is

____

6. Name the different coins in our money system that have a value of $1 or less. ________ ________ ________

________ ________ ________

7. How much money would you have if you had one of each coin?

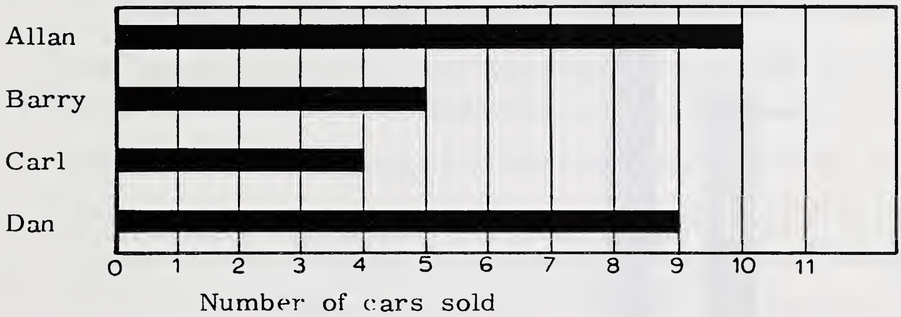
____

Topic Two: Bar graphs

A bar graph is made up of either vertical or horizontal bars.

Self-correcting Exercise 2

Average Weekly Sales of Four Used Car Salesmen



1. The title of the bar graph is ______________________________

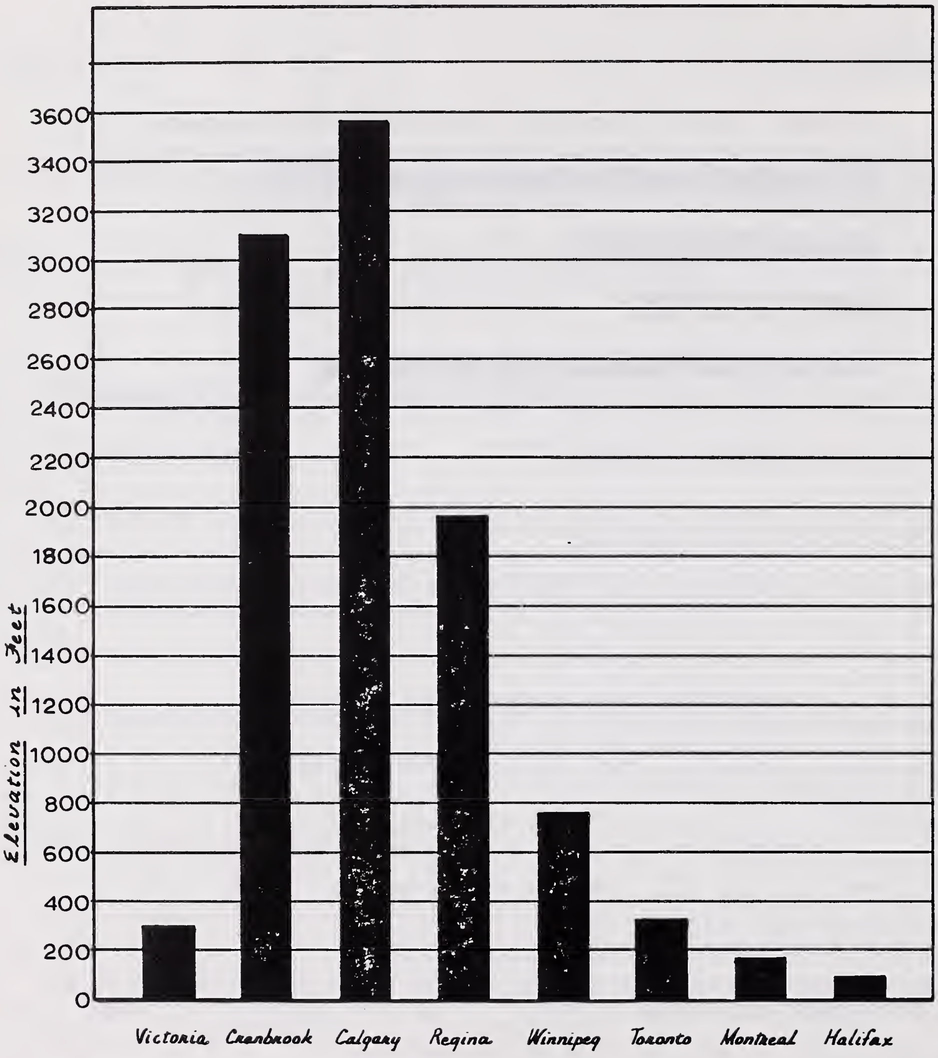
____

2. ____________ sold the most cars on the average.
3. ____________ sold the least cars on the average.
4. Barry sold ____________ fewer cars than Dan sold.
5. Barry sold ____________ more cars than Carl sold.
6. Dan sold fewer cars than ____________ sold.
7. Dan sold more cars than __________ and ____________ sold.

Corrections are on page 19.

Assignment 2

Elevation of Weather Stations in Canada



1. What is the title of this graph?

____

2. Is this a vertical bar graph or a horizontal bar graph?

____

3. Look at the values at the left of the graph. Each space represents ____________ feet. A half-space represents ________________ feet.

4. Estimate the elevation of each of the weather stations.

a. Calgary 3550 ft

b. Cranbrook ____________

c. Halifax ____________

d. Montreal ____________

e. Toronto ____________

f. Victoria ____________

g. Winnipeg ____________

h. Regina ____________

5. To the nearest one space, how many spaces would be needed to represent each of these elevations?

a. 750 feet 4 spaces

b. 1940 feet ____________

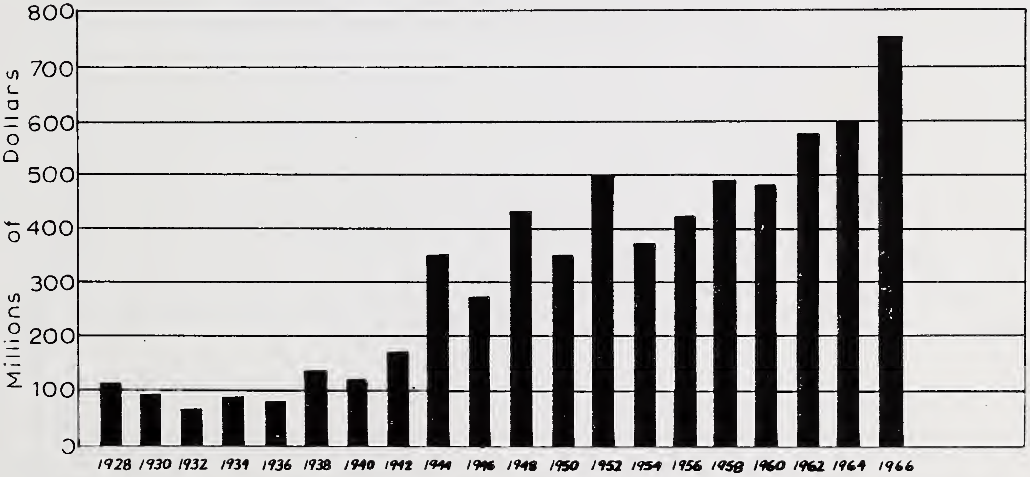
c. 3278 feet ____________

d. 147 feet ____________

e. 2172 feet ____________

f. 1194 feet ____________

Cash Income from the Sale of Farm Products in Alberta



6. The title of this graph is ____

____

7. The graph shows cash income from sale of farm products in Alberta for every second year from ______________ to ______________.

8. In ____________, the cash income from farm products was lowest.

9. In ____________, the cash income from farm products was highest.

10. Each space on the vertical scale of the graph represents

____

11. Give the cash income for each of the years.

a. 1966 $750,000,000

b. 1964 ______________________________

c. 1962 ______________________________

d. 1952 ______________________________

e. 1944 ______________________________

f. 1938 ______________________________

g. 1928 ______________________________

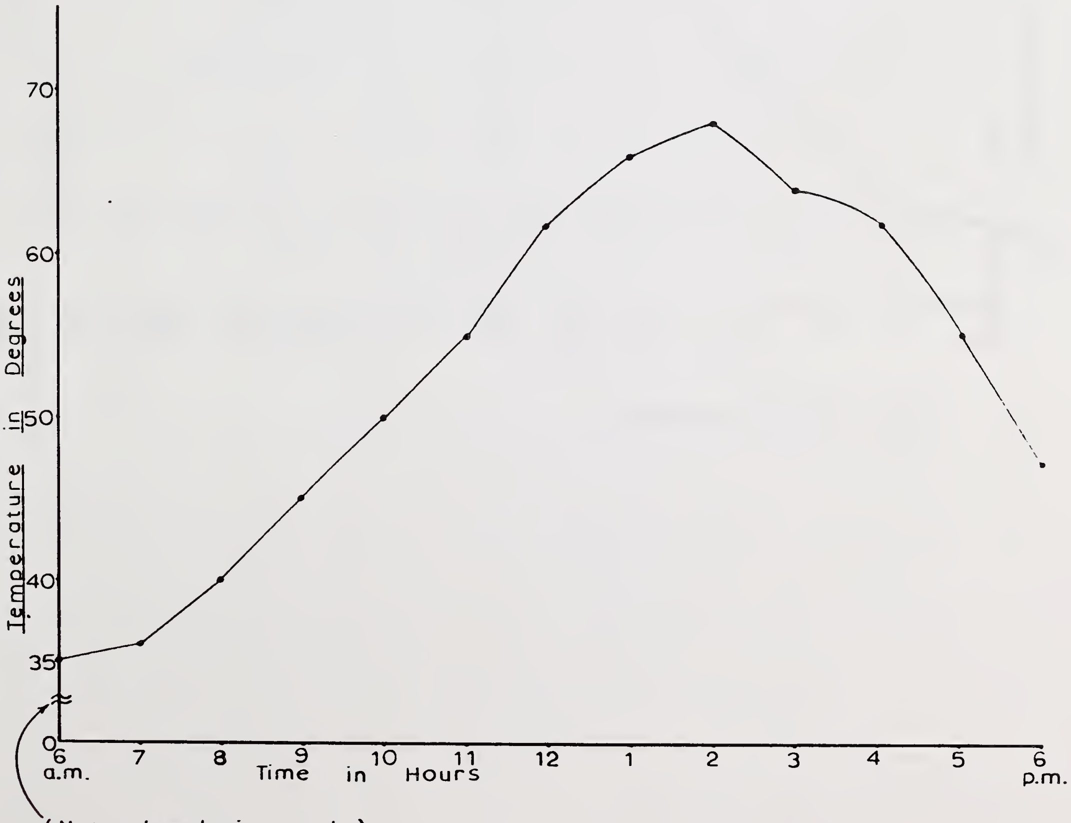
Topic Three: Line graphs

A line graph is different from a pictograph and a bar graph in that it shows continuous change. If the information given is related only to a small group of numbers, it is often impractical to show the vertical scale from 0 to the greatest amount required. To overcome this difficulty, we use a break in the scale to show that it really begins at 0 but that a large group of numbers are being omitted because they are not required to show the given information. To indicate the break we use $\underset{|}{\overset{|}{\approx}}$.

Two values correspond to each point on a line graph. One value is obtained from the scale along the bottom (the horizontal scale) of the graph and the other from the scale along the side (the vertical scale) of the graph.

EXAMPLE: a. What was the temperature of 12 o'clock?
b. At what time was the temperature 45° ?

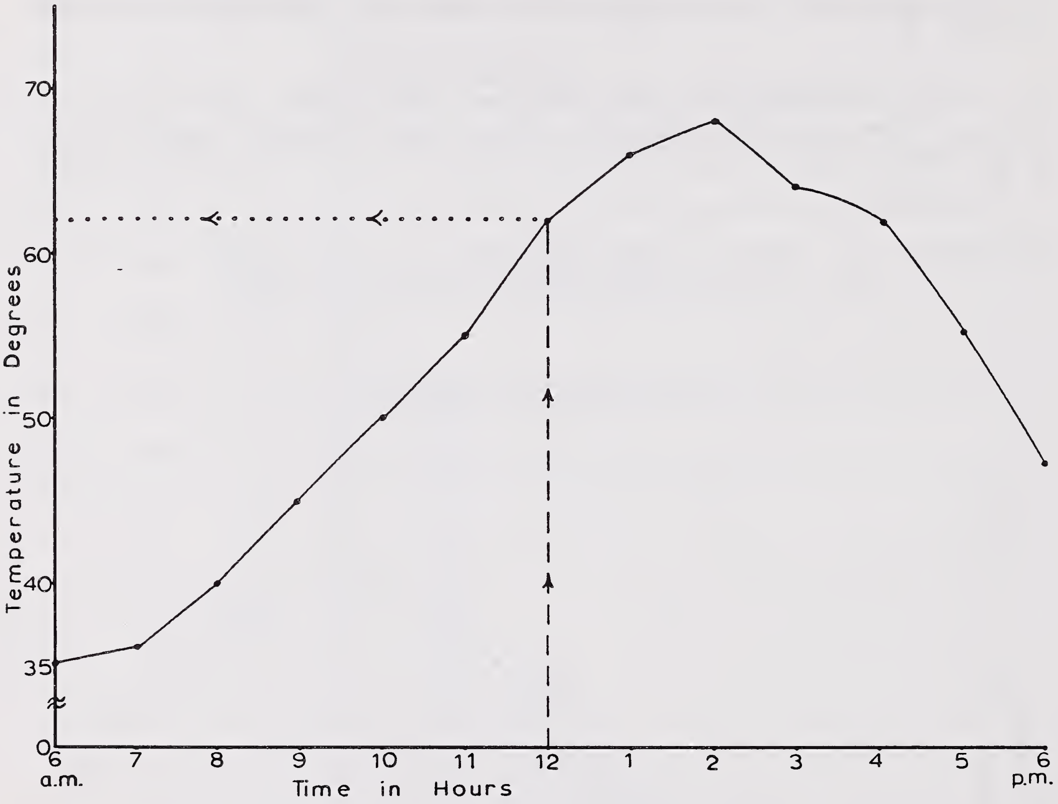
Temperature on June 23



(Note break in scale.)

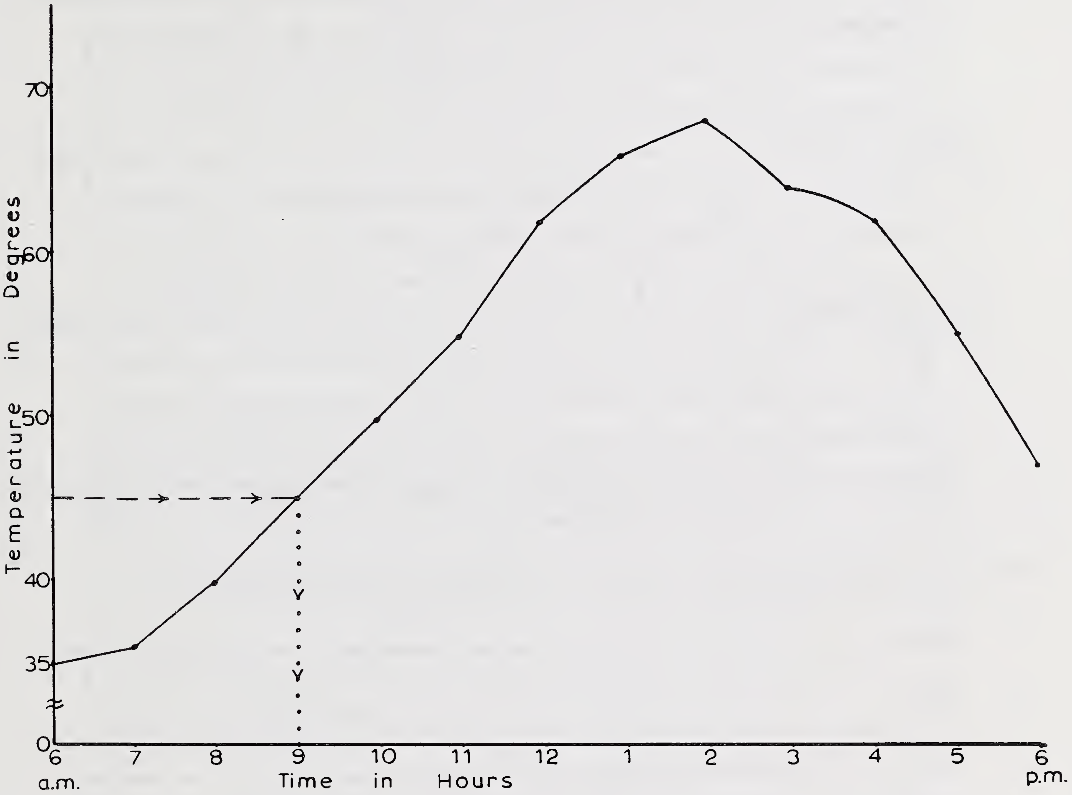
Solution

a. We must find the temperature that corresponds to 12 o'clock. Find 12 o'clock on the horizontal scale. Look up to the graph at a point above 12 (see dashed line on diagram below). Then from this point look left to the vertical scale (see dotted line). The answer is the point on the vertical scale.



At 12 o'clock it was 62°.

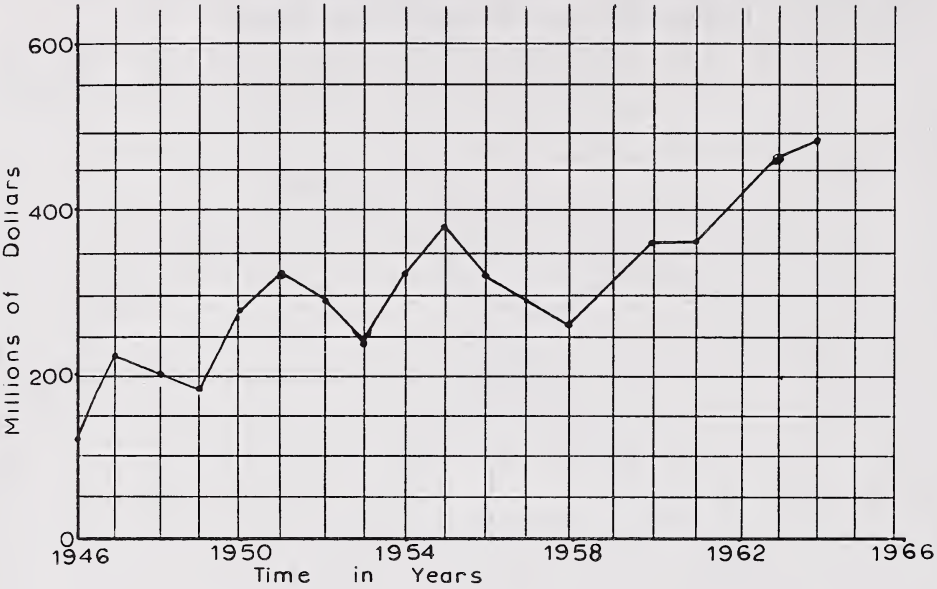
b. We must find the time that corresponds to a temperature of 45°. Find 45° on the vertical scale. Look across the graph at a point across from 45° (see dashed line on diagram below). Then from this point look down to the horizontal scale (see dotted line). The answer is the point on the horizontal scale.



It was 45° at 9 o'clock.

Self-correcting Exercise 3

Canada's Lumber Exports



For each year give the value of Canada's lumber exports.

1. 1946 ____________________

2. 1948 ____________________

3. 1949 ____________________

4. 1951 ____________________

5. 1953 ____________________

6. 1955 ____________________

7. 1957 ____________________

8. 1963 ____________________

9. 1964 ____________________

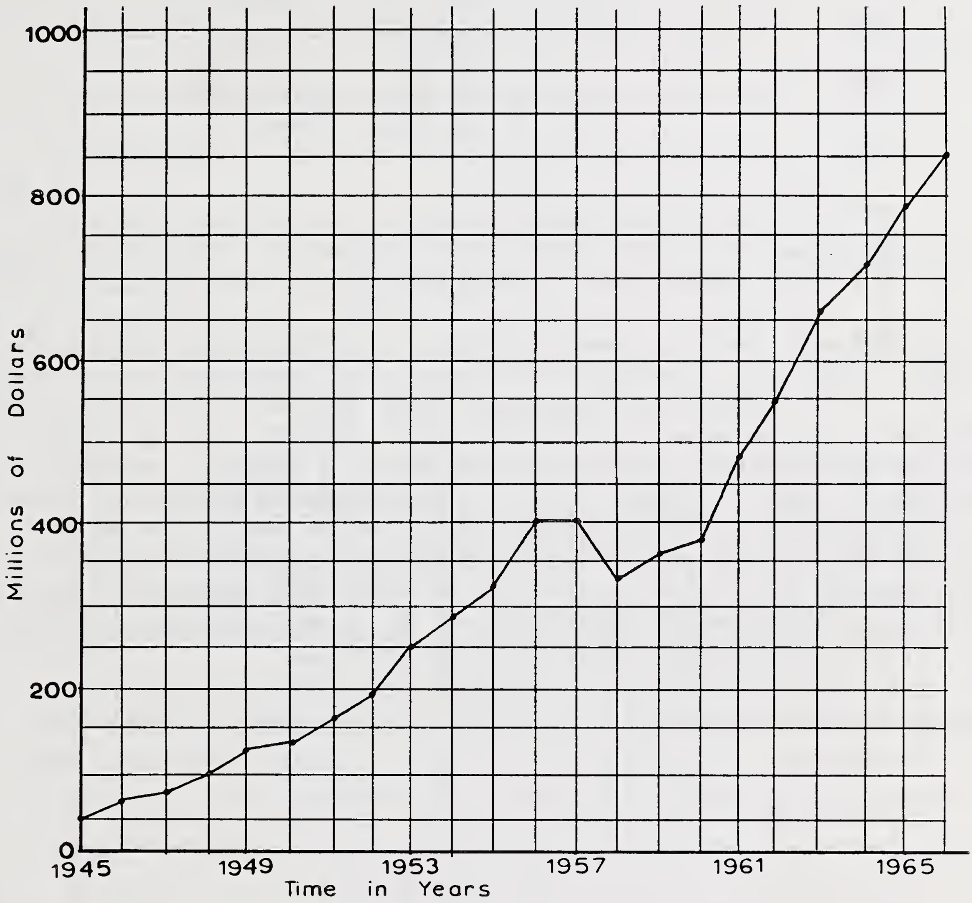
10. What trend does the graph indicate? ____________________

____

Corrections are on page 19.

Assignment 3

Value of Mineral Production in Alberta



1. The title of the graph is ______________________________

____

2. Each space shown on the vertical scale is ______________

3. Each space shown on the horizontal scale is ______________

4. What is the trend indicated by the graph?

____

5. Give the value of mineral production in Alberta in each of the following years. (See the graph on page 11.)

a. 1945 ______________________________

b. 1949 ______________________________

c. 1953 ______________________________

d. 1956 ______________________________

e. 1958 ______________________________

f. 1963 ______________________________

g. 1966 ______________________________

We will now have you draw some line graphs yourself. You probably noticed that the horizontal scale of a line graph is usually a measure of time (hours, years, months, etc.). The vertical scale can be some other measure such as money, temperature, weight, etc. You must decide what amount each space on the horizontal scale will represent and what amount each space on the vertical scale will represent. Then you must locate points which indicate the given information on the graph. Finally these points must be joined by lines.

The graph must be given a title to tell the reader what it is showing.

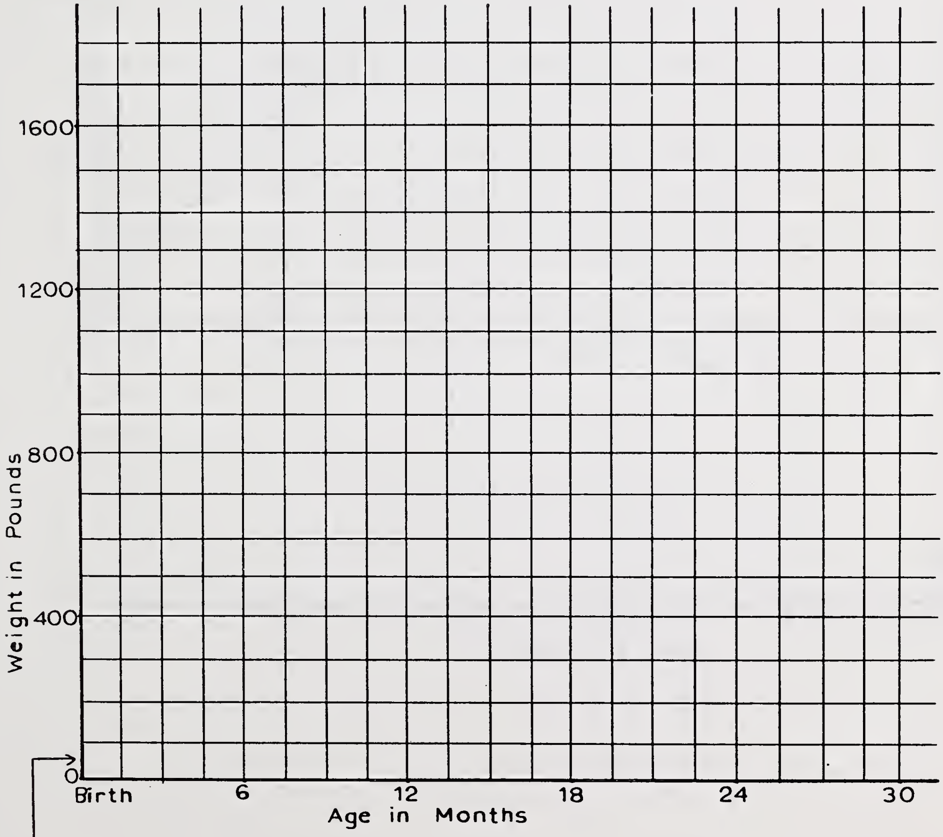
EXAMPLE: Use a line graph to show the following information about the average weights of well-fed steers of different ages.

Birth - 80 lb.
6 months - 440 lb.
12 months - 840 lb.
18 months - 1100 lb.
24 months - 1300 lb.
30 months - 1480 lb.

Solution

Since months are a measure of time, they will be on the horizontal scale. The number of pounds will be on the vertical scale.

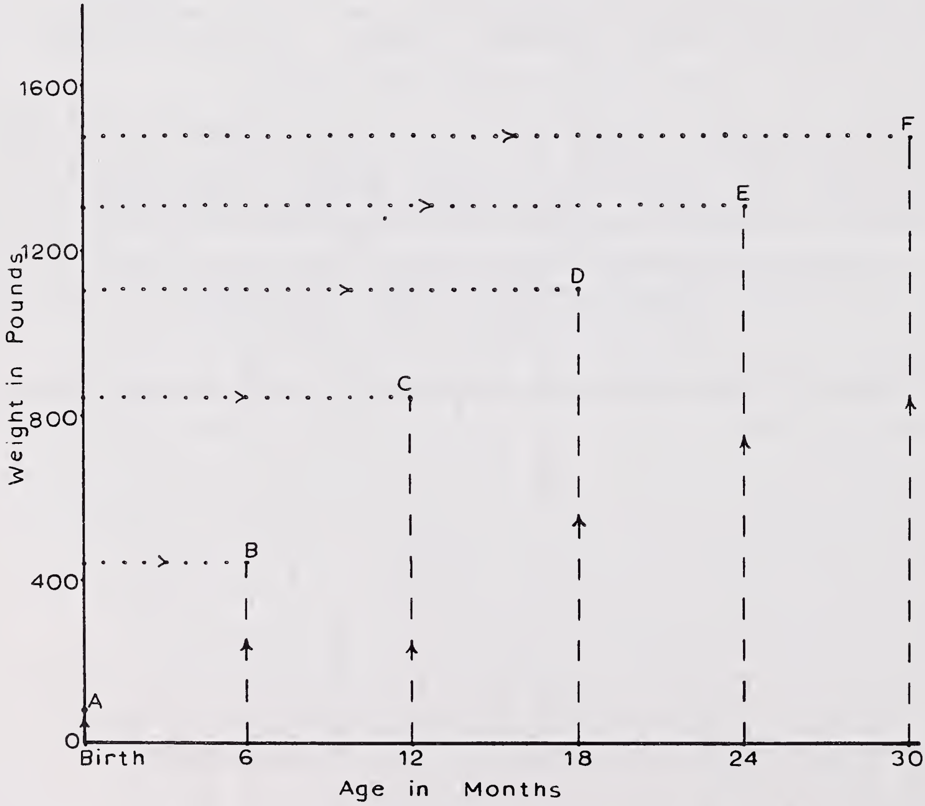
Since the age is indicated in 6 months steps, you can let each large space on the horizontal scale represent 6 months. You can let each large space on the vertical scale represent 400 pounds.



(NB. Each small square represents 100 lb.)

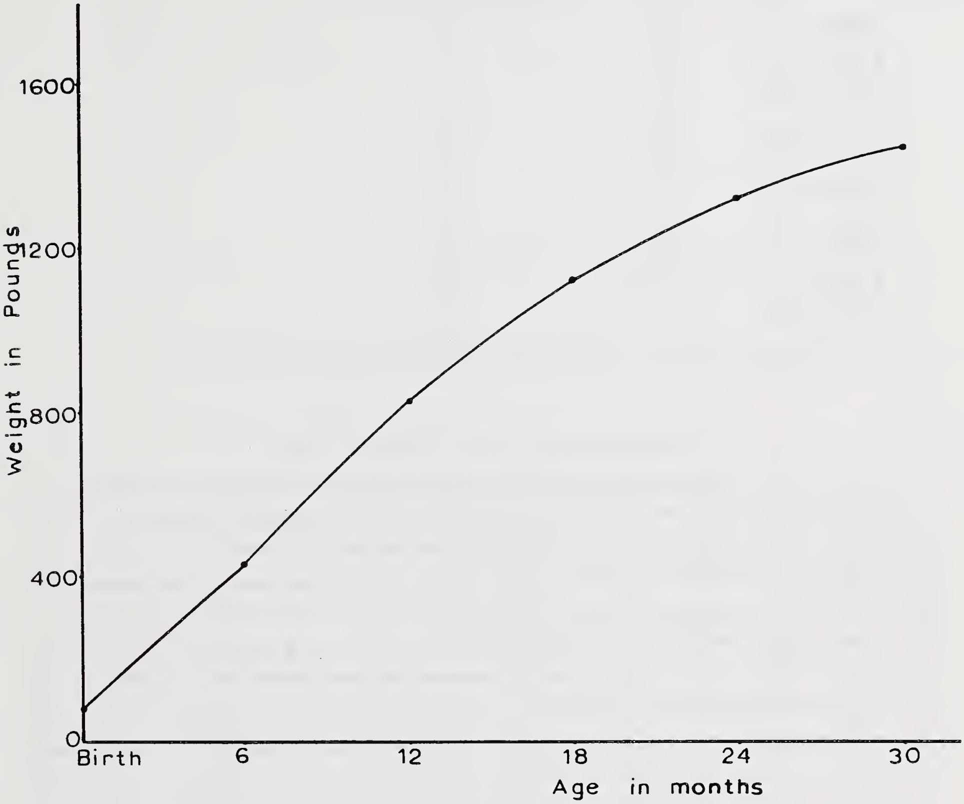
Now you must show the given information on the graph. You must look up from the given age and across to the right from the given weight. The point at which the imaginary lines meet shows the given information.

Look up from	Look right from	Point shown by
Birth	80	A
6	440	B
12	840	C
18	1100	D
24	1300	E
30	1480	F
(dashed lines)	(dotted lines)	



Now these points must be joined by a line. A title must be given to the graph.

Average Weight of Well-fed Steers at Different Ages

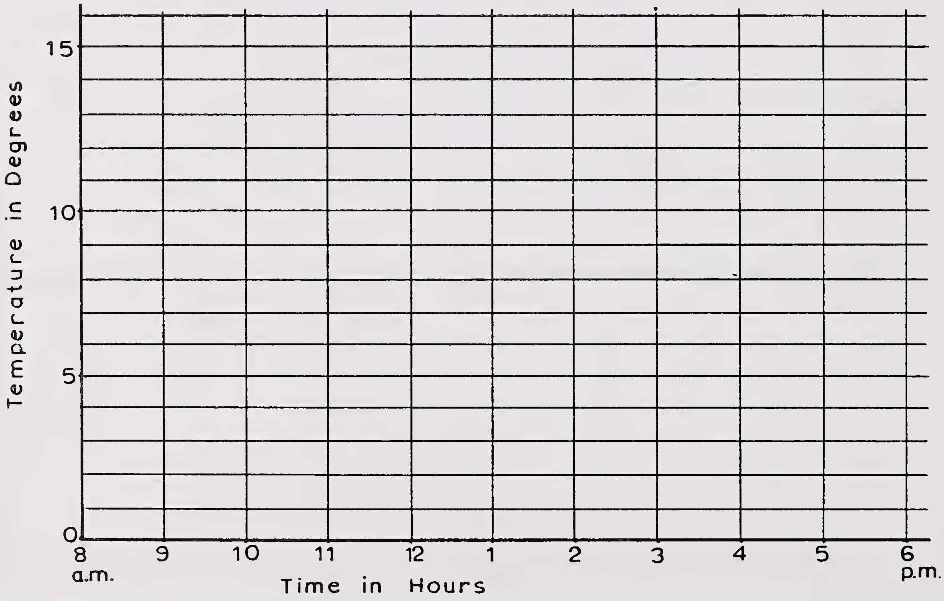


Assignment 4

1. The following temperature record was kept one winter day at an Air Force base. Make a line graph to show the information.

Hour of the day	Degrees Celsius
8:00 a.m.	0°
9:00 a.m.	1°
10:00 a.m.	1°
11:00 a.m.	2°
12:00 Noon	3°
1:00 p.m.	4°
2:00 p.m.	6°
3:00 p.m.	9°
4:00 p.m.	8°
5:00 p.m.	7°
6:00 p.m.	5°

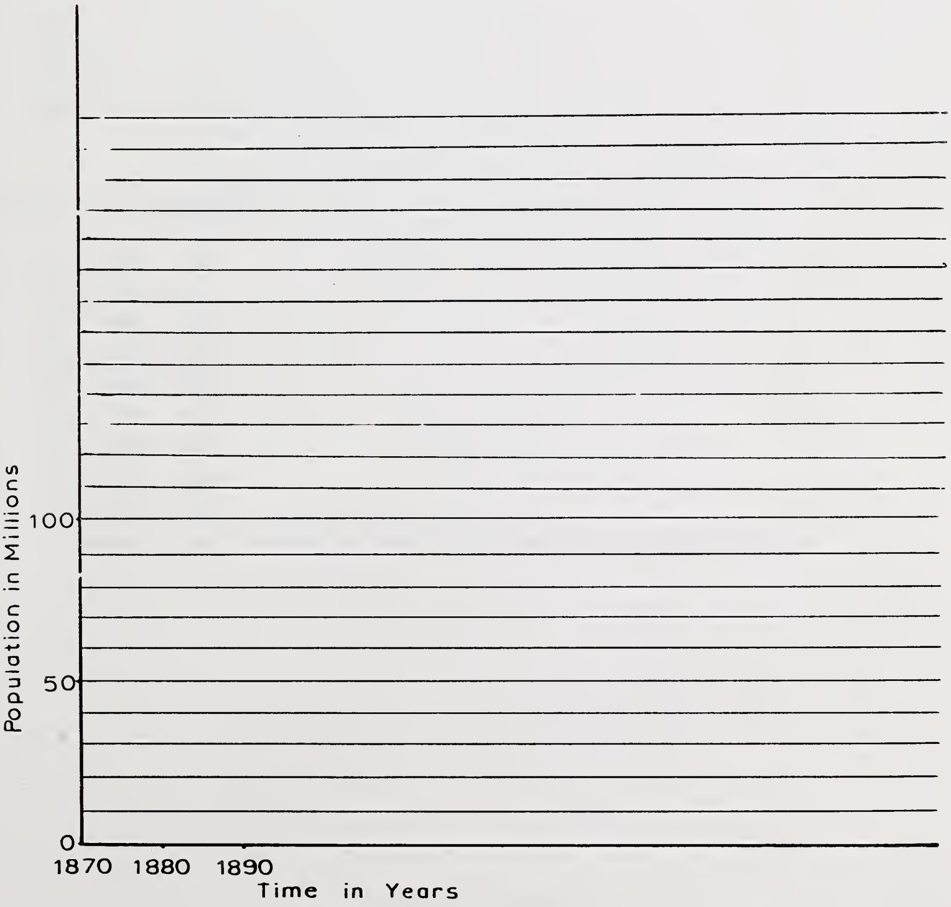
Temperature on a Winter Day



2. Make a line graph to show the population of the United States over a period of 100 years.

Year	Population to the Nearest Million
1870	39
1880	50
1890	63
1900	76
1910	92
1920	106
1930	123
1940	132
1950	151
1960	180
1970	210

Population of United States from 1870 to 1970



3. Use a line graph to show the total water used per day in the United States since 1910.

Year	Billions of Gallons of Water Used per Day
1910	64
1920	95
1930	110
1940	140
1950	200
1960	320
1970	400

(Use graph paper at end of lesson.)

Key to the Self-correcting Exercises in Lesson 9

Exercise 1, page 1

1. Number of Home Runs in a Season by Five Ballplayers
2. Hugh
3. Donald
4. Doug and George
5. two
6. Hugh
7. Frank and Hugh

Exercise 2, page 3

1. Average Weekly Sales of Four Used Car Salesman
2. Allan
3. Carl
4. four
5. one
6. Allan
7. Carl and Barry

Exercise 3, page 10

1. $120, 000, 000
2. $200, 000, 000
3. $180, 000, 000
4. $320, 000, 000
5. $240, 000, 000
6. $380, 000, 000
7. $290, 000, 000
8. $460, 000, 000
9. $480, 000, 000
10. It shows that Canada's lumber exports are increasing.

End of Lesson 9

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number __________

FOR SCHOOL USE ONLY

Assigned
Teacher: __________

Lesson Grading: __________

Additional Grading
E/R/P Code: __________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received:

____

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Review

Following each question, page references are given telling you where to go for help.

Marks

3 1. Use digits to write each of the numbers. (Lesson 1, pages 1-4)

a. Thirty thousand, three hundred thirty-three ____________________

b. Forty-four billion, four hundred four million, four hundred four thousand, four hundred four ______________________________

c. Two hundred seventy billion ________________________________

10 2. What place is held by each digit in 11,890,372,546? (Lesson 1, page 1)

6 is in the ________________________________ place.

4 is in the ________________________________ place.

5 is in the ________________________________ place.

0 is in the ________________________________ place.

8 is in the ________________________________ place.

9 is in the ________________________________ place.

2 is in the ________________________________ place.

3 is in the ________________________________ place.

7 is in the ________________________________ place.

1 is in the ________________________________ place.

3 3. When counting by hundreds you are actually adding _____________ to each number. e.g. 879, 979, _____, _____ (Lesson 1, page 8)

6 4. 5^2 means that the number _____ is used as a factor _____ times.

10^3 means that the number _____ is used as a factor _____ times.

8^6 means that the number _____ is used as a factor _____ times.

(Lesson 1, page 10)

Marks

4 5. Give the standard name of each power. (Lesson 1, page 11)

a. 4^2 ________________

b. 10^5 ________________

c. 7^3 ________________

d. 5^5 ________________

4 6. Use scientific notation to write each number. (Lesson 1, page 19)

a. 400 ________________

b. 2,000 ________________

c. 60 ________________

d. 10,000 ________________

1 7. Our number system is based on base ________. (Lesson 1, page 13)

3 8. Use words to write each number. (Lesson 1, page 17)

a. 397,205 ________________

b. 455_6 ________________

c. 30276_8 ________________

3 9. Base ten system has ________ digits. (Lesson 1, page 13)

Base eight system has ________ digits.

Base six system has ________ digits.

3 10. In 425_6, 5 is associated with ________ power of 6,

2 is associated with ________ power of 6 and

4 is associated with ________ power of 6. (Lesson 1, page 14)

3 11. In 425_8, 5 is associated with ________ power of ______,

2 is associated with ________ power of ______ and

4 is associated with ________ power of ____. (Lesson 1, page 15)

Marks

5 12. Write each number in expanded form. (Lesson 1, page 17)

a. 91,867 ____________________

b. 516_8 ____________________

c. 444_6 ____________________

d. 231_6 ____________________

e. 47_8 ____________________

3 13. Which is larger, 32_6 or 25_8? ______________ (Lesson 1, page 18)

4 14. Express each number as a base-ten numeral. (Lesson 1, page 18)

a. 1111_8 ____________________

b. 400_6 ____________________

c. 362_8 ____________________

d. 213_6 ____________________

10 15. Find the products. (Lesson 2)

a. $\begin{array}{r}128\\ \underline{80}\end{array}$ (p.6)

b. $\begin{array}{r}887\\ \underline{50}\end{array}$

c. $\begin{array}{r}4985\\ \underline{300}\end{array}$

d. $\begin{array}{r}5379\\ \underline{8000}\end{array}$

e. $\begin{array}{r}86\\ \underline{21}\end{array}$ (p.9)

f. $\begin{array}{r}837\\ \underline{64}\end{array}$

Marks

g. 56
49

h. 215
386

i. 305
257

j. 3784
596

20 16. Round each number as indicated. (Lesson 2, pages 13 - 15)

		Nearer ten	Nearer hundred	Nearer thousand	Nearer million
a.	1,568,485				
b.	8,423,647				
c.	4,397,862				
d.	7,762,526				
e.	2,425,916				

10 17. Find the quotients. (Lesson 3)

a. $9\overline{)4345}$

b. $3\overline{)2237}$

4345 ÷ 9 = _____ r. _____

2237 ÷ 3 = _____, r. _____

Marks

c. $52\overline{)\,3750}$

d. $16\overline{)\,1983}$

3750 ÷ 52 = _____ r. _____

1983 ÷ 16 = _____ r. _____

e. $27\overline{)\,2886}$

f. $81\overline{)\,4097}$

2886 ÷ 27 = _____ r. _____

4097 ÷ 81 = _____ r. _____

g. $406\overline{)\,9746}$

h. $981\overline{)\,184276}$

9746 ÷ 406 = _____ r. _____

184276 ÷ 981 = _____ r. _____

Marks

i. $32 \overline{)7496}$ j. $928 \overline{)38597}$

7496 ÷ 32 = _____ r. _____ 38597 ÷ 928 = _____ r. _____

10 18. Using the quotients you found in #17, round each quotient to the nearer one. (Lesson 3, pages 14 and 15)

a. ______________________ b. ______________________

c. ______________________ d. ______________________

e. ______________________ f. ______________________

g. ______________________ h. ______________________

i. ______________________ j. ______________________

5 19. Use the short division method to find these quotients. (Lesson 3)

a. $8 \overline{)5423}$ b. $3 \overline{)462}$ c. $7 \overline{)473}$

d. $8 \overline{)6006}$ e. $6 \overline{)777}$

10 20. Name the first 10 prime numbers. (Lesson 4, page 16)

_____ _____ _____ _____ _____ _____ _____ _____

_____ _____

arks

10 21. Name the first 10 composite numbers. (Lesson 4, page 11)

______ ______ ______ ______ ______ ______ ______ ______

______ ______

15 22. Name all the factors of each number. Circle the prime factors. Use curved lines to show the factor pairs. (Lesson 4, pages 4 and 14)

a. 75 ______________________________

b. 48 ______________________________

c. 28 ______________________________

d. 40 ______________________________

e. 54 ______________________________

2 23. Name two composite numbers which are odd. (Lesson 4, pages 9 and 11)

____

1 24. Name the prime numbers which are even. (Lesson 4, pages 8 and 14)

____

5 25. Find the complete factorization of each number. (Lesson 5, page 3)

a. 76 Show the steps.

____

____

____

Marks

b. 48 ______________________________

____

____

____

c. 91 ______________________________

____

____

____

d. 110 ______________________________

____

____

____

e. 1296 ______________________________

____

____

____

3 26. Using the complete factorization of each number from #25, find the greatest common factor of each group of numbers. (Lesson 5, page 8)

a. 48 and 1296 ______________________________

____

b. 76 and 48 ______________________________

____

c. 110 and 48 ______________________________

____

arks

5 27. Circle the numbers below which are common multiples of 3 and 4 :

12, 16, 18, 24, 28, 30, 36, 40, 42, 48. (Lesson 6, page 2)

5 28. Find the complete factorization of each number. (Lesson 5, page 3)

a. 36 ____________________

____

____

____

b. 48 ____________________

____

____

____

c. 16 ____________________

____

____

____

d. 21 ____________________

____

____

____

e. 56 ____________________

____

____

____

Marks

4 29. Using the complete factorization of each number from #28, find the least common multiple of each group of numbers. (Lesson 6, page 9)

a. 36 and 48 ______________________________

____

b. 56 and 16 ______________________________

____

c. 48 and 21 ______________________________

____

d. 36, 48 and 56 ______________________________

____

5 30. A manufacturer wishes to buy steel bars of such length that he can cut each bar into 6'' pieces, or 8'' pieces, or 10'' pieces without waste. What length of bar should he purchase? (Lesson 6, pages 11-13)

arks

5 31. Mr. Harry travelled by car for several days. He travelled 254 km, 498 km, 272 km, 395 km, 366 km, and 259 km to finish the journey. How many kilometres did he travel in all? (Lesson 1, p. 21)

Placeholder:

Equation: Computation

Answer:

Statement:

5 32. Redfarms Company wished to plant 2000 trees. The first day 192 trees were planted. How many more trees were yet to be planted? (Lesson 2, page 18)

Placeholder:

Equation: Computation

Answer:

Statement:

5 33. Fred was selling tickets for the school concert at 75¢ each. He sold 48 tickets. What was the amount of money that Fred collected for his sales?

Placeholder:

Equation: Computation

Answer:

Statement:

Marks

5 34. The Swim Club reported $31,990 for membership fees this year. If the fees were $35 for members, how many people are paid up members? (Lesson 4, page 19)

Placeholder:

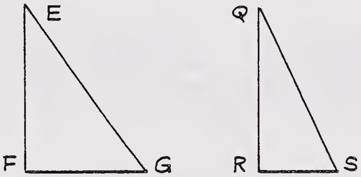
Equation: Computation

Answer:

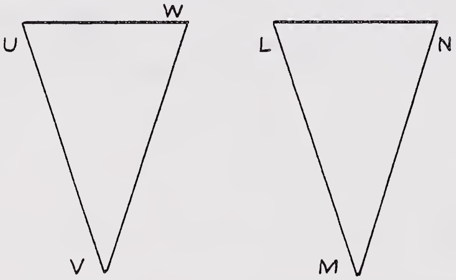
Statement:

11 35. Use the names of the angles and the congruent symbol to write statements which show that some pairs of angles below are congruent. (Use 3 letters to name each angle.) (Lesson 8, page 18)

a.



b.



____________________.

____________________.

____________________.

____________________.

____________________.

____________________.

c. ΔCDE is an obtuse triangle. Sketch it. Write a sentence naming the obtuse angle with 3 letters, and in your sentence tell why ΔCDE is called an obtuse triangle. (Lesson 8, page 10)

____

Marks

10 36. Construct with your protractor each of the following angles. In the blank provided, state what kind of angle you have constructed (right, acute, obtuse). (Lesson 7, pages 11-14)

a. 28°

______________________________.

b. 145°

____

c. 110°

____

d. 90°

____

Marks

e. 60°

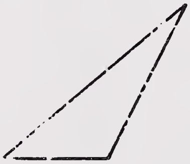
____

6 37. The measures of two angles of a triangle are given. Find the measure of the third angle. Classify the triangle according to the measures of its angles. (Lesson 8, pages 9 and 15)

a.	60°, 100°	Third angle	____________
		Type of triangle	____________
b.	90°, 30°	Third angle	____________
		Type of triangle	____________
c.	45°, 55°	Third angle	____________
		Type of triangle	____________

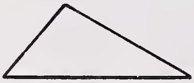
8 38. Classify each of the triangles according to the lengths of its sides. (Lesson 8, pages 4 and 5)

a.



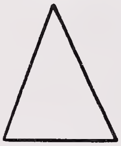
____

b.



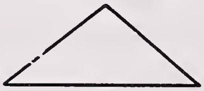
____

c.



____

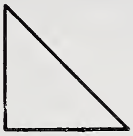
d.



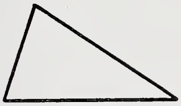
____

Marks

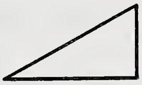
e.



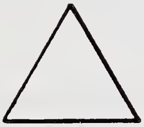
f.



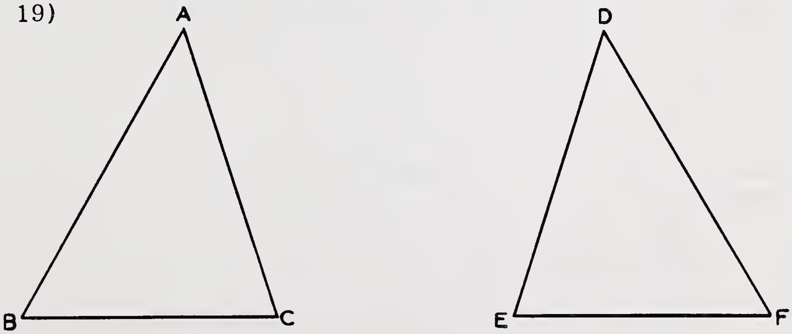
g.



h.

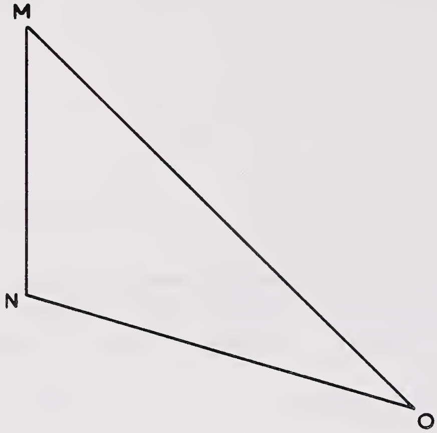


6 39. $\triangle ABC$ is congruent to $\triangle DEF$. Write six congruency statements that relate a part of $\triangle ABC$ to a part of $\triangle DEF$. (Lesson 8, pages 18 and 19)

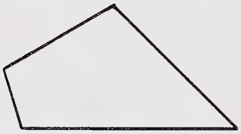


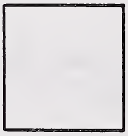
Marks

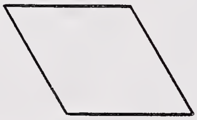
3 40. Construct a triangle which is congruent to △MNO using a straight edge and compass. (Lesson 8, pages 22 and 23)

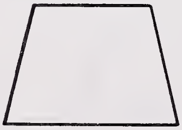


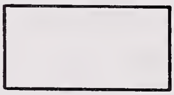
6 41. Classify each of the following quadrilaterals. Be as specific as possible. (Lesson 8, pages 27 and 28)

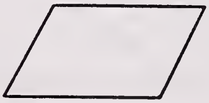
a.

____

b.

____

c.

____

d.

____

e.

____

f.

____

Marks

11 42. Make a line graph to show the following information about the number of pupils enrolled in public schools in a country, Southland. (Lesson 9, pages 12-15)

Year	Public School Enrollment (In millions)
1920	22
1930	24
1940	25
1950	30
1960	39
1970	49
1980	48

(Use graph paper at end of lesson.)

Did you put a title on the graph? ________

Did you put the number of years along one axis? ________

Did you put the enrollment along the other axis? ________

Did you join the points? ________

Total Marks
260

End of Lesson 10

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number __________

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned
Teacher: __________

Lesson Grading: __________

Additional Grading
E/R/P Code: __________

Mark: __________

Graded by: __________

Assignment Code: ________

Date Lesson Received:

____

Lesson Recorded ________

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

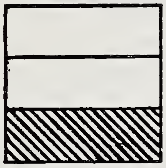
When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

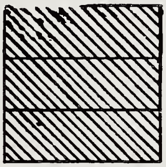
Fractions and Mixed Numerals

Topic One: Fractions

A fraction represents a part of a whole.



Looking at the above diagram, you can see that the whole object has been divided into three equal parts. One of these parts is shaded. If we compare the shaded part to the whole we are thinking about 1 part out of 3. We can express this by writing $\frac{1}{3}$. $\frac{1}{3}$ is a fraction.



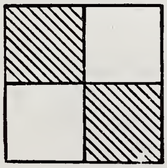
Again, the whole is divided into 3 parts. All 3 parts are shaded. If we compare the shaded parts to the whole, we are thinking about 3 parts out of 3. We can express this by writing $\frac{3}{3}$. $\frac{3}{3}$ is a fraction.

The upper part of a fraction is called the NUMERATOR. The lower part of a fraction is called the DENOMINATOR.

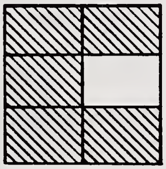
e.g. $\frac{2}{7}$ ← numerator ← denominator

Self-correcting Exercise 1

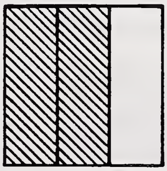
Below each diagram, use a fraction to compare the shaded parts to the whole.

1. 

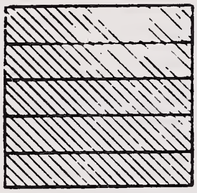
____

2. 

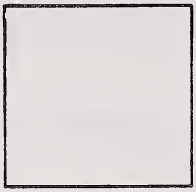
____

3. 

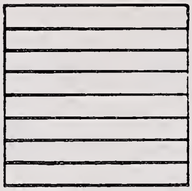
____

4.


____

5.


____

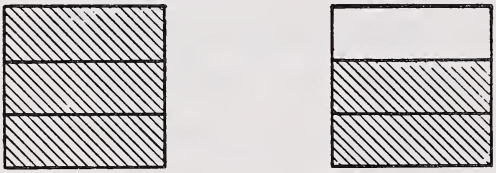
6.


____

Corrections are on page 28.

Topic Two: Mixed numerals

A mixed numeral is a natural number and a fraction.



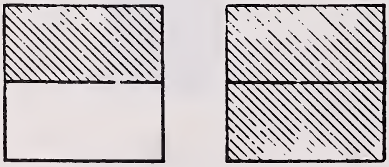
Now let us consider two whole objects, each of which is divided into 3 parts. If we compare the shaded parts of the first object to the whole of the first object, we can write the fraction $\frac{3}{3}$. If we compare the shaded parts of the second object to the whole of the second object, we can write the fraction $\frac{2}{3}$ of an object. If we consider these two objects together, we have a whole object and $\frac{2}{3}$ of an object.

$$1 + \frac{2}{3} = 1\frac{2}{3}$$

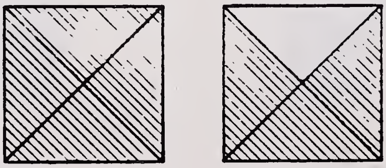
We call the number $1\frac{2}{3}$ a MIXED NUMERAL because it is composed of two different parts: a natural number and a fraction.

Assignment 1 Each 1" square represents a whole object.

Below each diagram, write a mixed numeral that tells how many wholes and parts of a whole are represented.

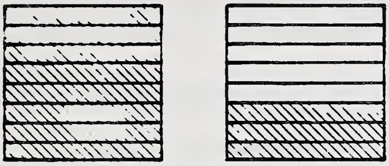
1.


$1\frac{1}{2}$

2.


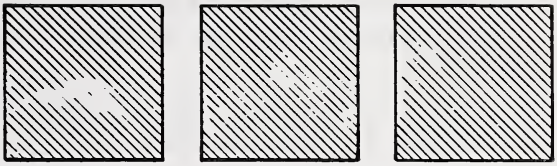
____

3.



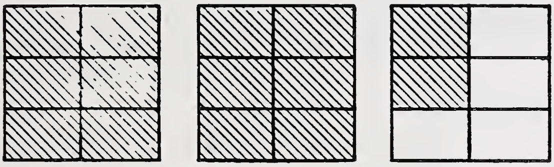
____

4.



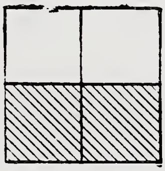
____

5.

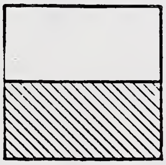


____

Topic Three: Equivalent fractions



$\frac{2}{4}$



$\frac{1}{2}$

Look at the two diagrams above. The fractions are different but the same amount of each object is shaded. We can say that $\frac{1}{2} = \frac{2}{4}$. $\frac{1}{2}$ and $\frac{2}{4}$ are called equivalent fractions.

Notice that the numerator of $\frac{1}{2}$ is multiplied by 2 to obtain the numerator of $\frac{2}{4}$; the denominator of $\frac{1}{2}$ is multiplied by 2 to obtain the denominator of $\frac{2}{4}$.

$$\frac{1 \times 2}{2 \times 2} = \frac{2}{4}$$

To obtain an equivalent fraction, both the numerator and denominator of the original fraction must be multiplied or divided by the same number.

e.g. $\frac{7}{6} = \frac{7 \times 4}{6 \times 4} = \frac{28}{24}$ $\left\{\begin{array}{l}\text{Both 7 and 6 are multiplied by 4.}\\ \frac{7}{6} \text{ and } \frac{28}{24} \text{ are equivalent fractions.}\end{array}\right.$

$\frac{30}{36} = \frac{30 \div 6}{36 \div 6} = \frac{5}{6}$ $\left\{\begin{array}{l}\text{Both 30 and 36 are divided by 6.}\\ \frac{30}{36} \text{ and } \frac{5}{6} \text{ are equivalent fractions.}\end{array}\right.$

We can use this idea of equivalent fractions to express a fraction with a different denominator.

EXAMPLE. Change $\frac{5}{6}$ to twenty-fourths.

Solution

$\frac{5}{6} = \frac{?}{24}$ We must multiply the denominator by 4 in order to change the fraction from sixths to twenty-fourths. To ensure that the fractions are equivalent, the numerator must also be multiplied by 4.

$\frac{5}{6} = \underline{\frac{20}{24}}$ $\left(\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}\right)$

Self-correcting Exercise 2

Find the missing numerators or denominators in the fractions below.

1. $\frac{1}{4} = \frac{\quad}{40}$
2. $\frac{12}{16} = \frac{\quad}{4}$
3. $\frac{9}{6} = \frac{3}{\quad}$
4. $\frac{5}{8} = \frac{\quad}{24}$
5. $\frac{35}{5} = \frac{7}{\quad}$
6. $\frac{8}{9} = \frac{32}{\quad}$

Corrections are on page 28.

Natural numbers can be written as fractions. Any natural number can be expressed as a fraction with a denominator of 1.

e.g.

1 can be written as the fraction $\frac{1}{1}$.

2 can be written as the fraction $\frac{2}{1}$.

3 can be written as the fraction $\frac{3}{1}$.

In fact, a natural number may be written as a fraction with any denominator you wish. First, you must write the natural number with a denominator of 1. Then, multiply the numerator and denominator by the same number.

EXAMPLE: Change 6 to eighths.

Solution

6 is the same as $\frac{6}{1}$.

$\frac{6}{1} = \frac{?}{8}$ — We must multiply the denominator by 8 to change the fraction to eighths. Therefore, the numerator, 6 must also be multiplied by 8.

$\frac{6}{1} = \underline{\frac{48}{8}}$ $\left(\frac{6}{1} = \frac{6 \times 8}{1 \times 8} = \frac{48}{8}\right)$

Assignment 2

Decide whether each of the following sentences expresses a true statement or a false statement.

1. $\frac{4}{3} = \frac{12}{9}$ true $\frac{4}{3} = \frac{4 \times 3}{3 \times 3} = \frac{12}{9}$

2. $\frac{20}{16} = \frac{5}{4}$ ____________________

3. $\frac{6}{8} = \frac{4}{6}$ ____________________

4. $\frac{2}{1} = \frac{1}{2}$ ____________________

5. $\frac{0}{5} = \frac{0}{8}$ ____________________

Tell how each fraction was obtained from $\frac{30}{50}$.

6. $\frac{15}{25}$ Divide 30 and 50 by 2. $\frac{30}{50} = \frac{30 \div 2}{50 \div 2} = \frac{15}{25}$

7. $\frac{6}{10}$ ____________________

8. $\frac{60}{100}$ ____________________

9. $\frac{3}{5}$ ____________________

10. $\frac{90}{150}$ ____________________

Fill in the numerator or denominator to produce a fraction which is equivalent to the given fraction.

11. $15 = \frac{15}{1}$

12. $6 = \frac{\quad}{2}$

13. $38 = \frac{\quad}{1}$

14. $59 = \frac{118}{\quad}$

15. $505 = \frac{1515}{\quad}$

Topic Four: Fractions and mixed numerals

For future work you need to know how to convert mixed numerals to fractions.

A mixed numeral is actually the sum of a natural number and a fraction.

e.g. $1\frac{2}{3} = 1 + \frac{2}{3}$

To change a mixed numeral to a fraction, we must think of the mixed numeral as being the sum of a natural number and a fraction. We change the natural number to a fraction with the same denominator as the fractional part of the mixed numeral. Then we add the two fractions.

EXAMPLE: Change $4\frac{1}{2}$ to a fraction.

Solution

Change the natural number 4 to halves.

$$4 = \frac{4}{1} = \frac{8}{2}$$

Now, both parts of the mixed numeral, $4\frac{1}{2}$, can be expressed in halves and we can find the sum of these two parts.

$$4\frac{1}{2} = \underbrace{4 + \frac{1}{2}}_{\text{Write as a sum.}} = \underset{\substack{\uparrow \\ \text{Change the natural number to halves.}}}{\frac{8}{2}} + \frac{1}{2} = \underline{\frac{9}{2}}$$

Thus $4\frac{1}{2} = \frac{9}{2}$. Note that the same result could be obtained in the following manner. Multiply the natural number 4 by the denominator 2 and add the numerator 1 in order to find the numerator of the resulting fraction. The denominator of the resulting fraction is the same as the denominator of the fractional part of the mixed numeral.

$$4\frac{1}{2} = \frac{(4 \times 2) + 1}{2} = \frac{9}{2}$$

Similarly, $2\frac{5}{6} = \frac{(2 \times 6) + 5}{6} = \frac{17}{6}$

$$1\frac{1}{8} = \frac{(1 \times 8) + 1}{8} = \frac{9}{8}$$

Self-correcting Exercise 3

Change each mixed numeral to a fraction.

1. $2\frac{1}{8}$ ____________________
2. $3\frac{3}{5}$ ____________________
3. $8\frac{1}{2}$ ____________________
4. $4\frac{4}{9}$ ____________________
5. $5\frac{3}{7}$ ____________________

Corrections are on page 29.

You also need to know how to change fractions (with numerators that are larger than the denominators) to mixed numerals.

To change a fraction to a mixed numeral, we must split the fraction into two parts. One part will equal a whole number and the other will equal a fraction less than 1.

EXAMPLE: Change $\frac{14}{3}$ to a mixed numeral.

Solution

Note that 12 is the largest multiple of 3 that is still less than 14.

$$\frac{14}{3} = \frac{12+2}{3}$$

$$= \frac{12}{3} + \frac{2}{3} \quad \begin{array}{l} \rightarrow \text{natural number} \\ \rightarrow \text{fraction less than 1} \end{array}$$

$$= 4 + \frac{2}{3}$$

$$= 4\frac{2}{3}$$

Thus $\frac{14}{3} = \underline{4\frac{2}{3}}$. The same result could be obtained by dividing the denominator 3 into the numerator 14. The quotient obtained gives the whole number part of the mixed numeral. The fractional part can be obtained by placing the remainder 2 over the denominator 3.

$\frac{14}{3} = 4\frac{2}{3}$ $\qquad$ 14 ÷ 3 = 4 remainder 2

Similarly,

$\frac{21}{4} = 5\frac{1}{4}$ $\qquad$ 21 ÷ 4 = 5 remainder 1

$\frac{36}{8} = 4\frac{4}{8}$ $\qquad$ 36 ÷ 8 = 4 remainder 4

Self-correcting Exercise 4

Change each fraction to a mixed numeral.

1. $\frac{14}{5}$ ____________________

2. $\frac{17}{13}$ ____________________

3. $\frac{10}{3}$ ____________________

4. $\frac{29}{9}$ ____________________

5. $\frac{37}{6}$ ____________________

Corrections are on page 29.

Assignment 3

Write each of the sums as a mixed numeral.

1. $5 + \frac{7}{8} =$ $5\frac{7}{8}$ ____________________

2. $3 + \frac{4}{5} =$ ____________________

3. $6 + \frac{2}{3} =$ ____________________

4. $\frac{7}{15} + 11 =$ ____________________

5. $10 + \frac{6}{8} =$ ____________________

Change each mixed numeral to a fraction.

6. $8\frac{6}{7} =$ $\frac{62}{7}$ __________ $8\frac{6}{7} = \frac{(8\times7)+6}{7} = \frac{62}{7}$

7. $10\frac{3}{4} =$ __________

8. $9\frac{2}{3} =$ __________

9. $4\frac{1}{16} =$ __________

10. $12\frac{5}{9} =$ __________

Change each fraction to a mixed numeral.

11. $\frac{11}{4}$ = $2\frac{3}{4}$ $4\overset{2}{\overline{)11}}$ remainder 3

12. $\frac{12}{5}$ = ________

13. $\frac{13}{8}$ = ________

14. $\frac{16}{7}$ = ________

15. $\frac{25}{11}$ = ________

Topic Five: Review of Addition and Subtraction

Fractions can only be added or subtracted if they have the same denominator. If two fractions that you are adding or subtracting have unlike denominators, you must rewrite them as equivalent fractions with the same denominator. This denominator should be the smallest number that both of the original denominators will divide evenly into. It is called the least common denominator (L.C.D.) of the two fractions. We can find this L.C.D. in the same way we found the least common multiple of two numbers in Lesson 6.

Once you have found the L.C.D., you can rewrite the given fractions so that they are expressed in terms of the L.C.D. Then, you can add or subtract as required.

EXAMPLE: Find the sum of $\frac{3}{4}$ and $\frac{5}{6}$.

Solution

Multiply the largest denominator, 6, by 1, 2, 3 and so on until you arrive at a number that the other denominator, 4, will divide into.

$6 \times 1 = 6$

$6 \times 2 = 12$ ← We can stop here, since 4 divides into 12.

Thus, the smallest number that both 4 and 6 will divide into is 12.

Now, both $\frac{3}{4}$ and $\frac{5}{6}$ must be written as twelfths.

$$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$$

$$\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{10}{12}$$

Finally, we can add.

$$\left.\begin{array}{rcl} \frac{3}{4} & = & \frac{9}{12} \\[1ex] \frac{5}{6} & = & \frac{10}{12} \end{array}\right\} \text{L.C.D. is 12. Rewrite both fractions as twelfths.}$$

$$\frac{19}{12} = \underline{1\frac{7}{12}}$$

The same procedure can be used for adding or subtracting mixed numerals.

EXAMPLE: Find the sum of $19\frac{5}{6}$ and $13\frac{3}{8}$.

Solution

First, find the least common multiple of the denominators, 6 and 8.

$8 \times 1 = 8$ 8 is not a multiple of 6.
$8 \times 2 = 16$ 16 is not a multiple of 6.
$8 \times 3 = 24$ 24 is a multiple of 6.

The least common multiple of 6 and 8 is 24.
The least common denominator of $\frac{5}{6}$ and $\frac{3}{8}$ is 24.

$\frac{5}{6}$ can be written in 24ths if its numerator and denominator are multiplied by 4. $\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}$

$\frac{3}{8}$ can be written in 24ths if its numerator and denominator are multiplied by 3. $\frac{3}{8} = \frac{3 \times 3}{8 \times 3} = \frac{9}{24}$

Now, we can add.

$$\begin{array}{rcl} 19\frac{5}{6} & = & 19\frac{20}{24} \\[1ex] 13\frac{3}{8} & = & 13\frac{9}{24} \\ \hline & & 32\frac{29}{24} = 32 + (1\frac{5}{24}) = 33\frac{5}{24} \end{array}$$

Sum of numerators

L.C.D.

$\frac{29}{24}$ is expressed as a mixed numeral.

The sum of $19\frac{5}{6}$ and $13\frac{3}{8}$ is $\underline{33\frac{5}{24}}$.

Self-correcting Exercise 5

Find the sums.

1. $\begin{array}{r} 10\frac{3}{8} \\[1ex] 7\frac{5}{8} \\ \hline \end{array}$

2. $\begin{array}{r} 20 \\[1ex] 3\frac{1}{2} \\ \hline \end{array}$

3. $$\begin{array}{r} 14\frac{4}{5} \\ \underline{13\frac{7}{8}} \end{array}$$

4. $$\begin{array}{r} 36\frac{3}{4} \\ \underline{18\phantom{\frac{3}{4}}} \end{array}$$

5. $$\begin{array}{r} 4\frac{2}{3} \\ \underline{1\frac{2}{5}} \end{array}$$

6. $$\begin{array}{r} 8\frac{1}{4} \\ \underline{6\frac{3}{8}} \end{array}$$

Corrections are on page 29.

The difference of two fractions is found in the same way as the sum except that the numerator of the difference is the difference between the numerators.

EXAMPLE: Find the difference between $11\frac{1}{2}$ and $7\frac{11}{12}$.

Solution

First, find the L.C.D. of $\frac{1}{2}$ and $\frac{11}{12}$.

$12 \times 1 = 12$ 12 is a multiple of 2.

12 is the least common denominator of $\frac{1}{2}$ and $\frac{11}{12}$.

$$\begin{array}{rcr} 11\frac{1}{2} & = & 11\frac{6}{12} \\ 7\frac{11}{12} & = & 7\frac{11}{12} \\ \hline \end{array}$$

We cannot subtract $\frac{11}{12}$ from $\frac{6}{12}$. We must change $11\frac{6}{12}$ to $10 + 1\frac{6}{12}$ which is the same as $10\frac{18}{12}$.

$$\begin{array}{rcrcr} 11\frac{1}{2} & = & 11\frac{6}{12} & = & 10\frac{18}{12} \\ 7\frac{11}{12} & = & 7\frac{11}{12} & = & 7\frac{11}{12} \\ \hline & & & & \underline{\underline{3\frac{7}{12}}} \end{array}$$

$7 \longleftarrow$ difference between the numerators

$12 \longleftarrow$ least common denominator

Assignment 4

Find the differences.

1. $$\begin{array}{rl} \frac{3}{4} = & \frac{9}{12} \\ \frac{1}{3} = & \frac{4}{12} \\ \hline & \frac{5}{12} \end{array}$$

2. $$\begin{array}{r} 5 \\ \frac{6}{12} \\ \hline \end{array}$$

3. $$\begin{array}{r} 1\frac{1}{4} \\ \frac{5}{8} \\ \hline \end{array}$$

4. $$\begin{array}{r} 7\frac{6}{7} \\ 3\frac{2}{3} \\ \hline \end{array}$$

5. $$\begin{array}{r} 17\frac{5}{6} \\ 15\frac{8}{9} \\ \hline \end{array}$$

6. $$\begin{array}{r} 17\frac{7}{10} \\ 14\frac{9}{10} \\ \hline \end{array}$$

Find the sums.

7. $$\begin{array}{rl} \frac{8}{12} = & \frac{8}{12} \\ \frac{9}{6} = & \frac{18}{12} \\ \hline & \frac{26}{12} = 2\frac{2}{12} = 2\frac{1}{6} \end{array}$$

8. $$\begin{array}{r} 1\frac{1}{2} \\ \frac{3}{10} \\ \hline \end{array}$$

9. $$\begin{array}{r} 2\frac{1}{3} \\ 4\frac{5}{9} \\ \hline \end{array}$$

10. $$\begin{array}{r} 4\frac{3}{5} \\ \frac{1}{3} \\ \hline \end{array}$$

Topic Six: Multiplication

We can use what we know about multiplying whole numbers to help us multiply fractions.

$$2 \times 3 = 6$$

We know that $\frac{4}{2}$ is equal to 2. We also know that $\frac{9}{3}$ is equal to 3. If we use these as replacements for 2 and 3, the product must still be 6. We can obtain a product of 6 if we multiply the numerators of $\frac{4}{2}$ and $\frac{9}{3}$ and divide this result by the product of the denominators.

$$\frac{4}{2} \times \frac{9}{3} = \frac{\overset{\xleftarrow{\text{Product of numerators}}}{4 \times 9}}{\underset{\xleftarrow{\text{Product of denominators}}}{2 \times 3}} = \frac{36}{6} \qquad \text{This product equals 6.}$$

To multiply fractions, we multiply the numerators to obtain the numerator of the fraction which is the product. We multiply the denominators to obtain the denominator of the fraction which is the product.

e.g. $\frac{5}{3} \times \frac{7}{6} = \frac{5 \times 7}{3 \times 6} = \frac{35}{18}$

Self-correcting Exercise 6

Write "T" after the statement if it is true or "F" after the statement if it is false. If the statement is false, correct it.

1. $\frac{7}{8} \times \frac{1}{4} = \frac{7}{32}$

2. $\frac{2}{3} \times \frac{1}{3} = \frac{2}{6}$

3. $\frac{16}{5} \times \frac{3}{10} = \frac{48}{50}$

4. $\frac{0}{6} \times \frac{6}{1} = \frac{6}{6}$

5. $\frac{4}{3} \times \frac{1}{9} = \frac{5}{12}$

6. $\frac{11}{12} \times \frac{12}{11} = \frac{132}{132}$

Corrections are on page 30.

Assignment 5

Find the product.

1. $\frac{7}{8} \times \frac{1}{2} =$ $\frac{7 \times 1}{8 \times 2} = \frac{7}{16}$

2. $\frac{4}{3} \times \frac{2}{5} =$ ______________________

3. $\frac{9}{10} \times \frac{0}{5} =$ ______________________

4. $\frac{18}{7} \times \frac{2}{5} =$ ______________________

5. $\frac{100}{1} \times \frac{3}{7} =$ ______________________

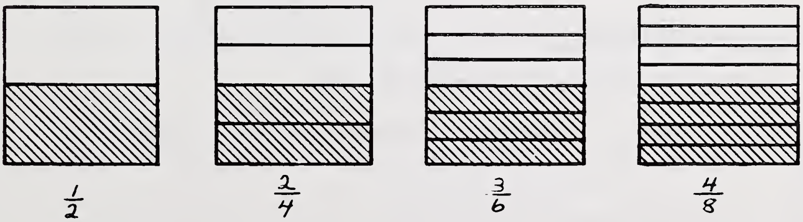
6. $\frac{15}{17} \times \frac{12}{7} =$ ______________________

7. $\frac{1}{3} \times \frac{1}{2} =$ ______________________

8. $\frac{4}{5} \times \frac{1}{5} =$ ______________________

9. $\frac{37}{3} \times \frac{1}{8} =$ ______________________

10. $\frac{5}{2} \times \frac{5}{2} =$ ______________________



Below each diagram is written the fraction which compares the shaded parts to the whole.

You will notice from the above diagrams that the same amount of the whole is shaded in each case. Therefore we can say that these fractions are equal. $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8}$

We say that a fraction such as $\frac{1}{2}$ is in its simplest form or that it is in its lowest terms. You will notice that the only common factor of 1 and 2 is 1. In other words when a numerator and a denominator of a fraction have no common factors other than 1, the fraction is in its lowest terms.

EXAMPLE: Is $\frac{7}{21}$ in its lowest terms?

Solution

The factors of 7 are 1 and 7.
The factors of 21 are 1, 3, 7 and 21.
The common factors of 7 and 21 are 1 and 7.

Since 7 and 21 have a common factor other than 1, $\frac{7}{21}$ is not in its lowest terms.

To find the simplest form of a fraction, we divide the numerator and denominator of the fraction by the greatest common factor of the numerator and denominator.

EXAMPLE: Express $\frac{4}{8}$ in its simplest form.

Solution

$$\left.\begin{aligned} 4 &= 2 \times 2 \\ 8 &= 2 \times 4 = 2 \times 2 \times 2 \end{aligned}\right\} \text{Complete factorization of the numerator and denominator}$$

The greatest common factor of 4 and 8 is 2 × 2 or 4. (See Lesson 5)
Divide the numerator and denominator by 4.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \underline{\frac{1}{2}}$$

$\frac{1}{2}$ is in lowest terms because the only common factor of 1 and 2 is 1.

Self-correcting Exercise 7

Express each fraction in its lowest terms.

1. $\frac{30}{50}$ = ____________
2. $\frac{70}{35}$ = ____________
3. $\frac{10}{4}$ = ____________
4. $\frac{5}{15}$ = ____________
5. $\frac{9}{25}$ = ____________

Corrections are on page 30.

There is a short-cut method for finding the product of two fractions that always results in a product that is expressed in lowest terms. Using this method, we can simplify the numerator and denominator of the product before the multiplication is completed.

EXAMPLE: Find the product of $\frac{3}{4}$ and $\frac{8}{9}$.

Solution

Write $\frac{3}{4}$ and $\frac{8}{9}$ in product form.

$$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times 8}{4 \times 9}$$

8 in the numerator and 4 in the denominator have a common factor of 4. Divide 8 and 4 by 4. Put a small 2 above the 8 to show that 8 ÷ 4 = 2. Put a small 1 below the 4 to show that 4 ÷ 4 = 1.

$$\frac{3}{4} \times \frac{8}{9} = \frac{3 \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times 9}$$

3 in the numerator and 9 in the denominator have a common factor of 3. Divide 3 and 9 by 3. Put a small 1 above the 3 to show that 3 ÷ 3 = 1. Put a small 3 below the 9 to show that 9 ÷ 3 = 3.

$$\frac{3}{4} \times \frac{8}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{3}{\cancel{9}}}$$

Now write the product in its simplified form and multiply.

$$\frac{3}{4} \times \frac{8}{9} = \frac{\overset{1}{\cancel{3}} \times \overset{2}{\cancel{8}}}{\underset{1}{\cancel{4}} \times \underset{3}{\cancel{9}}} = \frac{1 \times 2}{1 \times 3} = \frac{2}{3}$$

Assignment 6

Express each fraction in its lowest terms.

1. $\frac{3}{18}$ = $\frac{1}{6}$ $\frac{3}{18} = \frac{3 \div 3}{18 \div 3} = \frac{1}{6}$

2. $\frac{12}{16}$ = ____________

3. $\frac{18}{4}$ = ____________

4. $\frac{56}{64}$ = ____________

5. $\frac{60}{25}$ = ____________

6. $\frac{100}{10}$ = ____________________

7. $\frac{17}{70}$ = ____________________

8. $\frac{120}{30}$ = ____________________

Find each product. Make sure your answers are in simplest form.

9. $\frac{6}{7} \times \frac{14}{33}$ = $\frac{\overset{2}{\cancel{6}} \times \overset{2}{\cancel{14}}}{\underset{1}{\cancel{7}} \times \underset{11}{\cancel{33}}} = \frac{2 \times 2}{1 \times 11} = \frac{4}{11}$

10. $\frac{3}{4} \times \frac{7}{1}$ = ____________________

11. $\frac{25}{6} \times \frac{8}{15}$ = ____________________

12. $\frac{3}{9} \times \frac{9}{3}$ = ____________________

13. $\frac{50}{1} \times \frac{9}{30}$ = ____________________

14. $\frac{3}{6} \times \frac{9}{12}$ = ____________________

15. $\frac{4}{4} \times \frac{1}{5}$ = ____________________

16. $\frac{3}{4} \times \frac{2}{8}$ = ____________________

Since we know how to change mixed numerals to fractions and we know how to find the product of two fractions, we can now find the product of two mixed numerals.

To find the product of two mixed numerals, we change the mixed numerals to fractions and then multiply. The product can be expressed as a fraction or as a mixed numeral.

EXAMPLE: Find the product of $2\frac{5}{8}$ and $3\frac{1}{9}$.

Solution

$2\frac{5}{8} \times 3\frac{1}{9} = \frac{21}{8} \times \frac{28}{9}$ Change the mixed numerals to fractions.

$= \frac{\overset{7}{\cancel{21}} \times \overset{7}{\cancel{28}}}{\underset{2}{\cancel{8}} \times \underset{3}{\cancel{9}}}$ $\begin{cases} \text{3 is a common factor of 21 and 9.} \\ \text{4 is a common factor of 28 and 8.} \end{cases}$

$= \frac{7 \times 7}{2 \times 3}$

$= \underline{\frac{49}{6}}$

$= \underline{8\frac{1}{6}}$

The product is $\underline{\frac{49}{6}}$ or $\underline{8\frac{1}{6}}$.

Assignment 7

Express each fraction as a mixed numeral.

1. $\frac{31}{7}$ = $4\frac{3}{7}$ $7\overline{)31}^{\,4}$ remainder 3

2. $\frac{47}{8}$ = ______

3. $\frac{87}{12}$ = ______

4. $\frac{130}{30}$ = ______

Express each mixed numeral as a fraction.

5. $1\frac{7}{19}$ = $\frac{26}{19}$ $1\frac{7}{19} = \frac{(1 \times 19) + 7}{19} = \frac{26}{19}$

6. $3\frac{5}{12}$ = ______

7. $2\frac{4}{7}$ = ______

8. $12\frac{3}{5}$ = ______

Find each product. Express the product in its simplest form. If possible, express the product as a mixed numeral.

9. $2\frac{1}{2} \times 3\frac{1}{5}$ = $\frac{5}{2} \times \frac{16}{5} = \frac{\overset{1}{\cancel{5}} \times \overset{8}{\cancel{16}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{5}}} = \frac{1 \times 8}{1 \times 1} = \frac{8}{1} = 8$

10. $7 \times \frac{5}{6}$ = ____________________

11. $5 \times 1\frac{1}{4}$ = ____________________

12. $5\frac{2}{5} \times 1\frac{1}{8}$ = ____________________

13. $6\frac{3}{5} \times 2\frac{1}{10}$ = ____________________

14. $7 \times \frac{5}{11}$ = ____________________

15. $\frac{27}{9} \times \frac{14}{35}$ = ____________________

16. $3\frac{4}{7} \times 2\frac{4}{5}$ = ____________________

17. $\frac{19}{3} \times \frac{4}{6}$ = ____________________

18. $\frac{16}{15} \times \frac{27}{24}$ = ____________________

19. $3\frac{1}{8} \times 4\frac{4}{5}$ = ____________________

20. $2\frac{1}{3} \times 5\frac{1}{4}$ = ____________________

Topic Seven: Division

In order to understand division of fractions, we must understand division by one and the meaning of reciprocal.

Let's look at some examples of division by 1.

$$\begin{aligned} 13 \div 1 &= 13 \\ 6 \div 1 &= 6 \\ 263 \div 1 &= 263 \\ 1 \div 1 &= 1 \end{aligned}$$

You will notice that when you divide whole numbers by 1, the quotient is the same as the dividend. The same holds true for fractions. The quotient must be the same as the dividend when the divisor is 1.

e.g. $\frac{2}{3} \div 1 = \frac{2}{3}$

$\frac{3}{4} \div 1 = \frac{3}{4}$

We can write division expressions in another way. You know that $\frac{19}{6} = 3\frac{1}{6}$ because $19 \div 6 = 3$, remainder 1. $\frac{19}{6}$ can mean $19 \div 6$.

Therefore $\frac{\frac{2}{3}}{1}$ can mean $\frac{2}{3} \div 1$. $\frac{\frac{2}{3}}{1} = \frac{2}{3}$ because $\frac{2}{3} \div 1 = \frac{2}{3}$.

Let's see what is meant by reciprocal.

e.g. $\frac{3}{7} \times \frac{7}{3} = \frac{21}{21} = 1$

Since $\frac{3}{7} \times \frac{7}{3} = 1$, we can say that $\frac{3}{7}$ is the reciprocal of $\frac{7}{3}$ and $\frac{7}{3}$ is the reciprocal of $\frac{3}{7}$. Numbers are RECIPROCALS of each other, if their product is 1.

You will notice that to find the reciprocal of a fraction you need only to use the original denominator as the numerator and the original numerator as the denominator.

e.g $$\begin{tikzcd} 4 \arrow[dr, leftrightarrow] & 6 \\ 6 \arrow[ur, leftrightarrow] & 4 \end{tikzcd}$$ $\frac{4}{6}$ and $\frac{6}{4}$ are reciprocals because $\frac{4}{6} \times \frac{6}{4} = \frac{24}{24} = 1$.

Self-correcting Exercise 8

Name the reciprocal of each fraction.

1. $\frac{7}{8}$ ________

2. $\frac{14}{3}$ ______________________________

3. $\frac{27}{61}$ ______________________________

4. $\frac{652}{1865}$ ______________________________

5. $\frac{1}{100}$ ______________________________

Corrections are on page 31.

Now, let us see how we can apply these ideas about division by 1 and reciprocals to division of fractions.

EXAMPLE: $\frac{2}{3} \div \frac{4}{7}$

Solution

Write $\frac{2}{3} \div \frac{4}{7}$ as $\frac{\frac{2}{3}}{\frac{4}{7}}$.

We know that if we multiply the numerator and denominator of a fraction by the same number, the fraction is not changed. We will choose a number so that the product of it and $\frac{4}{7}$ is 1. This number must be the reciprocal of $\frac{4}{7}$. The reciprocal of $\frac{4}{7}$ is $\frac{7}{4}$.

$$\frac{\frac{2}{3}}{\frac{4}{7}} = \frac{\frac{2}{3} \times \frac{7}{4}}{\frac{4}{7} \times \frac{7}{4}} = \frac{\frac{2}{3} \times \frac{7}{4}}{1}$$

Using what we know about division by 1, we can change $\frac{\frac{2}{3} \times \frac{7}{4}}{1}$ to $\frac{2}{3} \times \frac{7}{4}$.

Now, we have turned the original division question, $\frac{2}{3} \div \frac{4}{7}$, into the multiplication question $\frac{2}{3} \times \frac{7}{4}$. Since we already know how to multiply fractions, our difficulties are over.

$$\underbrace{\frac{2}{3} \div \frac{4}{7} = \frac{2}{3} \times \frac{7}{4}} = \frac{\overset{1}{\cancel{2}} \times 7}{3 \times \underset{2}{\cancel{4}}} = \frac{1 \times 7}{3 \times 2} = \underline{\frac{7}{6}}$$

Note that we can get from the division question to the multiplication question simply by changing the operation to multiplication and giving the reciprocal of the divisor.

We can condense the procedure for dividing fractions to a short rule.

> In order to divide one fraction by another, change the operation to multiplication and give the reciprocal of the divisor.

EXAMPLE: $\frac{7}{8} \div \frac{21}{40}$

Solution

$$\frac{7}{8} \div \frac{21}{40} = \frac{7}{8} \times \frac{40}{21} = \underbrace{\frac{\overset{1}{\cancel{7}} \times \overset{5}{\cancel{40}}}{\underset{1}{\cancel{8}} \times \underset{3}{\cancel{21}}} = \frac{1 \times 5}{1 \times 3} = \frac{5}{3} = \underline{1\frac{2}{3}}}_{\text{Proceed as you would when multiplying fractions.}}$$

Change operation to multiplication.

Give reciprocal of divisor.

Self-correcting Exercise 9

Find the quotients.

1. $\frac{5}{2} \div \frac{3}{8} =$ ____________________
2. $\frac{0}{6} \div \frac{2}{3} =$ ____________________
3. $\frac{15}{4} \div \frac{7}{3} =$ ____________________
4. $\frac{5}{6} \div \frac{5}{6} =$ ____________________
5. $\frac{1}{9} \div \frac{9}{1} =$ ____________________

Corrections are on page 31.

Assignment 8

Name the reciprocal of each number.

1. $\frac{6}{12}$ $\frac{12}{6}$
2. $\frac{1}{9}$ ____________________
3. $\frac{5}{2}$ ____________________

4. $\frac{71}{4}$ ________________________________

5. $\frac{3}{20}$ ________________________________

Find each quotient. Express it in its simplest form. If possible, express it as a mixed numeral.

6. $\frac{4}{4} \div \frac{3}{3} = \frac{4}{4} \times \frac{3}{3} = \frac{\overset{1}{\cancel{4}} \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}} \times \underset{1}{\cancel{3}}} = \frac{1 \times 1}{1 \times 1} = \frac{1}{1} = 1$

7. $\frac{7}{2} \div \frac{6}{2} =$ ________________________________

8. $\frac{9}{10} \div \frac{9}{20} =$ ________________________________

9. $\frac{14}{3} \div \frac{6}{5} =$ ________________________________

10. $\frac{7}{8} \div \frac{1}{4} =$ ________________________________

Since we know how to change mixed numerals to fractions and we know how to find the quotient of two fractions, we can now find the quotient of two mixed numerals.

To find the quotient of two mixed numerals, we change the mixed numerals to fractions and multiply by the reciprocal of the divisor.

EXAMPLE: $5\frac{1}{2} \div 3\frac{2}{3}$

Solution

$$5\frac{1}{2} \div 3\frac{2}{3} = \underbrace{\frac{11}{2} \div \frac{11}{3}}_{\text{Change mixed numerals to fractions.}} = \frac{11}{2} \times \overbrace{\frac{3}{11}}^{\text{Multiply by reciprocal of divisor}} = \frac{\overset{1}{\cancel{11}} \times 3}{2 \times \underset{1}{\cancel{11}}} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2} \text{ or } 1\frac{1}{2}$$

The quotient is $\underline{\frac{3}{2} \text{ or } 1\frac{1}{2}}$.

Assignment 9

Name the reciprocal of each number.

1. 3 $3 = \frac{3}{1}$ Reciprocal is $\frac{1}{3}$.

2. $6\frac{7}{9}$ __________

3. $1\frac{2}{5}$ __________

4. $2\frac{1}{3}$ __________

5. $9\frac{1}{2}$ __________

6. $6\frac{4}{5}$ __________

Find each quotient. Make sure your answer is in simplest form. If possible, express the answer as a mixed numeral.

7. $\frac{2}{3} \div 1\frac{3}{4} =$ $\frac{2}{3} \div \frac{7}{4} = \frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$

8. $18 \div \frac{4}{9} =$ $\frac{18}{1} \div \frac{4}{9} =$

9. $\frac{1}{2} \div \frac{7}{8} =$ __________

10. $\frac{4}{5} \div 6 =$ __________

11. $1\frac{3}{4} \div \frac{1}{5} =$ __________

12. $8 \div 1\frac{2}{5} =$ __________

13. $2\frac{1}{11} \div 3 =$ __________

14. $9\frac{1}{2} \div 4\frac{3}{4} =$ __________

Assignment 10

For each problem:

- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. After Mrs. Jones had bought $\frac{1}{2}$ yd. of oilcloth, she had $2\frac{1}{4}$ yd. of oilcloth in all. How much oilcloth did she have before she bought the $\frac{1}{2}$ yd.?

Placeholder: Let x be the number of yards of oilcloth she had before she bought $\frac{1}{2}$ yd.

Equation: $x + \frac{1}{2} = 2\frac{1}{4}$

Computation

$$\begin{array}{rcccl} 2\frac{1}{4} & = & 2\frac{1}{4} & = & 1\frac{5}{4} \\ \frac{1}{2} & = & \frac{2}{4} & = & \frac{2}{4} \\ \hline & & & & 1\frac{3}{4} \end{array}$$

Answer: $x = 1\frac{3}{4}$

Statement: Mrs. Jones had $1\frac{3}{4}$ yd. before she bought $\frac{1}{2}$ yd.

2. Jill bought 2 cans of salted nuts. One can contained $\frac{7}{8}$ lb. of nuts. The other can contained $\frac{5}{8}$ lb. How many pounds of salted nuts did Jill buy?

Placeholder: Let y be ______________________

Equation:

Computation

Answer:

Statement: ______________________

3. Tom lost $9\frac{1}{2}$ lb. during the baseball season. At the end of the season he weighed $145\frac{3}{4}$ lb. How much did Tom weigh before he lost the $9\frac{1}{2}$ lb.?

Placeholder: Let w be ____________________

Equation: Computation

Answer:

Statement: ____________________

4. Mrs. Allen used $\frac{2}{3}$ cup of carrots, $\frac{2}{3}$ cup of potatoes and $\frac{1}{4}$ cup of onion for a meat and vegetable pie. How many cups of vegetables did she use for the pie?

Placeholder: Let n be ____________________

Equation: Computation

Answer:

Statement: ____________________

5. Kurt worked in his father's store $1\frac{1}{2}$ hours on Tuesday, $1\frac{3}{4}$ hours on Thursday and $4\frac{3}{4}$ hours on Saturday. How many hours did Kurt work in the store on these three days?

Placeholder: Let h be ____________________

Equation: Computation

Answer:

Statement: ____________________

Key to Self-correcting Exercises in Lesson 11

Exercise 1, page 1

1. $\frac{2}{4}$ 2. $\frac{5}{6}$ 3. $\frac{2}{3}$ 4. $\frac{5}{5}$ 5. $\frac{0}{1}$ 6. $\frac{0}{8}$

Exercise 2, page 4

1. $\frac{1}{4} = \frac{10}{40}$ $\left(\frac{1}{4} = \frac{1 \times 10}{4 \times 10} = \frac{10}{40}\right)$

2. $\frac{12}{16} = \frac{3}{4}$ $\left(\frac{12}{16} = \frac{12 \div 4}{16 \div 4} = \frac{3}{4}\right)$

3. $\frac{9}{6} = \frac{3}{2}$ $\left(\frac{9}{6} = \frac{9 \div 3}{6 \div 3} = \frac{3}{2}\right)$

4. $\frac{5}{8} = \frac{15}{24}$ $\left(\frac{5}{8} = \frac{5 \times 3}{8 \times 3} = \frac{15}{24}\right)$

5. $\frac{35}{5} = \frac{7}{1}$ $\left(\frac{35}{5} = \frac{35 \div 5}{5 \div 5} = \frac{7}{1}\right)$

6. $\frac{8}{9} = \frac{32}{36}$ $\left(\frac{8}{9} = \frac{8 \times 4}{9 \times 4} = \frac{32}{36}\right)$

Exercise 3, page 7

1. $2\frac{1}{8} = \frac{(2 \times 8) + 1}{8} = \frac{17}{8}$

2. $3\frac{3}{5} = \frac{(3 \times 5) + 3}{5} = \frac{18}{5}$

3. $8\frac{1}{2} = \frac{(8 \times 2) + 1}{2} = \frac{17}{2}$

4. $4\frac{4}{9} = \frac{(4 \times 9) + 4}{9} = \frac{40}{9}$

5. $5\frac{3}{7} = \frac{(5 \times 7) + 3}{7} = \frac{38}{7}$

Exercise 4, page 8

1. $\frac{14}{5} = 2\frac{4}{5}$ $\quad \begin{array}{r} 2 \\ 5\overline{)14} \end{array}$ remainder 4

2. $\frac{17}{13} = 1\frac{4}{13}$ $\quad \begin{array}{r} 1 \\ 13\overline{)17} \end{array}$ remainder 4

3. $\frac{10}{3} = 3\frac{1}{3}$ $\quad \begin{array}{r} 3 \\ 3\overline{)10} \end{array}$ remainder 1

4. $\frac{29}{9} = 3\frac{2}{9}$ $\quad \begin{array}{r} 3 \\ 9\overline{)29} \end{array}$ remainder 2

5. $\frac{37}{6} = 6\frac{1}{6}$ $\quad \begin{array}{r} 6 \\ 6\overline{)37} \end{array}$ remainder 1

Exercise 5, page 11

1. $$\begin{array}{r} 10\frac{3}{8} \\ 7\frac{5}{8} \\ \hline 17\frac{8}{8} \end{array} = 17 + 1 = 18$$

2. $$\begin{array}{r} 20 \\ 3\frac{1}{2} \\ \hline 23\frac{1}{2} \end{array}$$

3. $$\begin{array}{rcl} 14\frac{4}{5} & = & 14\frac{32}{40} \\ 13\frac{7}{8} & = & 13\frac{35}{40} \\ \hline & & 27\frac{67}{40} \end{array} = 27 + 1\frac{27}{40} = 28\frac{27}{40}$$

4. $$\begin{array}{r} 36\frac{3}{4} \\ 18 \\ \hline 54\frac{3}{4} \end{array}$$

5.
$$\begin{array}{rcl} 4\frac{2}{3} & = & 4\frac{10}{15} \\ 1\frac{2}{5} & = & 1\frac{6}{15} \\ \hline & & 5\frac{16}{15} = 5 + 1\frac{1}{15} = 6\frac{1}{15} \end{array}$$

6.
$$\begin{array}{rcl} 8\frac{1}{4} & = & 8\frac{6}{24} \\ 6\frac{3}{8} & = & 6\frac{9}{24} \\ \hline & & 14\frac{15}{24} \end{array}$$

Exercise 6, page 14

1. T

2. F $\frac{2}{3} \times \frac{1}{3} = \frac{2 \times 1}{3 \times 3} = \frac{2}{9}$

3. T

4. F $\frac{0}{6} \times \frac{6}{1} = \frac{0 \times 6}{6 \times 1} = \frac{0}{6} = 0$

5. F $\frac{4}{3} \times \frac{1}{9} = \frac{4 \times 1}{3 \times 9} = \frac{4}{27}$

6. T

Exercise 7, page 16

1. $\frac{30}{50} = \frac{3}{5}$

$30 = 2 \times 15 = \boxed{2} \times 3 \times \boxed{5}$
$50 = 2 \times 25 = \boxed{2} \times \boxed{5} \times 5$
Greatest common factor is $2 \times 5 = 10$.
$\frac{30}{50} = \frac{30 \div 10}{50 \div 10} = \frac{3}{5}$

2. $\frac{70}{35} = \frac{2}{1}$

$70 = 2 \times 35 = 2 \times \boxed{5 \times 7}$
$35 = \boxed{5 \times 7}$
Greatest common factor is $5 \times 7 = 35$.
$\frac{70}{35} = \frac{70 \div 35}{35 \div 35} = \frac{2}{1}$

3. $\frac{10}{4} = \frac{5}{2}$

$10 = \boxed{2} \times 5$
$4 = \boxed{2} \times 2$
Greatest common factor is 2.
$\frac{10}{5} = \frac{10 \div 2}{4 \div 2} = \frac{5}{2}$

4. $\frac{5}{15} = \frac{1}{3}$

$5 = \boxed{5}$
$15 = \boxed{5} \times 3$
Greatest common factor is 5.
$\frac{5}{15} = \frac{5 \div 5}{15 \div 5} = \frac{1}{3}$

5. $\frac{9}{25} = \frac{9}{25}$

$9 = 3 \times 3$
$25 = 5 \times 5$
Greatest common factor is 1.
$\frac{9}{25}$ is already in lowest terms.

Exercise 8, page 21

1. $\frac{8}{7}$ 2. $\frac{3}{14}$ 3. $\frac{61}{27}$ 4. $\frac{1865}{652}$ 5. $\frac{100}{1}$

Exercise 9, page 23

1. $\frac{5}{2} \div \frac{3}{8} = \frac{5}{2} \times \frac{8}{3} = \frac{5 \times \overset{4}{\cancel{8}}}{\underset{1}{\cancel{2}} \times 3} = \frac{5 \times 4}{1 \times 3} = \frac{20}{3} = 6\frac{2}{3}$

2. $\frac{0}{6} \div \frac{2}{3} = \frac{0}{6} \times \frac{3}{2} = \frac{0 \times \overset{1}{\cancel{3}}}{\underset{2}{\cancel{6}} \times 2} = \frac{0 \times 1}{2 \times 2} = \frac{0}{4} = 0$

3. $\frac{15}{4} \div \frac{7}{3} = \frac{15}{4} \times \frac{3}{7} = \frac{15 \times 3}{4 \times 7} = \frac{45}{28} = 1\frac{17}{28}$

4. $\frac{5}{6} \div \frac{5}{6} = \frac{5}{6} \times \frac{6}{5} = \frac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{6}}}{\underset{1}{\cancel{6}} \times \underset{1}{\cancel{5}}} = \frac{1 \times 1}{1 \times 1} = \frac{1}{1} = 1$

5. $\frac{1}{9} \div \frac{9}{1} = \frac{1}{9} \times \frac{1}{9} = \frac{1 \times 1}{9 \times 9} = \frac{1}{81}$

End of Lesson 11

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ____________

Address ____________

____

____

Postal Code ____________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Decimals

Topic One: Names of decimal numbers

You studied the base ten number system in Lesson 1. (You may want to go back and restudy this material.) When we speak of base-ten numbers we are actually talking about decimal numbers. "Decimal" means ten. You have already studied part of this number system. We shall attempt to extend your knowledge of this system by studying digits that occur to the right of the ones' place.

We use a dot, called a decimal point, to separate the digits in the ones' place from the digits to the right of the ones' place.

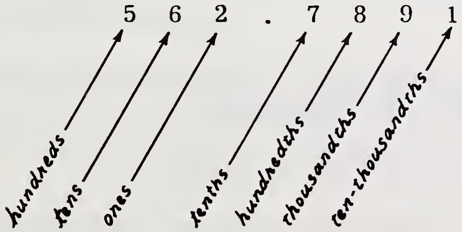
Just as with digits to the left of the decimal point, the digits to the right of the decimal point hold a specific position. Their position affects their value.

Since each digit holds a place, we must know the name of each place.

Going from right to left, the names of the places to the left of the decimal point are:	Going from left to right, the names of the places to the right of the decimal point are:
ones	tenths
tens	hundredths
hundreds	thousandths
thousands	ten-thousandths
ten thousands	hundred-thousandths
hundred thousands	etc.
etc.	

EXAMPLE: Name the place value of each digit in 562.7891.

Solution



You will notice that the names of the places to the right of the decimal point are similar to the names of the places to the left of the decimal point. They differ in that the places to the right of the decimal point end in "ths". Also there is no place to the right of the decimal point that compares with the ones' place.

To read the name of a number that contains a decimal point:

1. We read the part of the number to the left of the decimal point first. It is read as you learned to read whole numbers in Lesson 1.
2. For the decimal point we say "and".
3. For the part of the number to the right of the decimal point we read it just as if it were to the left of the decimal point. After reading this part of the number we say the name of the place held by the digit farthest to the right.

If there is one digit to the right of the decimal point, the last word used in naming the number is tenths.

EXAMPLE: Write the name for 8.2 in words.

Solution

8.2 is written as eight and two tenths.

The decimal point is read as "and".

Name of the place held by the digit farthest to the right of the decimal point

If there are two digits to the right of the decimal point, the last word used in naming the number is hundredths.

EXAMPLE: Write the name for 79.63 in words.

Solution

79.63 is written as seventy-nine and sixty-three hundredths.

The decimal point is read as "and".

The last digit, 3, is in the hundredths' position.

If there are three digits to the right of the decimal point, the last word used in naming the number is thousandths.

EXAMPLE: Write the name for 100.063 in words.

Solution

100.063 is written as one hundred and sixty-three thousandths.

The decimal point is read as "and".

The last digit, 3, is in the thousandths' position.

Self-correcting Exercise 1

Write the name for each number in words.

1. 4.6

____ ______

2. .007

____

3. 70.0501

____

____

4. 2789.563

____

____

____

Corrections are on page 16.

Now we know how to read decimals. We must also learn how to do the opposite - write the decimals using digits rather than words. Remember to use a zero if no other digit is given to hold a particular place in a number.

EXAMPLE: Write each number using digits.

1. twenty-seven hundredths

   Solution

   Twenty-seven is written 27. Since the decimal is hundredths we know that 7 must be in the second place to the right of the decimal point. Therefore 2 must be in the first place to the right of the decimal point.

   .27

2. eight hundred two and eight thousandths

   Solution

   Eight hundred two is written 802. "And" is the decimal point. Eight is written 8. Since the decimal is thousandths we know that 8 must be in the third place to the right of the decimal point. Since no other digits are given, we must use zeros in the tenths' and hundredths' places.

   802.008

Self-correcting Exercise 2

Use digits to write each number.

1. four thousand one and five ten-thousandths

2. thirty-five and six hundredths

3. five hundred seventy-six and two thousand fifty-four ten-thousandths

4. ninety-two and six tenths

5. ten thousandths

Corrections are on page 16.

Assignment 1

Write the name of each number in words.

1. 92.6

2. 42.005

3. .39

4. 9742.81

5. .75116

Use digits to write each number.

6. three thousand, three hundred thirty and thirty-three hundredths

____

7. nine hundred seventy-two and five hundred sixty-four thousandths

____

8. two hundred seventeen ten-thousandths

____

9. forty-one hundredths

____

10. thirty-four and four hundred eighty thousandths

____

____

11. In 27.613, the digit 6 is in the ____________________ place.

12. In 5.7642, the digit 4 is in the ____________________ place.

13. In 7.1681, the digit 6 is in the ____________________ place.

14. In 472.1, the digit 7 is in the ____________________ place.

15. In 836.30, the digit 0 is in the ____________________ place.

Topic Two: Value of digits in a decimal

The position of the digits in a number affects the value of the number.

The number expressed by each digit is the product of the digit and the value of the place it holds.

e.g. 2.36 The number expressed by the digit $\underline{3}$ depends on the place it holds. 3 is in the tenths' place. The number expressed by 3 is $3 \times \frac{1}{10}$ or $\frac{3}{10}$.

Using this idea we can use expanded form to write decimals just as we used expanded form to write whole numbers in Lesson 1.

EXAMPLE: Write 32.971 in expanded form.

Solution

3 in 32.971 expresses 3×10 or 30.
2 in 32.971 expresses 2×1 or 2.
9 in 32.971 expresses $9 \times \frac{1}{10}$ or $\frac{9}{10}$.
7 in 32.971 expresses $7 \times \frac{1}{100}$ or $\frac{7}{100}$.
1 in 32.971 expresses $1 \times \frac{1}{1000}$ or $\frac{1}{1000}$.

Therefore the expanded form of 32.971 is

$$(3\times10) + (2\times1) + (9\times\frac{1}{10}) + (7\times\frac{1}{100}) + (1\times\frac{1}{1000})$$

or $$30 + 2 + \frac{9}{10} + \frac{7}{100} + \frac{1}{1000}.$$

Since

There are 2 zeros in 100.
Thus, the exponent of 10 is 2.

$10 = 10^1$; $100 = 10^2$; $1000 = 10^3$; $10,000 = 10^4$; $100,000 = 10^5$; $1,000,000 = 10^6$;

we can write

$\frac{1}{10}$ as $\frac{1}{10^1}$; $\frac{1}{100}$ as $\frac{1}{10^2}$; $\frac{1}{1000}$ as $\frac{1}{10^3}$; $\frac{1}{10,000}$ as $\frac{1}{10^4}$; $\frac{1}{100,000}$ as $\frac{1}{10^5}$

and $\frac{1}{1,000,000}$ as $\frac{1}{10^6}$.

e.g. 9 in 32.971 expresses $9 \times \frac{1}{10}$ or $9 \times \frac{1}{10^1}$.

7 in 32.971 expresses $7 \times \frac{1}{100}$ or $7 \times \frac{1}{10^2}$.

1 in 32.971 expresses $1 \times \frac{1}{1000}$ or $1 \times \frac{1}{10^3}$.

Therefore, we can use exponential notation when writing the expanded form of a numeral. (Remember $10^0 = 1$. See Lesson 1.)

e.g. The expanded form of 32.971 using exponential notation is

$$(3 \times 10^1) + (2 \times 10^0) + (9 \times \frac{1}{10^1}) + (7 \times \frac{1}{10^2}) + (1 \times \frac{1}{10^3}).$$

You will notice that the exponent of 10 decreases from left to right until you reach 10^0. Then the power of 10 appears in the denominator and starts increasing again.

e.g. $135.264 = (1\times10^2) + (3\times10^1) + (5\times10^0) + (2\times\frac{1}{10^1}) + (6\times\frac{1}{10^2}) + (4\times\frac{1}{10^3})$

Exponents of 10 decrease as you move to the right.

Powers of 10 appear in the denominator. Exponents of 10 increase as you move to the right.

Self-correcting Exercise 3

Write each number in the expanded form. (Use exponential notation to write the powers of 10.)

1. 6005.4708

2. 33.006

3. 982.0808

Corrections are on page 16.

Assignment 2

Name the digit in each number that expresses the number of thousandths.

1. .125 5
2. 45.016
3. 2.004
4. 95.0001
5. 16.0025

Name the number expressed by 4 in each of the numerals.

6. .42 $4 \times \frac{1}{10} = \frac{4}{10}$
7. .040
8. 16.4
9. 75.0024
10. 346.17

Use the standard name of the number to express each of the number ideas below.

11. $4(10^1) + 9(10^0) + 6(\frac{1}{10^1}) + 3(\frac{1}{10^2}) + 0(\frac{1}{10^3}) + 1(\frac{1}{10^4}) =$ 49.6301

12. $5(10^2) + 0(10^1) + 0(10^0) + 0(\frac{1}{10^1}) + 6(\frac{1}{10^2}) =$ ____________

13. $6(\frac{1}{10^5}) + 0(\frac{1}{10^4}) + 8(\frac{1}{10^3}) =$ ____________

14. $5(10^2) + 0(10^1) + 0(10^0) + 3(\frac{1}{10^1}) + 8(\frac{1}{10^2}) =$ ____________

15. $9(10^3) + 0(10^2) + 0(10^1) + 9(10^0) + 0(\frac{1}{10^1}) + 9(\frac{1}{10^2}) =$ ____________

Write each number in expanded form. Use two different ways. Firstly, write it without using exponential notation. Secondly, write it using exponential notation.

16. 2.53

$= (2 \times 1) + (5 \times \frac{1}{10}) + (3 \times \frac{1}{100})$

$= (2 \times 10^0) + (5 \times \frac{1}{10^1}) + (3 \times \frac{1}{10^2})$

17. .005

____

____

18. 4.623

____

____

19. 580.04

____

____

20. 100.001

____

____

Topic Three: Increasing and decreasing decimals

To increase or decrease a number by .1, the digit in the tenths' place must be increased or decreased by 1.

$$\text{e.g.}\quad \left.\begin{array}{l} .843726 + .1 = .\overset{8+1=9}{\boxed{9}}43726 \\ .843726 - .1 = .\underset{8-1=7}{\boxed{7}}43726 \end{array}\right\} \begin{array}{l}\text{We need to consider only the digit} \\ \text{in the tenths' place.}\end{array}$$

To increase or decrease a number by .01, the digit in the hundredths' place must be increased or decreased by 1.

$$\text{e.g.}\quad \left.\begin{array}{l} .843726 + .01 = .8\overset{4+1=5}{\boxed{5}}3726 \\ .843726 - .01 = .8\underset{4-1=3}{\boxed{3}}3726 \end{array}\right\} \begin{array}{l}\text{We need to consider only the digit} \\ \text{in the hundredths' place.}\end{array}$$

To increase or decrease a number by .001, the digit in the thousandths' place must be increased or decreased by 1.

$$\text{e.g.}\quad \left.\begin{array}{l} .843726 + .001 = .84\overset{3+1=4}{\boxed{4}}726 \\ .843726 - .001 = .84\underset{3-1=2}{\boxed{2}}726 \end{array}\right\} \begin{array}{l}\text{We need to consider only the} \\ \text{digit in the thousandths' place.}\end{array}$$

To increase or decrease a number by .0001, the digit in the ten-thousandths' place must be increased or decreased by 1.

$$\text{e.g.}\quad \left.\begin{array}{l} .843726 + .0001 = .843\overset{7+1=8}{\boxed{8}}26 \\ .843726 - .0001 = .843\underset{7-1=6}{\boxed{6}}26 \end{array}\right\} \begin{array}{l}\text{We need to consider only the digit} \\ \text{in the ten-thousandths' place.}\end{array}$$

A special change occurs when the digit 9 is increased by 1.

e.g. $.4972 + .01 = .\underset{49+1=50}{\boxed{50}}72$

Increasing the digit in the hundredths' place, by 1, we get 9 + 1 = 10. Since each place can contain only 1 digit, 10 HUNDREDTHS is regrouped to 1 TENTH, 0 HUNDREDTHS. 0 is written in the hundredths' place. The digit in the tenths' place is increased by 1.

A special change occurs when the digit 0 is decreased by 1.

e.g. .3204 - .001 = .3194

20-1=19

Decreasing the digit in the thousandths' place by 1, we get 0-1 which has no answer. We must regroup 2 HUNDREDTHS to 1 HUNDREDTH, 10 THOUSANDTHS. 10 - 1 = 9. 1 is written in the hundredths' place. 9 is written in the thousandths' place.

Assignment 3

Complete each of the following sentences.

1. 3.672 + .01 = ________________

2. .234 + .001 = ________________

3. 96.81 + .1 = ________________

4. .006379 + .000001 = ________________

5. 46.083 + .01 = ________________

6. .1670 + .0001 = ________________

7. 3.7821 - .0001 = ________________

8. .609 - .01 = ________________

9. 142.783 - .001 = ________________

10. 6.897 - .1 = ________________

11. 9.873659 - .000001 = ________________

12. 13.601 - .01 = ________________

Topic Four: Decimals and fractions

For future work, you need to know how to change decimals to fractions.

To do this, we need only to consider the way in which we read the name of a decimal. It is read in the same way that a fraction is read.

e.g.

.9 is read "nine tenths". $\frac{9}{10}$ is read "nine tenths".

$$\text{Thus} \quad \underset{\nearrow \atop \text{decimal}}{.9} = \underset{\nwarrow \atop \text{fraction}}{\frac{9}{10}} .$$

To change a decimal to a fraction we must remember that the place value of the digit farthest to the right in the decimal is used as the denominator of the fraction. All the digits to the right of the decimal point make up the numerator.

EXAMPLE: Change .25 to a fraction.

Solution

.25 is read twenty-five hundredths.

$$.25 = \frac{25 \longleftarrow \text{Numerator is composed of the digits to the right of the decimal.}}{100 \longleftarrow \text{Denominator is 100 since 5 is in the hundredths' place.}}$$

Note that the fraction $\frac{25}{100}$ is not written in simplest form since the numerator and denominator have a common factor of 25.

$$.25 = \frac{25}{100} = \frac{25 \div 25}{100 \div 25} = \frac{1}{4}$$

.25 is equal to the fraction $\underline{\frac{1}{4}}$.

Self-correcting Exercise 4

Write each decimal as a fraction.

1. .423 = ____________________

2. .0217 = ____________________

3. .003 = ____________________

4. .7651 = ____________________

Corrections are on page 16.

Assignment 4

Write each decimal as a fraction in simplest form.

1. .4 = $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$
2. .0002 = ____________________
3. .00045 = ____________________
4. .047 = ____________________
5. 0.721 = ____________________
6. .8383 = ____________________
7. .4080 = ____________________
8. .00041 = ____________________
9. .000009 = ____________________
10. .55 = ____________________
11. .013 = ____________________
12. .0025 = ____________________

Decimals which contain some digits to the left of the decimal point can be changed to mixed numerals. As you know a mixed numeral can then be changed to a fraction.

EXAMPLE: Write 6.3 as a mixed numeral and then as a fraction.

Solution

6.3 is the same as 6 + .3. .3 can be written as $\frac{3}{10}$.

$$6.3 = \underset{\substack{\uparrow \\ \text{mixed numeral}}}{6\frac{3}{10}} = \underset{\substack{\uparrow \\ \text{fraction}}}{\frac{63}{10}}$$

Assignment 5

Write each decimal as a mixed numeral in which the fraction part is in its simplest form.

1. 2.36 = $2\frac{36}{100} = 2\frac{9}{25}$
2. 5.10 =
3. 4.0009 =
4. 356.7 =
5. 34.05 =

Write each decimal as a mixed numeral in which the fraction part is in its simplest form. Then change it to a fraction.

	MIXED NUMERAL	FRACTION
6. 7.005 =	$7\frac{5}{1000} = 7\frac{1}{200}$	$\frac{1401}{200}$
7. 32.9710 =		
8. 14.386 =		
9. 9.702 =		
10. 127.8 =		

Assignment 6

For each problem:
- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. Tom lost $9\frac{1}{2}$ lb. during the baseball season. At the end of the season he weighed $145\frac{3}{4}$ lb. How much did Tom weigh before he lost $9\frac{1}{2}$ lb.?

Placeholder: Let w represent the number of pounds he weighed before he lost $9\frac{1}{2}$ lb.

Equation: $w - 9\frac{1}{2} = 145\frac{3}{4}$

Computation

$$\begin{array}{rl} 145\frac{3}{4} & = 145\frac{3}{4} \\ +\ 9\frac{1}{2} & = \ \ \ 9\frac{2}{4} \\ \hline & \ \ 154\frac{5}{4} = 155\frac{1}{4} \end{array}$$

Answer: $w = 155\frac{1}{4}$

Statement: Tom weighed $155\frac{1}{4}$ pounds before he lost $9\frac{1}{2}$ pounds.

2. Henry had a board that was $5\frac{1}{2}$ ft. long. He used $2\frac{3}{4}$ ft. of this board to build a shelf. How much of the board did he have left after he had built the shelf?

Placeholder: Let b be ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

3. Precipitation is moisture in the form of rain, snow or dew. The normal yearly precipitation in Halifax is $54\frac{1}{4}$ inches. In Calgary it is $17\frac{1}{2}$ inches. Halifax receives how much more precipitation than Calgary?

Placeholder: Let p be ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

4. A passenger ship, the "France" is $45\frac{1}{6}$ ft. longer than the passenger ship, the "United States". The "France" is $1035\frac{1}{6}$ ft. long. What is the length of the "United States"?

Placeholder: Let y be ____________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

5. Mr. Holmes worked in his garden for $4\frac{2}{3}$ hr. on Saturday morning. He worked a total of $6\frac{3}{4}$ hr. during the morning and the afternoon. How many hours did he work that afternoon?

Placeholder: Let x be ____________________.

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

Key to Self-correcting Exercises in Lesson 12

Exercise 1, page 3

1. four and six tenths
2. seven thousandths
3. seventy and five hundred one ten-thousandths
4. two thousand seven hundred eighty-nine and five hundred sixty-three thousandths

Exercise 2, page 4

1. 4001.0005
2. 35.06
3. 576.2054
4. 92.6
5. .010

Exercise 3, page 7

1. $(6\times10^3) + (0\times10^2) + (0\times10^1) + (5\times10^0) + (4\times\frac{1}{10^1}) + (7\times\frac{1}{10^2}) + (0\times\frac{1}{10^3}) + (8\times\frac{1}{10^4})$
2. $(3\times10^1) + (3\times10^0) + (0\times\frac{1}{10^1}) + (0\times\frac{1}{10^2}) + (6\times\frac{1}{10^3})$
3. $(9\times10^2) + (8\times10^1) + (2\times10^0) + (0\times\frac{1}{10^1}) + (8\times\frac{1}{10^2}) + (0\times\frac{1}{10^3}) + (8\times\frac{1}{10^4})$

Exercise 4, page 11

1. $\frac{423}{1000}$
2. $\frac{217}{10000}$
3. $\frac{3}{1000}$
4. $\frac{7651}{10000}$

End of Lesson 12

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________
Address ____________________
____
____
Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Multiplication of Decimals

Topic One: Multiplication of decimals

Review multiplication of whole numbers (Lesson 2) and multiplication of fractions (Lesson 11).

Multiplication of decimals requires an understanding of multiplication of whole numbers and multiplication of fractions. Since you are multiplying decimals to obtain a product, the product must contain a decimal point. Your understanding of multiplication of fractions will help you to decide where the decimal point should go.

EXAMPLE: Find the product of .63 and .7.

Solution

If we change the decimals to fractions, we can obtain the product (since we know how to find the product of fractions).

$$.63 \times .7 = \frac{63}{100} \times \frac{7}{10}$$ Write the decimals as fractions.

$$= \frac{63 \times 7}{100 \times 10}$$ Find the product of the fractions.

$$= \frac{441}{1000}$$

We could also multiply the decimals as if they were whole numbers and then in the final step decide where the decimal should be placed.

$$\begin{array}{r} .63 \\ \underline{\times\ .7} \\ \underline{.441} \end{array}$$

The product of 63 and 7 is 441.

.63 is a number of hundredths and .7 is a number of tenths. Since $\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$, the product of these two numbers should be a number of thousandths. Insert the decimal to the left of the 3 digits to show that 441 is a number of thousandths. ($.441 = \frac{441}{1000}$)

Self-correcting Exercise 1

Find each product. Explain how you decide where to place the decimal point.

1. $\begin{array}{r} .5 \\ \times .9 \\ \hline .45 \end{array}$

2. $\begin{array}{r} .008 \\ \times\ .03 \\ \hline \end{array}$

3. $\begin{array}{r} .66 \\ \times\ .5 \\ \hline \end{array}$

$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$

The product is a number of hundredths.
(There are 2 decimal places in the answer.)

Corrections are on page 13.

Decimals that involve whole number parts can be multiplied in a similar manner.

EXAMPLE: Find the product of 9.2 and 1.4.

Solution

We could find the product by changing the decimals first to mixed numerals and then to fractions. Then we could multiply these fractions.

$$\begin{aligned} 9.2 \times 1.4 &= 9\tfrac{2}{10} \times 1\tfrac{4}{10} && \text{Mixed numerals} \\ &= \frac{92}{10} \times \frac{14}{10} && \text{Fractions} \\ &= \frac{92 \times 14}{10 \times 10} && \text{Product} \\ &= \frac{1288}{100} \text{ or } 12\tfrac{88}{100} \end{aligned}$$

Or we could multiply the decimals as if they were whole numbers and then decide where the decimal point goes.

$$\begin{array}{r} 9.2 \\ \times 1.4 \\ \hline 368 \\ 920 \\ \hline 12.88 \end{array}$$

The decimal parts of 9.2 and 1.4 are both a number of tenths. Since $\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$, the decimal part of the product must be a number of hundredths. Insert the decimal point in front of the last 2 digits to show that 88 is a number of hundredths.

$(12.88 = 12\frac{88}{100})$

Self-correcting Exercise 2

Find each product. Explain how you decided where to place the decimal point.

1. $\begin{array}{r} 4.12 \\ \underline{9.6} \end{array}$

2. $\begin{array}{r} 236.5 \\ \underline{1.79} \end{array}$

3. $\begin{array}{r} 4.1 \\ \underline{6.5} \end{array}$

Corrections are on page 13.

You have probably realized that there is a short method for finding the position of the decimal point in a product. If you find the sum of the number of digits to the right of the decimal point in the multiplicand and multiplier, it is equal to the number of digits to the right of the decimal point in the product.

EXAMPLE: Find the product of 23.35 and .071.

Solution

$$\begin{array}{r} 23.\boxed{35} \\ \underline{\boxed{.071}} \\ 2335 \\ \underline{163450} \\ 1.\boxed{65785} \end{array}$$

There are 2 digits to the right of the decimal point in 23.35.

There are 3 digits to the right of the decimal point in .071.

2 + 3 = 5

There are 5 digits to the right of the decimal point in the product. (You can count these by starting at the digit farthest to the right, 5, and counting 5 places to the left.)

Assignment 1

Find the products. Show how you found the position of the decimal point.

1. $$\left.\begin{array}{r} 1.2 \\ \underline{.14} \end{array}\right\}\text{ 3 digits to the right of the decimals}$$
$$\begin{array}{r} 48 \\ \underline{120} \\ .168 \end{array}$$

2. $$\begin{array}{r} 6.84 \\ \underline{.236} \end{array}$$

3. $$\begin{array}{r} 35.1 \\ \underline{.875} \end{array}$$

4. $$\begin{array}{r} 693.0 \\ \underline{.47} \end{array}$$

5. $$\begin{array}{r} 1.05 \\ \underline{.0003} \end{array}$$

6. $$\begin{array}{r} 13.6 \\ \underline{2.4} \end{array}$$

7. $$\begin{array}{r} 9.2 \\ \underline{1.45} \end{array}$$

8. $$\begin{array}{r} 213.2 \\ \underline{.029} \end{array}$$

9. 1.007
.009

10. 783
.0012

11. 76.3
.08

12. .44
1.5

Topic Two: Multiplying by powers of 10

You learned, in Lesson 2, how to multiply whole numbers by a number of tens, a number of hundreds or a number of thousands.

Now you will learn how to multiply decimals by 10, 100, 1000 etc. We shall multiply by the long method and then try to arrive at a short-cut method for multiplying by powers of 10.

EXAMPLE: Multiply 27.8 by 100.

Solution

27.8 ← 27.8 has 1 digit to the right of the decimal point.
100 ← 100 has 0 digits to the right of the decimal point.
2780.0 ← The product must have 1 + 0 or 1 digit to the right of the decimal point.

There is one place to the right of the decimal.

Thus, 27.8 × 100 = 2780.

Note that 100 has two zeros and that when you are multiplying another number by 100, we move the decimal point two places to the right.

> To multiply a number by 10, 100, 1000 and so on, move the decimal point to the right according to the number of zeros in the multiplier.

e.g. $0.33 \times 1\boxed{0}$ ← One zero in multiplier, move decimal point one place to the right
= 3.3

$12.54 \times 1\boxed{000}$ ← Three zeros in multiplier, move decimal point three places to the right
= 12,540

Self-correcting Exercise 3

Find each product.

1. 3.0 × 100 = ____ 2. 15.6 × 100 = ____ 3. 1.065 × 100 = ____

4. 1000 × .005 = ____ 5. 10 × .001 = ____ 6. 100 × 2.031 = ____

7. 1.12 × 10 = ____ 8. 11.5 × 100 = ____ 9. .01 × 1000 = ____

Corrections are on page 13.

Assignment 2

Find the products. Try to use the short method.

1. 2736.05 × 10,000 = 27,360,500.00
2. .0009 × 1000 = = ____
3. 2.457 × 100 = ____
4. 2.001 × 10 = ____
5. .5 × 100,000 = ____
6. .75 × 1,000,000 = ____
7. 437 × 100 = ____
8. 1.8 × 10 = ____
9. 97.32 × 100 = ____
10. 3.5876 × 10,000 = ____

Topic Three: Rounding decimals

In Lesson 2, you studied the rounding of whole numbers. We will use a similar process for the rounding of decimals.

To round a number to the nearer tenth, we must consider the digits in the hundredths' place. If the digit in the hundredths' place is less than 5, then the digit in the tenths' place does not change.

EXAMPLE: Round 1.43 to the nearer tenth.

Solution

Look at the digit in the hundredths' place.
Since 3 is less than 5, the 4 in the tenths' place does not change.

1.43 rounded to the nearer tenth is 1.4. (This means that 1.43 is nearer to 1.4 than 1.5.)

If the digit in the hundredths' place is 5 or greater than 5, then the digit in the tenths' place is increased by 1.

EXAMPLE: Round 1.47 to the nearer tenth.

Solution

Look at the digit in the hundredths' place.
Since 7 is greater than 5, the 4 in the tenths' place is increased by 1.

1.47 rounded to the nearer tenth is 1.5. (This means that 1.47 is nearer to 1.5 than 1.4).

Self-correcting Exercise 4

Name the digit in the tenths' place and the hundredths' place in each number.

	Tenths' place	Hundredths' place
1. .0123		
2. 327.15		
3. 8.307		
4. .162		

Round each of the numbers to the nearer tenth.

5. 38.57 ______________________________

6. 4.809 ______________________________

7. .954 ______________________________

8. 3.5876 ______________________________

For each number, name 10 numbers having 2 decimal places which, when rounded to the nearer tenth, are rounded to the given number.

9. .5 ______________________________

10. 1.8 ______________________________

11. 3.6 ______________________________

12. 11.5 ______________________________

Corrections are on page 14.

To round a number to the nearer hundredth, we must consider the digit in the thousandths' place. If the digit in the thousandths' place is less than 5, the digit in the hundredths' place does not change. If the digit in the thousandths' place is 5 or greater than 5, then the digit in the hundredths' place increases by 1.

e.g. .029 The digit in the thousandths' place is 9. Therefore 2 must be increased by 1. .029 rounded to the nearer hundredth is $\underline{.03}$.

To round a number to the nearer thousandth, we must consider the digit in the ten-thousandths' place. If the digit in the ten-thousandths' place is less than 5, the digit in the thousandths' place does not change. If the digit in the ten-thousandths' place is 5 or greater than 5, then the digit in the thousandths' place increases by 1.

e.g. .0012 The digit in the ten-thousandths' place is 2. Therefore the digit in the thousandths' place does not change. .0012 rounded to the nearer thousandth is $\underline{.001}$.

Assignment 3

Round each number to the nearer tenth.

1. 14.62 14.6

2. .862 __________

3. .372 __________

4. 1.003 __________

5. .2145 __________

Round each number to the nearer hundredth.

6. .086 .09

7. .019 __________

8. .835 __________

9. .792 __________

10. .096 __________

Round each number to the nearer thousandth.

11. .04956 .050

12. 2.1075 __________

13. .9086 __________

14. .1254 __________

15. .5009 __________

Find the product. Below each product, round the answer to the nearer tenth.

16.
$$\begin{array}{r} 13 \\ \underline{.87} \\ 91 \\ \underline{1040} \\ 11.31 \end{array}$$

Rounded answer is 11.3.

17.
$$\begin{array}{r} 4.9 \\ \underline{.65} \end{array}$$

Rounded answer is ________.

18.
$$\begin{array}{r} .71 \\ \underline{.42} \end{array}$$

Rounded answer is ________.

19. $\begin{array}{r} .692 \\ \underline{.83} \end{array}$

Rounded answer is ______.

20. $\begin{array}{r} .382 \\ \underline{9} \end{array}$

Rounded answer is ______.

21. $\begin{array}{r} .429 \\ \underline{13} \end{array}$

Rounded answer is ______.

Name 3 numbers which when rounded to the nearer hundredth would equal the given number.

22. .76 ____________________

23. .82 ____________________

24. .12 ____________________

25. .11 ____________________

Assignment 4

For each problem:

- tell what the variable represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. Lorraine worked in a bakery 4 days last week. She worked an average of $1\frac{1}{2}$ hr. per day. How many hours in all did she work during the 4 days?

Placeholder: Let h be the number of hours Lorraine worked during the week.

Equation: $1\frac{1}{2} \times 4 = h$

Computation

$1\frac{1}{2} \times 4 = \frac{3}{2} \times \frac{4}{1} = \frac{3 \times \overset{2}{\cancel{4}}}{\underset{1}{\cancel{2}} \times 1} = 6$

Answer: $h = 6$

Statement: Lorraine worked six hours during that week.

2. Each volume of a set of books weighs $2\frac{1}{2}$ lb. At this rate, what does a set of 8 volumes weigh?

Placeholder: Let w be ______________________________

____

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

3. One year Mr. Nichols sold $\frac{3}{5}$ of his corn crop and kept the rest for cattle feed. His corn crop that year was 10,000 bushels. How many bushels of corn did Mr. Nichols sell?

Placeholder: Let b be ______________________________

____

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

4. The speed of sound in steel is 15,000 ft. per second. The speed of sound in a certain kind of wood is approximately $\frac{2}{3}$ of the speed of sound in steel. What is the approximate speed of sound in this kind of wood?

Placeholder: Let s be ____________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

5. A cake recipe that Jean has calls for $2\frac{1}{2}$ cups of flour. Jean made a cake using $\frac{1}{2}$ of the ingredients called for in the recipe. How many cups of flour should she have used?

Placeholder: Let c be ____________________

____

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

Key to Self-correcting Exercises in Lesson 13

Exercise 1, page 2

2.
$$\begin{array}{r} .008 \\ .03 \\ \hline .00024 \end{array}$$

$$\frac{1}{1000} \times \frac{1}{100} = \frac{1}{100,000}$$

The product is a number of hundred-thousandths. (There are 5 digits to the right of the decimal in the answer.)

3.
$$\begin{array}{r} .66 \\ \times .5 \\ \hline .330 \end{array}$$

$$\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$$

The product is a number of thousandths. (There are 3 digits to the right of the decimal in the answer.)

Exercise 2, page 3

1.
$$\begin{array}{r} 4.12 \\ 9.6 \\ \hline 2472 \\ 37080 \\ \hline 39.552 \end{array}$$

$$\frac{1}{100} \times \frac{1}{10} = \frac{1}{1000}$$

The product is a number of thousandths. (There are 3 digits to the right of the decimal in the answer.)

2.
$$\begin{array}{r} 236.5 \\ 1.79 \\ \hline 21285 \\ 165550 \\ 236500 \\ \hline 423.335 \end{array}$$

$$\frac{1}{10} \times \frac{1}{100} = \frac{1}{1000}$$

The product is a number of thousandths. (There are 3 digits to the right of the decimal in the answer.)

3.
$$\begin{array}{r} 4.1 \\ 6.5 \\ \hline 205 \\ 2460 \\ \hline 26.65 \end{array}$$

$$\frac{1}{10} \times \frac{1}{10} = \frac{1}{100}$$

The product is a number of hundredths. (There are 2 digits to the right of the decimal in the answer.)

Exercise 3, page 6

Move decimal 2 places to the right.

1. 3.0 × 100 = 300
2. 15.6 × 100 = 1560
3. 1.065 × 100 = 106.5

Move decimal 3 places to the right.

4. 1000 × .005 = 5
5. 10 × .001 = .01
6. 100 × 2.031 = 203.1
7. 1.12 × 10 = 11.2

Move decimal 1 place to the right.

8. 11.5 × 100 = 1150
9. .01 × 1000 = 10

Exercise 4, page 7

1. In the number .0123, 0 is in the tenths' place and 1 is in the hundredths' place.

2. In the number 327.15, 1 is in the tenths' place and 5 is in the hundredths' place.

3. In the number 8.307, 3 is in the tenths' place and 0 is in the hundredths' place.

4. In the number .162, 1 is in the tenths' place and 6 is in the hundredths' place.

5. 38.57 to the nearer tenth is 38.6.
   Digit in hundredths' place is greater than 5.

6. 4.809 to the nearer tenth is 4.8.
   Digit in hundredths' place is less than 5.

7. .954 to the nearer tenth is 1.0.
   Digit in hundredths' place is 5 or greater.
   Increase tenths' digit by 1. (.9 + .1 = 1.0)

8. 3.5876 to the nearer tenth is 3.6.
   Digit in hundredths' place is greater than 5.

9. .45, .46, .47, .48, .49, .50, .51, .52, .53, .54 would all be rounded to .5.

10. 1.75, 1.76, 1.77, 1.78, 1.79, 1.80, 1.81, 1.82, 1.83, 1.84 would all be rounded to 1.8.

11. 3.55, 3.56, 3.57, 3.58, 3.59, 3.60, 3.61, 3.62, 3.63, 3.64 would all be rounded to 3.6.

12. 11.45, 11.46, 11.47, 11.48, 11.49, 11.50, 11.51, 11.52, 11.53, 11.54 would all be rounded to 11.5.

End of Lesson 13

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received:

____

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Division of Decimals

Topic One: Dividend as a decimal

Review division of whole numbers (Lesson 3) and division of fractions (Lesson 11).

Division of decimals requires an understanding of division of whole numbers and division of fractions. Since you are dividing decimals to obtain a quotient, the quotient must contain a decimal point. Your understanding of division of fractions will help you to decide where the decimal point should go.

EXAMPLE: Divide 62.5 by 25.

Solution

If we change the decimals to fractions, we can obtain the quotient (since we know how to find the quotient of fractions).

$$\left.\begin{aligned} 62.5 \div 25 &= 62\tfrac{5}{10} \div \frac{25}{1} \\ &= \frac{625}{10} \div \frac{25}{1} \end{aligned}\right\}$$

Write decimals as fractions.

$$\left.\begin{aligned} &= \frac{625}{10} \times \frac{1}{25} \\ &= \frac{\overset{25}{\cancel{625}} \times 1}{10 \times \underset{1}{\cancel{25}}} \end{aligned}\right\}$$

Change operation to multiplication and give the reciprocal of the divisor.

$$\begin{aligned} &= \frac{25}{10} \\ &= 2\tfrac{5}{10} \\ &= 2.5 \end{aligned}$$

We could also divide the decimals as if they were whole numbers and then in the final step decide where the decimal should be placed.

$$\begin{array}{r|l} & 2.5 \\ \hline 25 & 62.5 \\ & \underline{50} \\ & 125 \\ & \underline{125} \\ & 0 \end{array}$$

Decimal point goes between the 2 and the 5. (Since the dividend is tenths, the quotient must also be tenths.)

62.5 ÷ 25 = 2.5

You could also use a process of estimation to help you place the decimal in the quotient. We know that 50 ÷ 25 = 2, therefore 62.5 ÷ 25 = 2.5.

In Lesson 3, you learned a short-cut method of division to use when the divisor is a 1-digit number.

EXAMPLE: Divide 32785 by 5, using the short-cut method.

Solution

Step 1. $$\begin{array}{r@{\,}l} & 6 \\ 5 & \overline{)32785} \\ & {}_{2} \end{array}$$

Divide 5 into 32. The quotient is 6 and the remainder is 2.

Step 2. $$\begin{array}{r@{\,}l} & 65 \\ 5 & \overline{)32785} \\ & {}_{2}{}_{2} \end{array}$$

Divide 5 into 27. The quotient is 5 and the remainder is 2.

Step 3. $$\begin{array}{r@{\,}l} & 655 \\ 5 & \overline{)32785} \\ & {}_{2}{}_{2}{}_{3} \end{array}$$

Divide 5 into 28. The quotient is 5 and the remainder is 3.

Step 4. $$\begin{array}{r@{\,}l} & 6557 \\ 5 & \overline{)32785} \\ & {}_{2}{}_{2}{}_{3} \end{array}$$

Divide 5 into 35. The quotient is 7 with no remainder.

Thus, 32785 ÷ 5 = 6557.

We can apply this short-cut method to division of decimals when the divisor is a 1-digit number. The only added rule that you must remember is that the decimal must be positioned so that the digit farthest to the right in the quotient has the same place value as the digit farthest to the right in the dividend.

e.g. If the dividend is tenths, the quotient must be tenths.
If the dividend is thousandths, the quotient must be thousandths.

EXAMPLE: Divide 583.602 by 6.

Solution

Step 1. $$\begin{array}{r@{\,}l} & 9 \\ 6 & \overline{)583.602} \\ & {}_{4} \end{array}$$

Divide 6 into 58. The quotient is 9 and the remainder is 4.

Step 2. $$\begin{array}{r@{\,}l} & 97 \\ 6 & \overline{)583.602} \\ & {}_{4}{}_{1} \end{array}$$

Divide 6 into 43. The quotient is 7 and the remainder is 1.

Step 3. $$\begin{array}{r@{\,}l} & 972 \\ 6 & \overline{)583.602} \\ & {}_{4}{}_{1}{}_{4} \end{array}$$

Divide 6 into 16. The quotient is 2 and the remainder is 4.

Step 4. $$\begin{array}{r@{\,}l} & 9726 \\ 6 & \overline{)583.602} \\ & {}_{4}{}_{1}{}_{4}{}_{4} \end{array}$$

Divide 6 into 40. The quotient is 6 and the remainder is 4.

Step 5.

$$\begin{array}{r} 97\;\;267 \\ 6\;\overline{\big)\,583.602} \\ {\scriptstyle 4\quad 1\,44} \end{array}$$

Divide 6 into 42. The quotient is 7 and there is no remainder.

Step 6.

$$\begin{array}{r} 97.267 \\ 6\;\overline{\big)\,583.602} \end{array}$$

decimals should line up.

Place the decimal. It should appear in the quotient directly above the decimal in the dividend. The dividend is thousandths, so the quotient is also thousandths.

Thus, 583.602 ÷ 6 = 97.267.

Self-correcting Exercise 1

Find the quotients.

1. $150\;\overline{\big)\,15.6600}$

2. $58\;\overline{\big)\,83.346}$

Corrections are on page 18.

Assignment 1

Find the quotients.

1. $9\overline{)\,.0036}$ with quotient $\boxed{.000}4$

0×××
0
0
3
0
36
36
0

Note that zeros have been inserted as place holders.

.0036 ÷ 9 = .0004

2. $6\overline{)\,24.06}$

24.06 ÷ 6 = ________

3. $5\overline{)\,15.5}$

15.5 ÷ 5 = ________

4. $15\overline{)\,.645}$

.645 ÷ 15 = ________

5. $12\overline{)\,7.56}$

7.56 ÷ 12 = ________

6. $5\overline{)\,29.30}$

29.30 ÷ 5 = ________

Topic Two: Divisor as a decimal

You know that $10 \div 2 = 5$, $20 \div 4 = 5$, $30 \div 6 = 5$ and $40 \div 8 = 5$. Notice that all of the quotients are the same even though the dividend and divisor are different.

Compare $10 \div 2 = 5$ and $20 \div 4 = 5$. You will notice that both the dividend (10) and divisor (2) are multiplied by the same number (2) to obtain the second division fact.
$10 \div 2 = 5$
$10 \times 2 = 20$ $\quad 2 \times 2 = 4$ $\quad 20 \div 4 = 5$

Compare $10 \div 2 = 5$ and $30 \div 6 = 5$. 10 is multiplied by 3 to get 30.
2 is multiplied by 3 to get 6.

Compare $10 \div 2 = 5$ and $40 \div 8 = 5$. 10 is multiplied by 4 to get 40.
2 is multiplied by 4 to get 8.

Compare $20 \div 4 = 5$ and $40 \div 8 = 5$. 20 is multiplied by 2 to get 40.
4 is multiplied by 2 to get 8.

From these comparisons you can see that we can multiply the dividend and the divisor by the same number without changing the quotient (answer).

Using this principle, we can develop a simple method for dividing by decimals. If we multiply the divisor and dividend by a power of ten, we will not affect the quotient. If we choose the power of ten carefully, we will then have a divisor which is a whole number. You learned how to divide decimals by whole numbers in Topic One of this lesson.

EXAMPLE: Divide 45 by .75.

Solution

If we multiply .75 by 10^2 or 100 it becomes 75 which is a whole number. To ensure that the quotient remains the same, we must also multiply 45 by 10^2 or 100. $45 \times 10^2 = 4500$.
The division problem is now $4500 \div 75$.

```
      60
75 )4500
    450
      0
      0
      0
```

$45 \div .75 = \underline{60}$

Self-correcting Exercise 2

Multiply the divisor and dividend by a power of ten so that the divisor is expressed as a whole number. Then find the quotient.

1. 4545 ÷ .009

2. 4545 ÷ 45.45

Corrections are on page 19.

Assignment 2

Find the quotients. To do this you must first multiply the divisor and dividend by a power of ten.

1. 153 ÷ .025 = (multiply dividend and divisor by 1,000.)

153000 ÷ 25

$$\begin{array}{r} 6120 \\ 25\overline{)153\overset{\times}{0}\overset{\times}{0}\overset{\times}{0}} \\ \underline{150} \\ 30 \\ \underline{25} \\ 50 \\ \underline{50} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

153 ÷ .025 = 6120

2. $12 \div .16$

$12 \div .16 =$ ______

3. $65 \div .13$

$65 \div .13 =$ ______

4. $70 \div .56$

$70 \div .56 =$ ______

5. 268 ÷ 6.7

268 ÷ 6.7 = ______

Topic Three. Division of decimals

If you combine what you learned in Topic One and Topic Two, you should be able to do any division problem.

You know, from your work with fractions, that $\frac{25}{100}$ is the same as $\frac{250}{1000}$. Therefore, it is true of decimals that .25 is the same as .250. Also it is true that .250 is the same as .2500. From this you can see that we can use zeros to hold places to the right of the digits which are to the right of the decimal point without affecting the value of the number.

Sometimes division problems do not work out "evenly", that is, there is a remainder. Knowing that zeros to the right of the digits to the right of the decimal point do not change the value of the number, we can insert zeros to hold places and continue to divide. Often it is indicated in brackets as to how many places you should divide. When dividing to a specific place, we must remember to take the remainder in into consideration. Review rounding of quotients (Lesson 3).

The principles you must remember when dividing are:

1. Multiply the divisor and dividend by a power of ten to obtain a whole number as a divisor.
2. Insert zeros in the dividend so that you can find the required quotient.
3. Insert the decimal in the quotient so that the place value of the digit farthest to the right in the quotient is the same as the place value of the digit farthest to the right in the dividend.

EXAMPLE: Find the quotient of 5.73 ÷ 2.3 (hundredths).

Solution

First multiply the divisor and dividend by a power of 10 so that the divisor is a whole number. (In this case, multiply both the divisor and dividend by 10.)

5.73 ÷ 2.3 = 57.3 ÷ 23

Since the quotient must be in hundredths, the dividend must also be in hundredths.

57.3 ÷ 23 = 57.30 ÷ 23

```
        2.49
23 ) 57.30
     46  × ×
     11 3
      9 2
      2 10
      2 07
         3
```

5.73 ÷ 2.3 = 2.49 (hundredths)

Since the remainder (3) is less than half of the divisor (23), the digit in the hundredths' place does not change.

Self-correcting Exercise 3

Find the quotients.

1. 26.734 ÷ 32.6 (hundredths)

2. 14.82 ÷ 1.78 (hundredths)

26.734 ÷ 32.6 = ______ (hundredths)

14.82 ÷ 1.78 = ______ (hundredths)

Corrections are on page 19.

Assignment 3

Following each statement write "T" if the statement is true or "F" if the statement is false.

1. .05 = .0500 T
2. 450 = 4500 ________
3. 1.5 = 1.05 ________
4. 143.2 = 1430.2 ________
5. 16.1 = 016.1 ________
6. 3.06 = 3.060 ________
7. 4.20 = 42.00 ________
8. 76.806 = 76.8006 ________
9. 1789 = 1789.0 ________
10. .2709 = .02709 ________

How many digits must be shown to the right of the decimal point in each quotient?

11. 3256 ÷ 1.3 (tenths) 1
12. 463 ÷ 35 (ones) ________
13. 3482 ÷ 11 (tenths) ________
14. 26.734 ÷ 32.6 (hundredths) ________
15. 1 ÷ 3 (thousandths) ________

Find the quotients.

16. 225 ÷ 1.3 (tenths)

= 2250 ÷ 13

= 2250.0 ÷ 13

$$\begin{array}{r} 173.0 \\ 13\overline{)2250.0} \\ 13 \\ \hline 95 \\ 91 \\ \hline 40 \\ 39 \\ \hline 10 \\ 0 \\ \hline 10 \end{array}$$

Be sure the decimals are in line.

225 ÷ 1.3 = 173.1 (tenths)

17. 463 ÷ 35 (ones)

463 ÷ 35 = ________ (ones)

18. 3482 ÷ 11 (tenths)

3482 ÷ 11 = ________ (tenths)

19. 62.881 ÷ 51.6 (hundredths)

62.881 ÷ 51.6 = ________ (hundredths)

20. 24.81 ÷ 2.76 (hundredths)

21. 1 ÷ 3 (ten-thousandths)

24.81 ÷ 2.76 = ________ (hundredths)

1 ÷ 3 = __________ (ten-thousandths

Topic Four: Quotients with remainder zero

As you know, not all division problems "come out evenly". Some have a remainder. Since we can insert zeros in places to the right of the digits to the right of the decimal point, we can divide further - thus obtaining a quotient with remainder zero.

EXAMPLE: Find the quotient of 62 ÷ 25. (Keep dividing until you obtain a remainder of zero.)

Solution

```
      2
25 ) 62
     50
     12
```

62 ÷ 25 does not "come out evenly".

```
      2.4
25 ) 62.(0) <--
     50  x
     12  0
     10  0
      2  0
```

Place a zero to the right of the decimal point in the dividend. Since 62 is actually 62., we know that 62.0 is the same as 62. 62.0 ÷ 25 also does not "come out evenly".

```
      2.48
25 ) 62.0(0) <--
     50  xx
     12  0
     10  0
      2  00
      2  00
          0
```

Place another zero to the right of the zero in the tenths' place. 62.00 is the same as 62.0 or 62. 62.00 ÷ 25 <u>does</u> "come out evenly".

EXAMPLE: Find the quotient of 126 ÷ 24. (Keep dividing until you obtain a remainder of zero.)

Solution

$$\begin{array}{rr} & 5 \\ 24 & \overline{)\,126} \\ & \underline{120} \\ & 6 \end{array}$$

126 ÷ 24 does not "come out evenly".

$$\begin{array}{rr} & 5.2 \\ 24 & \overline{)\,126.\underset{x}{0}} \\ & \underline{120} \\ & 60 \\ & \underline{48} \\ & 12 \end{array}$$

126.0 is the same as 126.

126.0 ÷ 24 does not "come out evenly".

$$\begin{array}{rr} & 5.25 \\ 24 & \overline{)\,126.\underset{xx}{00}} \\ & \underline{120} \\ & 60 \\ & \underline{48} \\ & 120 \\ & \underline{120} \\ & 0 \end{array}$$

126.00 is the same as 126.

126.00 ÷ 24 does "come out evenly".

Self-correcting Exercise 4

Find the quotients (with remainder zero).

1. 228 ÷ 16

2. 1224 ÷ 144

Corrections are on page 20.

Assignment 4

Find the quotient (with remainder zero).

1. $2820 \div 75$

2. $3728 \div 128$

3. $4053 \div 56$

4. $799 \div 85$

5. $2058 \div 96$

6. $2457 \div 104$

Topic Five: Dividing by powers of 10

It is convenient to use a short method for dividing by powers of ten.

We can use long division to show how the short method was developed.

EXAMPLE: 273.16 ÷ 100 (ten-thousandths)

Solution

$$
\begin{array}{l}
\qquad\quad 2.7316 \\
100\,\overline{)\,273.1600} \\
\qquad\; \underline{200}\,{}^{\times\times\times\times} \\
\qquad\quad 73\;1 \\
\qquad\quad \underline{70\;0} \\
\qquad\qquad 3\;16 \\
\qquad\qquad \underline{3\;00} \\
\qquad\qquad\quad 160 \\
\qquad\qquad\quad \underline{100} \\
\qquad\qquad\qquad 600 \\
\qquad\qquad\qquad \underline{600} \\
\qquad\qquad\qquad\quad 0
\end{array}
$$

273.16 ÷ 100 = 2.7316

You know that $100 = 10^2$. You will also notice that the decimal point moved 2 places to the left of the position it held in the dividend.

To divide a decimal by a power of 10, move the decimal point to the left a number of places equal to the number which has been used as the exponent.

To distinguish between multiplying and dividing by a power of ten remember that when you multiply the number increases in size; when you divide the number decreases in size.

Self-correcting Exercise 5

Find the quotients using the short method.

1. 248.36 ÷ 1000 = ____________________

2. 327.16 ÷ 10 = ____________________

3. 97.32 ÷ 10,000 = ____________________

4. 3.5876 ÷ 100 = ____________________

Corrections are on page 20.

Assignment 5

1. 2736.05 ÷ 10,000 = ____________
2. .0008 ÷ 1000 = ____________
3. 2.475 ÷ 100 = ____________
4. 2.001 ÷ 10 = ____________
5. .5 ÷ 100,000 = ____________
6. .75 ÷ 1,000,000 = ____________
7. 437 ÷ 100 = ____________
8. 1.8 ÷ 10 = ____________
9. 25.91 ÷ 100 = ____________
10. 251.8 ÷ 10,000 = ____________

Assignment 6

For each problem:

- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. Four books weigh a total of $6\frac{9}{25}$ lb. Suppose that all the books were the same weight. How much would each book weigh?

Placeholder: Let w be the weight of each book.

Equation: $6\frac{9}{25} \div 4 = w$

Answer: $w = 1\frac{59}{100}$

Statement: Each book weighs $1\frac{59}{100}$ lb.

Computation

$$6\frac{9}{25} \div 4 = \frac{159}{25} \div \frac{4}{1}$$

$$= \frac{159}{25} \times \frac{1}{4}$$

$$= \frac{159 \times 1}{25 \times 4}$$

$$= \frac{159}{100}$$

$$= 1\frac{59}{100}$$

2. The pencils that Mark wants to buy cost 8¢ each. He has 25¢ to spend for these pencils. How many pencils can he buy?

Placeholder: Let p be ____________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

3. The area of Lake Nipissing in Ontario is 350 sq. mi. The area of Lake Dauphin in Manitoba is 200 sq. mi. Lake Nipissing is how many times larger than Lake Dauphin?

Placeholder: Let x be ____________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

4. Mary had $5\frac{1}{2}$ yd. of printed cotton. She made as many aprons as possible out of this cloth. Each apron took $1\frac{1}{2}$ yd. of cloth. How many aprons did she make?

Placeholder: Let a be ____________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

5. John has 225 bu. of wheat. He seeds $2\frac{1}{2}$ bu. for each acre. How many acres will 225 bu. of wheat seed?

Placeholder: Let y be ______________________________

Equation: ______________________________ Computation

Answer: ______________________________

Statement: ______________________________

____

Key to Self-correcting Exercises in Lesson 14

Exercise 1, page 3

```
               .1044
1.   150 ) 15.6600
           15 0 xxx
              66
               0
              660
              600
               600
               600
                 0
```

15.6600 ÷ 150 = .1044

```
             1.437
2.   58 ) 83.346
          58  xxx
          25 3
          23 2
           2 14
           1 74
             406
             406
               0
```

83.346 ÷ 58 = 1.437

Exercise 2, page 6

1. 4545 ÷ .009 (Multiply dividend and divisor by 1,000.)
= 4545000 ÷ 9

505000
9) 4545000

2. 4545 ÷ 45.45 (Multiply dividend and divisor by 100.)
= 454500 ÷ 4545

100
4545) 454500
4545 x x
00
0
00
0
0

Exercise 3, page 9

1. 26.734 ÷ 32.6 (Multiply the divisor and dividend by 10.)
= 267.34 ÷ 326

.82
326) 267.34
260 8 x
6 54
6 52
2

Remainder is less than half the divisor. Digit in the hundredth's place is left unchanged.

26.734 ÷ 32.6 = .82 (hundredths)

2. 14.82 ÷ 1.78
= 1482 ÷ 178
= 1482.$\textcircled{00}$ ÷ 178

Add two zeros to the dividend since the quotient must be in hundredths.

8.32
178) 1482.00
1424 x x
580
534
460
356
104

Since the remainder, 104, is more than half of the divisor, 178, the digit in the hundredth's place is increased by 1.

14.82 ÷ 1.78 = 8.33 (hundredths)

Exercise 4, page 13

1.
$$\begin{array}{r}
14.25 \\
16\;\overline{)\,228.00} \\
\underline{16}^{\times\;\times\times} \\
68 \\
\underline{64} \\
40 \\
\underline{32} \\
80 \\
\underline{80} \\
0
\end{array}$$

$228 \div 16 = 14.25$

2.
$$\begin{array}{r}
8.5 \\
144\;\overline{)\,1224.0} \\
\underline{1152}^{\times} \\
72\ 0 \\
\underline{72\ 0} \\
0
\end{array}$$

$1224 \div 144 = 8.5$

Exercise 5, page 15

1. .24836 (Decimal has been moved 3 places to the left since 1000 has 3 zeros.)

2. 32.716 (Decimal has been moved 1 place to the left since 10 has 1 zero.)

3. .009732 (Decimal has been moved 4 places to the left since 10,000 has 4 zeros.)

4. .035876 (Decimal has been moved 2 places to the left since 100 has 2 zeros.)

End of Lesson 14

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ________________

Time Spent on Lesson ________________

(If label is missing
or incorrect)

File Number ________________

Lesson Number ________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ________

Address ________

Postal Code ________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail **each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Terminating and Repeating Decimals

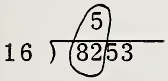
Topic One: Review of division

Work with decimals involves many division calculations so you must be able to divide quickly and accurately. You studied division in Lesson 3 but let's review the shortened method of division.

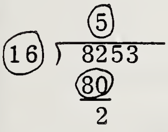
The quotient is placed above the dividend. An important point to remember in short division is that when you subtract, the remainder must be less than the divisor. If it is not you must repeat the last step you performed by increasing the quotient.

EXAMPLE: Divide 8253 by 16.

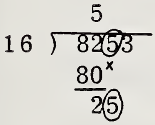
Solution



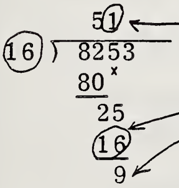
16 does not divide into 8. Therefore we must try to divide 16 into 82. The greatest number of times that 16 divides into 82 is 5.
Place 5 above the 2 in 82.



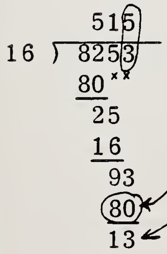
Now multiply 16 by 5. Write the product below 82 and subtract. The remainder, 2, is less than the divisor, 16, so we can continue.



Now we "bring down" the 5 from the dividend and place it beside remainder 2. See the small "x" below 5.



Divide 16 into 25. It divides into 25 only 1 time. Multiply 16 by 1. Subtract. The remainder is less than 16 so we can continue.



"Bring down" the 3 from the dividend and place it beside 9 remainder. Divide 16 into 93. It goes only 5 times. Place 5 above the 3 in the dividend. Multiply 16 by 5. Then subtract. Since the remainder is less than the divisor, we have completed this division problem.

8253 ÷ 16 = 515 remainder 13

Self-correcting Exercise 1

Use the short method of division for the following problems.

```
1.  89 |4450          2.  54 |5184          3.  127 |29718
```

Corrections are on page 26.

This short method of division is suited to decimals too. We divide as if there were no decimal. Finally the decimal is placed so that the digit farthest to the right in the quotient has the same place value as the digit farthest to the right in the dividend.

EXAMPLE: 49.22 ÷ 36.8

Solution

Multiply dividend and divisor by 10 to remove the decimal in the divisor. 49.22 ÷ 36.8 = 492.2 ÷ 368

```
       1.3
368 |492.2    Divide 368 into 492.  Multiply.  Since the remainder,
     368 x    124, is less than the divisor, we can continue.
     1242     Divide 368 into 1242.  Multiply and subtract.
     1104
      138     The remainder, 138, is less than the divisor.
```

We can place zeros to the right of the decimal without changing the value of the dividend. By doing this, we can continue to divide.

```
       1.3375
368 |492.2000      The dividend is ten-thousandths so the quotient
     368  xxxx     must also be ten-thousandths.
     1242
     1104
      1380
      1104
       2760
       2576
        1840
        1840
           0
```

49.22 ÷ 36.8 = 1.3375

Self-correcting Exercise 2

Use the short method of division. Find the quotient with remainder zero.

1. .66 ÷ .44
2. 32.64 ÷ 2.4
3. 13.34 ÷ 1.45

Corrections are on page 26.

If division problems do not "work out evenly", you can round the answers.

Assignment 1

Use the short method of division.

1. 76.16 ÷ 136
2. 2385 ÷ 7.5

3. $2.048 \div 3.2$

4. $696 \div 29$

Topic Two: Terminating decimals

You will remember that in order to change a fraction whose numerator is larger than its denominator to a mixed numeral, you divide the denominator into the numerator.

e.g. $\frac{24}{5}$ means $24 \div 5$ which equals $4\frac{4}{5}$.

$$\begin{array}{r} 4 \\ 5\,\overline{)\,24} \\ \underline{20} \\ 4 \end{array}$$

We can use the same idea to change fractions to decimals. Divide the denominator into the numerator. Here we can apply some of the principles that you learned for division of decimals (Lesson 14). Remember to insert zeros when necessary.

EXAMPLE: Change $\frac{1}{4}$ to a decimal.

Solution

$$\begin{array}{r} .25 \\ 4\,\overline{)\,1.00} \\ \underline{8}^{x} \\ 20 \\ \underline{20} \\ 0 \end{array} \qquad \text{or} \qquad \begin{array}{r} .25 \\ 4\,\overline{)\,1.0{}_{2}0} \end{array}$$

$\frac{1}{4}$ is the same as .25.

Assignment 2

Change each fraction to a decimal. (Divide until you obtain a remainder of 0.)

1. $\frac{1}{2}$

2. $\frac{1}{5}$

3. $\frac{3}{8}$

4. $\frac{3}{10}$

5. $\frac{5}{16}$

6. $\frac{7}{20}$

7. $\frac{6}{25}$

8. $\frac{7}{32}$

In each of the above cases you should have found that the division "came out evenly" if you continue to divide and insert zeros correctly. If a fraction can be converted to a decimal with remainder zero, it is called a TERMINATING decimal. A terminating decimal will always be equal to a fraction whose denominator is a power of 2, or 5, or 2 and 5.

Self-correcting Exercise 3

Give the complete factorization of each number. Tell whether the number is a power of 2, a power of 5 or a power of 2 and 5.

1. 2 = ______________________________ ______________
2. 4 = ______________________________ ______________
3. 5 = ______________________________ ______________
4. 8 = ______________________________ ______________
5. 10 = ______________________________ ______________
6. 16 = ______________________________ ______________
7. 20 = ______________________________ ______________
8. 25 = ______________________________ ______________
9. 32 = ______________________________ ______________
10. 40 = ______________________________ ______________
11. 50 = ______________________________ ______________
12. 64 = ______________________________ ______________

State whether the fraction will be a terminating decimal.

13. $\frac{12}{24}$ ______________ Why? ____

____

____

14. $\frac{1}{6}$ ______________ Why? ____

____

____

15. $\frac{3}{10}$ ____________ Why? ____

____

____

Corrections are on page 27.

Topic Three: Repeating decimal

Sometimes division problems do not "come out evenly" even if we use decimals.

EXAMPLE: Change $\frac{1}{3}$ to a decimal.

Solution

```
       .3333                                      .3333   remainder 1
  3 ) 1.0000                   or        3 ) 1.0000
      9                                            
     10
      9
      10
       9
       10
        9
        1
```

You can see from the above example that the digits in the quotient are reoccurring or repeating. It is obvious that if we continue to insert zeros, the quotient will still be all threes. Also you will notice that the remainder, before you inserted the next zero, was the same for each step (1). This type of decimal, in which the digits in the quotient repeat, is called a REPEATING decimal.

Sometimes more than one digit will reoccur in the quotient.

```
                                  .2727
e.g.   3       or          11 ) 3.0000
      11                        2 2
                                  80
                                  77
                                   30
                                   22
                                    80
                                    77
                                     3
```

In this particular case two digits are repeating in a special order.

The number of repeating digits will always be at least one less than the divisor.

Self-correcting Exercise 4

Give the maximum number of repeating digits that could be obtained in a decimal which expresses each fraction.

1. $\frac{2}{3}$ ____________________

2. $\frac{1}{6}$ ____________________

3. $\frac{4}{9}$ ____________________

4. $\frac{2}{7}$ ____________________

5. $\frac{5}{11}$ ____________________

Corrections are on page 27.

There are many different ways of writing repeating decimals. We shall use only one method. The digits which repeat are called the REPETEND of the decimal. A bar is placed over the repeating digits to show that they reoccur again and again.

e.g. $\frac{1}{3}$ was .3333 remainder 1. Using the above method, we say $\frac{1}{3} = .\overline{3}$. This is read "one third is equal to point three repetend three".

$\frac{3}{11}$ was .2727 remainder 3. We say $\frac{3}{11} = .\overline{27}$ which is read "three elevenths is equal to point two seven repetend two seven".

Assignment 3

Change each fraction to a decimal. Below your work write the decimal using the repetend symbol.

1. $\frac{3}{7}$

$$\begin{array}{r@{}l} & .428571 \\ 7\,\big) & \overline{3.000000} \\ & \times\times\times\times\times \\ & \underline{2\,8} \\ & 20 \\ & \underline{14} \\ & 60 \\ & \underline{56} \\ & 40 \\ & \underline{35} \\ & 50 \\ & \underline{49} \\ & 10 \\ & \underline{7} \\ & 3 \end{array}$$

or

$$\begin{array}{r@{}l} & .428571 \text{ remainder } 3 \\ 7\,\big) & \overline{3.0_{2}0_{6}0_{4}0_{5}0_{1}0} \end{array}$$

$$\frac{3}{7} = .\overline{428571}$$

2. $\frac{2}{3}$

3. $\frac{2}{6}$

4. $\frac{2}{9}$

5. $\frac{4}{11}$

6. $\frac{5}{13}$

Sometimes the repetend does not include all of the digits to the right of the decimal.

e.g. $\frac{5}{6}$

$$6\,\overline{)\,5.000}^{\;.833}$$

4 8ˣˣ
20
18
20
18
2

or

.833 remainder 2
6 ⟌ 5.0₂0₂0

$\frac{5}{6} = .8\overline{3}$ The bar is shown above the three because only the three reoccurs.

Assignment 4

Change each fraction to a decimal. Below your work write the decimal using the repetend symbol.

1. $\frac{5}{12}$

.4166
12 ⟌ 5.0000
48
20
12
80
72
80
72
8

$\frac{5}{12} = .41\overline{6}$

2. $\frac{1}{6}$

3. $\frac{4}{15}$

4. $\frac{7}{18}$

Change each fraction to a decimal. Write the decimal in its correct form below the work you did. Also state whether it is a repeating or terminating decimal.

5. $\frac{3}{5}$ $5 \overset{.6}{\overline{)3.0}}$

6. $\frac{4}{7}$

$\frac{3}{5} = .6$

terminating decimal

7. $\frac{4}{9}$

8. $\frac{5}{8}$

9. $\frac{7}{6}$

10. $\frac{12}{11}$

11. $\frac{1}{13}$

12. $\frac{7}{15}$

Arrange these numbers in order, beginning with the least number.

13. .375 .037 .073 .307 .357 .370

14. .4086 .406 .4 .468 .4068 .46

15. $.\overline{6}$ .6 .63 .601 $.1\overline{6}$ .61

16. .12 $.\overline{12}$ $.1\overline{2}$ .122 .21 .012

Marks

REVIEW

Carefully restudy Lessons 11, 12, 13, 14 and the first part of Lesson 15. Notice where you made errors. Try to eliminate the errors (similar to those you made earlier) from this review.

Express each fraction as a mixed numeral.

1 1. $\frac{3}{2} =$ ______________________________

1 2. $\frac{5}{3} =$ ______________________________

1 3. $\frac{11}{4} =$ ______________________________

1 4. $\frac{23}{5} =$ ______________________________

1 5. $\frac{17}{6} =$ ______________________________

1 6. $\frac{77}{8} =$ ______________________________

1 7. $\frac{31}{9} =$ ______________________________

1 8. $\frac{317}{100} =$ ______________________________

Express each mixed numeral as a fraction.

1 9. $3\frac{1}{2} =$ ______________________________

1 10. $5\frac{2}{3} =$ ______________________________

1 11. $2\frac{3}{4} =$ ______________________________

1 12. $1\frac{2}{5} =$ ______________________________

1 13. $8\frac{1}{6} =$ ______________________________

1 14. $6\frac{1}{7} =$ ______________________________

1 15. $46\frac{7}{8} =$ ______________________________

1 16. $5\frac{9}{20} =$ ______________________________

rks

Put the numbers in order of size beginning with the least one.

5 17. $1\frac{7}{8}$ $\frac{22}{7}$ $2\frac{1}{3}$ $\frac{99}{30}$ $\frac{64}{25}$

5 18. $7\frac{1}{9}$ $\frac{47}{6}$ $8\frac{1}{4}$ $\frac{67}{8}$ $\frac{153}{20}$

Give the reciprocal.

1 19. 8

1 20. $3\frac{1}{3}$

1 21. $\frac{9}{5}$

1 22. $\frac{6}{7}$

1 23. $\frac{1}{9}$

1 24. $\frac{9}{15}$

1 25. $2\frac{5}{16}$

1 26. $4\frac{7}{8}$

Give the standard name of each expression. Express the answer in its simplest form or as a mixed numeral.

2 27. $\frac{5}{8} \times \frac{3}{10} =$

2 28. $\frac{16}{5} \times \frac{1}{4} =$

2 29. $\frac{8}{3} \times \frac{3}{8} =$

2 30. $\frac{14}{5} \times \frac{15}{6} =$

Marks

2 31. $6 \times \frac{1}{3} =$ ____________________

2 32. $6\frac{2}{3} \times \frac{9}{10} =$ ____________________

2 33. $6\frac{1}{2} \times 2 =$ ____________________

2 34. $4\frac{1}{14} \times 2\frac{11}{14} =$ ____________________

2 35. $\frac{5}{12} \div \frac{1}{2} =$ ____________________

2 36. $\frac{11}{12} \div \frac{4}{3} =$ ____________________

2 37. $\frac{22}{5} \div \frac{1}{10} =$ ____________________

2 38. $\frac{9}{4} \div \frac{8}{3} =$ ____________________

2 39. $3 \div \frac{6}{10} =$ ____________________

2 40. $2\frac{7}{8} \div \frac{3}{4} =$ ____________________

2 41. $8\frac{1}{3} \div 10 =$ ____________________

2 42. $1\frac{5}{9} \div 3\frac{1}{3} =$ ____________________

Write the following numbers using words.

1 43. .003

____

1 44. .0527

____

____

rks

45. .461

46. .8

47. 3.72

48. 9.707

Write each number using digits.

1 49. thirty-five thousandths

1 50. one hundred and one hundredth

1 51. one hundred sixty-two thousandths

1 52. eighteen ten-thousandths

1 53. two hundred eighty-five thousand

Marks

10 Consider the number 3267.580149

54. The digit in the hundreds' place is __________.

55. The digit in the hundred-thousandths' place is __________.

56. The digit in the hundredths' place is __________.

57. The digit in the millionths' place is __________.

58. The digit in the ones' place is __________.

59. The digit in the tens' place is __________.

60. The digit in the ten-thousandths' place is __________.

61. The digit in the tenths' place is __________.

62. The digit in the thousands' place is __________.

63. The digit in the thousandths' place is __________.

Express each fraction as a decimal.

2 64. $\frac{7}{8}$ 65. $\frac{3}{40}$

2 66. $\frac{80}{9}$ 67. $\frac{9}{10000}$

arks

2 68. $\frac{21}{320}$ 69. $\frac{17}{5}$

Express each decimal as a fraction. Then express it in its lowest terms.

2 70. 3.1416 = ______________________________

2 71. .002 = ______________________________

2 72. 16.4 = ______________________________

2 73. 3.12 = ______________________________

2 74. 59 = ______________________________

2 75. .00040 = ______________________________

Write the numbers in expanded form.

1 76. 1900.002

____

1 77. .459

____

1 78. 15.67

____

Marks

1 79. .000409

____

Name the decimal expressed by each expanded form.

1 80. $0(10^0) + 6\left(\frac{1}{10^1}\right)$

____

1 81. $0(10^0) + 3\left(\frac{1}{10^1}\right) + 5\left(\frac{1}{10^2}\right)$

____

1 82. $2\left(\frac{1}{10^1}\right) + 2\left(\frac{1}{10^2}\right) + 5\left(\frac{1}{10^3}\right)$

____

1 83. $3(10^0) + 4\left(\frac{1}{10^1}\right) + 8\left(\frac{1}{10^2}\right)$

____

1 84. $4(10^4) + 0(10^3) + 0(10^2) + 1(10^1) + 0(10^0) + 0\left(\frac{1}{10^1}\right) + 0\left(\frac{1}{10^2}\right)$

____

Round each number to the nearer thousandth.

1 85. 3.14159 ____________________

1 86. .0213566 ____________________

1 87. $.\overline{6}$ ____________________

1 88. $.0\overline{152}$ ____________________

1 89. $.00\overline{4}$ ____________________

1 90. 167.035 ____________________

Marks

State the maximum possible number of digits in the repetend of each of these repeating decimals.

1 91. $\frac{4}{7}$ ____________________

1 92. $\frac{10}{13}$ ____________________

1 93. $\frac{5}{71}$ ____________________

1 94. $\frac{3}{98}$ ____________________

10 95. Name ten fractions which express terminating decimals.

____

Indicate whether the following are repeating or terminating decimals. For each repeating decimal, name the repetend.

			Type of decimal	Repetend
2	96.	.59	______________	______________
2	97.	$146.3\overline{2}$	______________	______________
2	98.	.754	______________	______________
2	99.	$5.\overline{734}$	______________	______________
2	100.	$93.\overline{78}$	______________	______________

Find each quotient rounded to the nearer hundredth.

2 101. $.967 \div 1.3$ 102. $.9372 \div .18$

Marks

2 103. 6.4798 ÷ .63 104. 87.35 ÷ 24

Divide until the quotient is exact or until the hundredths' digit is obtained.

2 105. 723 ÷ 317 106. 674 ÷ 25

2 107. 17.8 ÷ 2.53 108. 1053 ÷ .032

Marks

Give the standard name for each expression.

1 109. 1.49×10^2 = ______________________

1 110. 36.002×10^4 = ______________________

1 111. 3.14159×10^4 = ______________________

1 112. 171.33×10^3 = ______________________

1 113. $.00081 \times 10^3$ = ______________________

1 114. $49.72 \div 100$ = ______________________

1 115. $3.001 \div 10^2$ = ______________________

1 116. $37.04 \div 10^4$ = ______________________

1 117. $.312 \div 10^3$ = ______________________

1 118. $3157 \div 10^5$ = ______________________

Find the products.

4 119. $\begin{array}{r}1645\\ \underline{32}\end{array}$ 120. $\begin{array}{r}4.25\\ \underline{3.67}\end{array}$ 121. $\begin{array}{r}5.731\\ \underline{3.43}\end{array}$ 122. $\begin{array}{r}3.9\\ \underline{.08}\end{array}$

4 123. $\begin{array}{r}1.11\\ \underline{.909}\end{array}$ 124. $\begin{array}{r}.956\\ \underline{.105}\end{array}$ 125. $\begin{array}{r}.0316\\ \underline{2.196}\end{array}$ 126. $\begin{array}{r}.039\\ \underline{.01}\end{array}$

Total - 170 Marks

Key to Self-correcting Exercises in Lesson 15

Exercise 1, page 2

1.
$$\begin{array}{r} 50 \\ 89\,\overline{)\,445\underset{\times}{0}} \\ \underline{445} \\ 00 \end{array}$$

Divide 89 into 445 because 89 does not divide into 4 or 44.
Multiply 5 × 89. Then subtract.
Bring down 0. 89 divides into 0 zero times.

$4450 \div 89 = 50$

2.
$$\begin{array}{r} 96 \\ 54\,\overline{)\,518\underset{\times}{4}} \\ \underline{486} \\ 324 \\ \underline{324} \\ 0 \end{array}$$

Divide 54 into 518 because 54 does not divide into 5 or 51. Multiply 54 × 9. Subtract.
Bring down 4. Divide 54 into 324.
Multiply and subtract.

$5184 \div 54 = 96$

3.
$$\begin{array}{r} 234 \\ 127\,\overline{)\,297\underset{\times}{1}\underset{\times}{8}} \\ \underline{254} \\ 431 \\ \underline{381} \\ 508 \\ \underline{508} \\ 0 \end{array}$$

Bring down 1.

Bring down 8.

$29718 \div 127 = 234$

Exercise 2, page 3

1. $.66 \div .44 = 66 \div 44$

$$\begin{array}{r} 1.5 \\ 44\,\overline{)\,66.\underset{\times}{0}} \\ \underline{44} \\ 220 \\ \underline{220} \\ 0 \end{array}$$

Add 1 zero.

$.66 \div .44 = 1.5$

2. $32.64 \div 2.4 = 326.4 \div 24$

$$\begin{array}{r} 13.6 \\ 24\,\overline{)\,32\underset{\times}{6}.\underset{\times}{4}} \\ \underline{24} \\ 86 \\ \underline{72} \\ 144 \\ \underline{144} \\ 0 \end{array}$$

No zeros required.

$32.64 \div 2.4 = 13.6$

3. $13.34 \div 1.45 = 1334 \div 145$

$$\begin{array}{r@{}l} & 9.2 \\ 145\big) & \overline{1334.\underset{x}{0}} \\ & \underline{1305} \\ & 290 \\ & \underline{290} \\ & 0 \end{array}$$

One zero required.

$13.34 \div 1.45 = 9.2$

Exercise 3, page 6

1. $2 = 2$ a power of 2
2. $4 = 2 \times 2$ a power of 2
3. $5 = 5$ a power of 5
4. $8 = 2\times2\times2$ a power of 2
5. $10 = 2\times5$ a power of 2 and 5
6. $16 = 2\times2\times2\times2$ a power of 2
7. $20 = 2\times2\times5$ a power of 2 and 5
8. $25 = 5\times5$ a power of 5
9. $32 = 2\times2\times2\times2\times2$ a power of 2
10. $40 = 2\times2\times2\times5$ a power of 2 and 5
11. $50 = 2\times5\times5$ a power of 2 and 5
12. $64 = 2\times2\times2\times2\times2\times2$ a power of 2
13. yes, a terminating decimal
    $\frac{12}{24}$ is the same as $\frac{1}{2}$. 2 is a power of 2.
14. no, not a terminating decimal
    $6 = 2 \times 3$ 6 is a power of 2 and 3.
15. yes, a terminating decimal
    $10 = 2 \times 5$ 10 is a power of 2 and 5.

Exercise 4, page 8

1. 2 digits
2. 5 digits
3. 8 digits
4. 6 digits
5. 10 digits

End of Lesson 15

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number __________

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned Teacher: __________

Lesson Grading: __________

Additional Grading E/R/P Code: __________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Per Cent

Topic One: Ratios

In your lifetime you have probably heard rates quoted in many different ways. For example, you have often heard expressions such as: 25 miles per hour, 4 cans of soup for 89¢ and 3 cups of sugar to 1 cup of water.

There are many different mathematical ways to write ratios which express rates. Let's consider the rate "25 miles per hour". 25 miles per hour is actually 25 miles per 1 hour. This ratio could be written $\frac{25}{1}$, 25/1 or 25:1. All of these ratios are read "twenty-five to one". Notice that the units of measure are omitted when writing ratios. Only the numbers from the rate are used. Since it is necessary to use these ratios in computation, in the future, we will use the form shown by $\frac{25}{1}$ (which resembles a fraction).

Self-correcting Exercise 1

Write ratios to express the following rates.

1. 54 correct answers to 60 questions ________
2. 35 gallons of water to $4\frac{2}{5}$ bags of cement ________
3. 35 gallons of water to 4.4 bags of cement ________
4. 85 hits to 190 times at bat ________
5. $4\frac{1}{4}$ inches for each 5 feet ________
6. 280 miles for 7 hours travel ________
7. $6\frac{1}{2}$ oranges to $2\frac{1}{4}$ lemons ________
8. $.03 to $2.25 ________
9. 8 feet to 312 pounds ________
10. $15.00 for 250 pounds ________

Corrections are on page 19.

Just as fractions can be changed to their simplest form, so can ratios.

e.g. If there are 4 boys for every 8 girls in a class, this ratio is the same as 1 boy for every 2 girls.

$$\frac{4}{8} = \frac{1}{2}$$

Thus, a ratio can be written in its simplest form by dividing both numbers in the ratio by their greatest common factor.

$$\frac{4}{8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$$ Divide both parts by the greatest common factor, 4.

Assignment 1

First give the ratio; then give the simplest ratio.

1. A class has 28 boys and 7 girls.

   a. The ratio of the boys to the girls is $\frac{28}{7} = \frac{4}{1}$.

   b. The ratio of the girls to the boys is ______

   c. The ratio of the boys to the whole class is ______

   d. The ratio of the girls to the whole class is ______

2. During target practice, Peter scored 22 hits and 10 misses.

   a. The ratio of the hits to the misses is ______

   b. The ratio of the hits to the total shots is ______

   c. The ratio of the misses to the total shots is ______

3. An automobile travels 112 miles and uses 7 gallons of gasoline. What is the ratio of miles to gallons?

   ______

4. A typist was paid $6.00 for typing 12 letters. What is the ratio of dollars to letters?

   ______

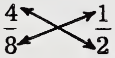
5. A jet-prop passenger plane cruises at 350 miles per hour. A piston engine airplane cruises at 250 miles per hour. What is the ratio of the speed of the jet-prop plane to the speed of the piston engine plane?

   ______

The upper part of a ratio (similar to the numerator of a fraction) is called the FIRST COMPONENT of the ratio. The lower part of a ratio (similar to the denominator of a fraction) is called the SECOND COMPONENT of the ratio.

Let's go back to an earlier example "4 boys to 8 girls is the same ratio as 1 boy to 2 girls". This can be shown by $\frac{4}{8} = \frac{1}{2}$. To show that this is true, we can use the ratio test. The ratio test states that for two ratios to be equal the product of the first component of the first ratio and the second component of the second ratio must equal the product of the first component of the second ratio and the second component of the first ratio.

e.g. $\frac{4}{8}$ and $\frac{1}{2}$



$4 \times 2 \boxed{\overset{?}{=}} 1 \times 8$ ← Use this symbol until you are sure the 2 sides are equal.

$8 = 8$

Since 8 does equal 8, we know that $\frac{4}{8}$ does equal $\frac{1}{2}$. Therefore we know that these two ratios are the same (or equal).

The order in which we multiply the components does not matter. It is only necessary that we write the cross-products and decide whether they are equal.

e.g. Use $\frac{4}{8}$ and $\frac{1}{2}$ again.

We could write:

$2 \times 4 \overset{?}{=} 8 \times 1$ or $4 \times 2 \overset{?}{=} 8 \times 1$ or $2 \times 4 \overset{?}{=} 1 \times 8$.

The above statements are all the same.

Self-correcting Exercise 2

Perform the ratio test, then tell whether or not the ratios are equal.

1. $\frac{55}{22}$ and $\frac{5}{2}$

2. $\frac{9}{12}$ and $\frac{54}{72}$

3. $\frac{5}{9}$ and $\frac{35}{54}$

4. $\frac{2}{3}$ and $\frac{20}{30}$

5. $\frac{1}{7}$ and $\frac{7}{42}$

6. $\frac{63}{45}$ and $\frac{7}{5}$

Corrections are on page 19.

The ratio test can be used to find an equal ratio if you know either the first component or the second component of the second ratio.

EXAMPLE: What is the cost of 5 cans of soup if 3 cans of this soup cost 54¢?

Solution

The ratio of the number of cans of soup to the cost is $\frac{3}{54}$.

If we let x be the cost of 5 cans of soup, the second ratio comparing the number of cans of soup to the cost is $\frac{5}{x}$.

These ratios must be equal.

$$\xrightarrow{\text{Ratio of no. of cans to cost}} \boxed{\frac{3}{54}} = \frac{5 \leftarrow \text{no. of cans}}{x \leftarrow \text{cost}}$$

Using the ratio test we know that $3 \times x = 5 \times 54$.

$3x = 270$ (3x is the same as 3 times x.)

(Note that both ratios give the no. of cans as the first component and the cost as the second component.)

Because of the relationship between multiplication and division we know that 3x = 270 is the same as

$x = 270 \div 3$. (Factor × factor = product is the same as product ÷ factor = factor.)

$x = 90$

5 cans of soup will cost 90¢ if 3 cans of soup cost 54¢.

Self-correcting Exercise 3

Use the ratio test to find the missing component in each pair of ratios.

1. $\frac{25}{10} = \frac{15}{x}$

2. $\frac{x}{3} = \frac{10}{5}$

3. $\frac{7}{21} = \frac{x}{3}$

4. $\frac{50}{x} = \frac{1}{5}$

5. $\frac{9}{15} = \frac{x}{50}$

6. $\frac{x}{1} = \frac{55}{11}$

Corrections are on page 20.

Assignment 2

Use the ratio test to decide whether each pair of ratios are equal. Write "true" if they are equal; write "false" if they are not equal.

1. $\frac{1}{2} = \frac{6}{8}$ false

$1 \times 8 \overset{?}{=} 6 \times 2$

$8 \neq 12$

(This symbol is used to express "is not equal to.")

2. $\frac{21}{35} = \frac{24}{40}$ ________

3. $\frac{9}{1} = \frac{99}{11}$ ________

4. $\frac{10}{4} = \frac{15}{6}$ ________

5. $\frac{101}{1} = \frac{202}{2}$ ________

6. $\frac{1}{3} = \frac{9}{27}$ ________

Use the ratio test to find the missing component in each pair of ratios.

7. $\frac{x}{100} = \frac{1}{2}$

$x \times 2 = 100 \times 1$

$2x = 100$

$x = 100 \div 2$

$x = 50$

8. $\frac{13}{26} = \frac{5}{x}$

9. $\frac{x}{90} = \frac{21}{14}$

10. $\frac{x}{9} = \frac{40}{12}$

11. $\frac{1}{3} = \frac{8}{x}$

12. $\frac{12}{x} = \frac{15}{40}$

For each problem:

- tell what the placeholder represents,
- write an equation,
- show how you find the answer and
- write a sentence answering the question asked.

13. Allen received 98 votes in a student council election. For each 2 votes that Allen received, Roger received 3 votes. How many votes did Roger receive in the election?

Placeholder: Let x be the number of votes Roger received.

Equation: $\frac{2}{3} = \frac{98 \leftarrow \text{Allen's votes}}{x \leftarrow \text{Roger's votes}}$

Ratio of Allen's votes to Roger's

$2 \times x = 98 \times 3$

$2x = 294$

$x = 294 \div 2$

Answer: $x = 147$

Computation

$\begin{array}{r} 98 \\ \underline{3} \\ 294 \end{array}$

$\begin{array}{r} 147 \\ 2\overline{)294} \end{array}$

Sentence: Roger received 147 votes.

14. Last year the Gibson Auto Company sold 72 used cars. The company sold 8 used cars for each 6 new cars that they sold. How many new cars did the company sell last year?

Placeholder: Let n be ____________________

Equation: $\frac{8}{6} = \frac{\quad \leftarrow \text{used cars}}{\quad \leftarrow \text{new cars}}$

Ratio of used to new cars

Computation

Answer: ____________________

Sentence: ____________________

15. Jerry was paid $6 for delivering 24 grocery orders. At this rate, how many dollars would he be paid for delivering 16 orders?

Placeholder: ______________________________

Equation: ____________________ Computation

____

____

____

Answer: ______________________________

Sentence: ______________________________

16. Mrs. Adams bought 18 ears of sweet corn priced at 6 ears for 29¢. How many cents did she pay for the sweet corn?

Placeholder: ______________________________

Equation: ____________________ Computation

____

____

____

Answer: ______________________________

Sentence: ______________________________

Topic Two: Changing fractions and decimals to per cent

Per cents are obtained by finding an equal ratio whose second component is 100. The first component of this ratio is written in front of the per cent symbol. Per cent actually means per 100.

e.g. $\frac{24}{100}$ is 24%.

EXAMPLE: Write $\frac{1}{5}$ as a per cent.

Solution

Find a ratio, equal to $\frac{1}{5}$, whose second component is 100.

$$\frac{1}{5} = \frac{x}{100}$$

$1 \times 100 = x \times 5$

$100 = 5x$

$100 \div 5 = x$

$20 = x$

$\frac{1}{5}$ is the same as $\frac{20}{100}$ which is 20%. $\frac{1}{5}$ = 20%

Self-correcting Exercise 4

Write each of the following as a per cent.

1. $\frac{78}{100}$ = ____________

2. $\frac{125}{100}$ = ____________

3. $\frac{1}{100}$ = ____________

4. $\frac{10}{100}$ = ____________

For each ratio find an equal ratio that has a second component of 100. Then write it as a per cent.

5. $\frac{4}{5}$

6. $\frac{9}{50}$

7. $\frac{32}{64}$

8. $\frac{90}{30}$

Corrections are on page 21.

Decimals can also be converted to per cent. First write the decimal as a fraction. This fraction is the same as a ratio. Find an equal ratio whose second component is 100. Then change it to a per cent.

EXAMPLE: Write 1.024 as a per cent.

Solution

Change 1.024 to a fraction. $1.024 = 1\frac{24}{1000} = \frac{1024}{1000}$

Find an equal ratio whose second component is 100.

$$\frac{1024}{1000} = \frac{x}{100}$$

$$\begin{aligned} 1024 \times 100 &= x \times 1000 \\ 102400 &= 1000x \\ 102400 \div 1000 &= x \\ 102.4 &= x \end{aligned}$$

$$\frac{1024}{1000} = \frac{102.4}{100}$$

Therefore 1.024 = 102.4%.

Note that in changing the decimal 1.024 to a per cent the decimal moved two places to the right and a per cent sign was added.

An easy-to-remember rule for changing decimals to per cents is to move the decimal two places to the right of its original position and place a per cent sign to the right of the number.

e.g. 4.732 = 473.2%

Assignment 3

Write each ratio as a per cent.

1. $\frac{25\frac{1}{2}}{100}$ = $25\frac{1}{2}\%$

2. $\frac{1.72}{100}$ = ______

3. $\dfrac{\frac{1}{10}}{100}$ = __________

4. $\dfrac{100.6}{100}$ = __________

5. $\dfrac{.06}{100}$ = __________

6. $\dfrac{133\frac{1}{3}}{100}$ = __________

Change each decimal to a per cent.

7. .28 = 28 %

8. .06 = __________

9. 1 = __________

10. .9 = __________

11. 2.05 = __________

12. .1056 = __________

You will remember that some fractions are equal to repeating decimals. When we change this type of fraction to a per cent, the remainder is expressed as a fraction.

EXAMPLE: Change $\frac{2}{3}$ to a per cent.

Solution

Find a ratio, equal to $\frac{2}{3}$, whose second component is 100.

$$\frac{2}{3} = \frac{x}{100}$$

$$2 \times 100 = 3 \times x$$

$$200 = 3x$$

$$200 \div 3 = x$$

$$66\frac{2}{3} = x$$

$$\begin{array}{r} 66\frac{2}{3} \\ 3\overline{)200} \end{array}$$

$\frac{2}{3}$ is the same as $\dfrac{66\frac{2}{3}}{100}$ which is $66\frac{2}{3}\%$. $\frac{2}{3} = \underline{66\frac{2}{3}\%}$

Self-correcting Exercise 5

Write each fraction as a per cent.

1. $\frac{1}{6}$

2. $\frac{7}{11}$

3. $\frac{4}{9}$

4. $\frac{6}{7}$

Corrections are on page 21.

Topic Three: Changing per cents to fractions and decimals

To change a per cent to a fraction, we need only to write the per cent as a ratio whose second component is 100. We can then change it to a fraction in its simplest form.

EXAMPLE: Change 120% to a fraction.

Solution

120% is the same as $\frac{120}{100}$.

$$\frac{120}{100} = \frac{6}{5} = 1\frac{1}{5}$$

120% is the same as $\underline{1\frac{1}{5}}$.

Assignment 4

Change each per cent to a fraction in its simplest form.

1. 2% = $\frac{2}{100} = \frac{1}{50}$

2. 500% = __________

3. 71% = __________

4. 5% = __________

5. 90% = __________

6. 1000% = __________

7. 98% = __________

8. 43% = __________

To change a per cent to a decimal, we need only to divide the number by 100.

e.g. $25\% = \frac{25}{100}$ which is the same as 25 ÷ 100.

To divide by 100 we move the decimal two places to the left.

e.g. $25\% = \frac{25}{100} = .25$

An easy-to-remember rule for changing per cents to decimals is to move the decimal two places to the left of its original position and drop the per cent sign.

e.g. 63.2% = .632
5% = .05
.3% = .003

Note that zeros must be inserted when the decimal is moved 2 places to the left.

Assignment 5

Change each per cent to a decimal.

1. 17% = .17

2. 150% = __________

3. 12.5% = __________

4. .015% = __________

5. .75% = __________

6. 8% = __________

Topic Four: Problems involving per cent

You have learned that a per cent can be written as a ratio whose second component is always 100. In solving problems, we can always write the given per cent as a ratio of some amount over 100. From the other information given in the problem, we can make another ratio which is equal to the per cent ratio. We can then find the number represented by the placeholder in one of the ratios.

We can always condense per cent problems to a single, concise sentence.

EXAMPLE: Write this problem as a single sentence.
Jack took a mathematics test that contained 40 problems. He answered 85% of the questions correctly. How many of the 40 questions did he answer correctly?

Solution

We will let x be the number of questions answered correctly.

x is 85% of 40. This means that some number of questions is the same as 85% of 40 questions.

Self-correcting Exercise 6

Condense each problem to one sentence.

1. 540 of the 720 students enrolled at Lee School live more than five blocks from school. What per cent of the students live more than five blocks from the school?

____

2. Last summer Doris saved 54% of her earnings. She saved $81. How many dollars did Doris earn last summer?

____

3. Harold saves $6 out of each $10 that he earns. The money Harold saves is what per cent of the money he earns?

____

4. Dolores was reading a 320-page book. She read 30% of the pages in one evening. How many pages did she read that evening?

____

5. A weather bureau reported that 40% of the days in April were cloudy. How many days in April were cloudy?

____

Corrections are on page 22.

From the single sentence we can make two ratios which are equal to each other. One ratio will always represent the per cent which is written as some amount over 100. The other ratio will always compare the "part" to the "whole". The number which is the "part" is written before "is" in the sentence. The number which is the "whole" is written after "of" in the sentence. Since one part of the ratios is a placeholder, we must find the number it represents.

EXAMPLE: Last year Mr. Roberts saved 5% of his take-home pay. He saved \$270. What was his take-home pay?

Solution

Placeholder: Let x be the take-home pay in dollars.

Simple sentence: 270 is 5% of x.

$$\text{Equation:}\quad \frac{\overset{\text{savings}\searrow}{270}}{\underset{\text{take-home pay}\rightarrow}{x}} = \boxed{\frac{5}{100}} \leftarrow \text{Ratio of savings to take-home pay}$$

$$\begin{aligned} 270 \times 100 &= 5 \times x \\ 27000 &= 5x \\ 27000 \div 5 &= x \end{aligned}$$

Answer: $5400 = x$

Computation

$$\begin{array}{r} 5400 \\ 5\overline{)\,27_{2}000} \end{array}$$

Statement: Mr. Roberts' take-home pay was \$5400.

Self-correcting Exercise 7

For each problem:

- tell what the placeholder represents,
- condense the problem into one simple sentence and
- write an equation.

1. June sold 8 tickets to the school play and Martin sold 12 tickets. The number of tickets Martin sold is what per cent of the number of tickets June sold?

Placeholder: ______________________________

Simple sentence: ______________________________

Equation: __________________________

2. One year the Toronto Maple Leafs had 16 players on their hockey team. 25% of these players played center ice. How many center ice players did the Toronto Maple Leafs have that year?

Placeholder:

Simple sentence:

Equation:

3. 20% of all the students at Long School are in the seventh grade. 92 students are enrolled in the seventh grade. How many students in all are enrolled at Long School?

Placeholder:

Simple sentence:

Equation:

Corrections are on page 22.

Assignment 6

For each problem:

- tell what the placeholder represents,
- condense the problem into one simple sentence,
- write an equation,
- find the answer and
- write a sentence answering the question asked.

1. During a basketball game, David made 7 baskets out of 20 tries. The number of baskets David made was what per cent of the number of tries?

Placeholder: Let x be the per cent comparing baskets to tries.

Simple sentence: 7 is $x\%$ of 20.

Equation: $\frac{7}{20} = \frac{x}{100}$

$7 \times 100 = x \times 20$

$700 = 20x$

$700 \div 20 = x$

Computation

$$\begin{array}{r} 35 \\ 20\overline{)700} \\ \underline{60} \\ 100 \\ \underline{100} \\ 0 \end{array}$$

Answer: $35 = x$

Statement: The number of baskets were 35% of the number of tries.

2. Henry worked 15 mathematics problems and had correct answers for 100% of them. How many correct answers did he have?

Placeholder: ____________________

Simple sentence: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

3. Bill sold his bicycle for $39. This was 65% of the price he had paid for it. How many dollars had he paid for the bicycle?

Placeholder: ____________________

Simple sentence: ____________________

Equation: ___ ________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

4. During a sale, Marilyn paid $30 for a coat that was regularly priced at $40. The sale price of the coat is what per cent of the regular price?

Placeholder: ____________________

Simple sentence: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

5. In 1940, a British Columbia newspaper estimated that the province's population by 1960 would be 175% of its population in 1940. In 1940, the population was 800,000. What was the estimated population of British Columbia for 1960?

Placeholder: ____________________

Simple sentence: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

6. 5% of the science club members have paid their yearly dues of 50¢. Four members have paid their dues. How many members does the science club have?

Placeholder: ____________________

Simple sentence: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

Key to Self-correcting Exercises in Lesson 16

Exercise 1, page 1

1. $\frac{54}{60}$
2. $\frac{35}{4\frac{2}{5}}$
3. $\frac{35}{4.4}$
4. $\frac{85}{190}$
5. $\frac{4\frac{1}{4}}{5}$
6. $\frac{280}{7}$
7. $\frac{6\frac{1}{2}}{2\frac{1}{4}}$
8. $\frac{.03}{2.25}$
9. $\frac{8}{312}$
10. $\frac{15.00}{250}$

Exercise 2, page 3

1.
$$\begin{tikzcd} 55 \arrow[dr, <->] & 5 \arrow[dl, <->] \\ 22 & 2 \end{tikzcd}$$
$$55 \times 2 \overset{?}{=} 5 \times 22$$
$$110 = 110$$
$\frac{55}{22}$ does equal $\frac{5}{2}$.

2.
$$\begin{tikzcd} 9 \arrow[dr, <->] & 54 \arrow[dl, <->] \\ 12 & 72 \end{tikzcd}$$
$$9 \times 72 \overset{?}{=} 54 \times 12$$
$$648 = 648$$
$\frac{9}{12}$ does equal $\frac{54}{72}$.

3. $$\begin{tikzcd} 5 \arrow[dr, leftrightarrow] & 35 \\ 9 \arrow[ur, leftrightarrow] & 54 \end{tikzcd}$$

$5 \times 54 \overset{?}{=} 35 \times 9$

$270 \neq 315$

$\frac{5}{9}$ does not equal $\frac{35}{54}$.

4. $$\begin{tikzcd} 2 \arrow[dr, leftrightarrow] & 20 \\ 3 \arrow[ur, leftrightarrow] & 30 \end{tikzcd}$$

$2 \times 30 \overset{?}{=} 20 \times 3$

$60 = 60$

$\frac{2}{3}$ does equal $\frac{20}{30}$.

5. $$\begin{tikzcd} 1 \arrow[dr, leftrightarrow] & 7 \\ 7 \arrow[ur, leftrightarrow] & 42 \end{tikzcd}$$

$1 \times 42 \overset{?}{=} 7 \times 7$

$42 \neq 49$

$\frac{1}{7}$ does not equal $\frac{7}{42}$.

6. $$\begin{tikzcd} 63 \arrow[dr, leftrightarrow] & 7 \\ 45 \arrow[ur, leftrightarrow] & 5 \end{tikzcd}$$

$63 \times 5 \overset{?}{=} 7 \times 45$

$315 = 315$

$\frac{63}{45}$ does equal $\frac{7}{5}$.

Exercise 3, page 5

1. $\frac{25}{10} = \frac{15}{x}$

$25 \times x = 15 \times 10$

$25x = 150$

$x = 150 \div 25$

$x = 6$

2. $\frac{x}{3} = \frac{10}{5}$

$x \times 5 = 10 \times 3$

$5x = 30$

$x = 30 \div 5$

$x = 6$

3. $\frac{7}{21} = \frac{x}{3}$

$7 \times 3 = x \times 21$

$21 = 21x$

$21 \div 21 = x$

$1 = x$

4. $\frac{50}{x} = \frac{1}{5}$

$50 \times 5 = 1 \times x$

$250 = 1x$

$250 = x$ (1x is the same as x.)

5. $\frac{9}{15} = \frac{x}{50}$

$9 \times 50 = x \times 15$

$450 = 15x$

$450 \div 15 = x$

$30 = x$

6. $\frac{x}{1} = \frac{55}{11}$

$x \times 11 = 55 \times 1$

$11x = 55$

$x = 55 \div 11$

$x = 5$

Exercise 4, page 9

1. 78%
2. 125%
3. 1%
4. 10%
5. $$\begin{aligned} \frac{4}{5} &= \frac{x}{100} \\ 4 \times 100 &= x \times 5 \\ 400 &= 5x \\ 400 \div 5 &= x \\ 80 &= x \\ \frac{4}{5} &= 80\% \end{aligned}$$

6. $$\begin{aligned} \frac{9}{50} &= \frac{x}{100} \\ 9 \times 100 &= x \times 50 \\ 900 &= 50x \\ 900 \div 50 &= x \\ 18 &= x \\ \frac{9}{50} &= 18\% \end{aligned}$$

7. $$\begin{aligned} \frac{32}{64} &= \frac{x}{100} \\ 32 \times 100 &= x \times 64 \\ 3200 &= 64x \\ 3200 \div 64 &= x \\ 50 &= x \\ \frac{32}{64} &= 50\% \end{aligned}$$

8. $$\begin{aligned} \frac{90}{30} &= \frac{x}{100} \\ 90 \times 100 &= x \times 30 \\ 9000 &= 30x \\ 9000 \div 30 &= x \\ 300 &= x \\ \frac{90}{30} &= 300\% \end{aligned}$$

Exercise 5, page 12

1. $$\begin{aligned} \frac{1}{6} &= \frac{x}{100} \\ 1 \times 100 &= x \times 6 \\ 100 &= 6x \\ 100 \div 6 &= x \\ 16\frac{2}{3} &= x \\ \frac{1}{6} &= 16\frac{2}{3}\% \end{aligned}$$

$$\begin{array}{rl} & 16\frac{4}{6} = 16\frac{2}{3} \\ 6 & \overline{)\,100} \\ & 4 \end{array}$$

2. $$\begin{aligned} \frac{7}{11} &= \frac{x}{100} \\ 7 \times 100 &= x \times 11 \\ 700 &= 11x \\ 700 \div 11 &= x \\ 63\frac{7}{11} &= x \\ \frac{7}{11} &= 63\frac{7}{11}\% \end{aligned}$$

$$\begin{array}{rr} & 63 \\ 11 & \overline{)\,700} \\ & \underline{66} \\ & 40 \\ & \underline{33} \\ & 7 \end{array}$$

3.
$$\begin{aligned}\frac{4}{9} &= \frac{x}{100}\\ 4 \times 100 &= x \times 9\\ 400 &= 9x\\ 400 \div 9 &= x\\ 44\tfrac{4}{9} &= x\\ \frac{4}{9} &= 44\tfrac{4}{9}\%\end{aligned}$$

$$\begin{array}{r} 44\frac{4}{9} \\ 9\overline{)400} \end{array}$$

4.
$$\begin{aligned}\frac{6}{7} &= \frac{x}{100}\\ 6 \times 100 &= x \times 7\\ 600 &= 7x\\ 600 \div 7 &= x\\ 85\tfrac{5}{7} &= x\\ \frac{6}{7} &= 85\tfrac{5}{7}\%\end{aligned}$$

$$\begin{array}{r} 85\frac{5}{7} \\ 7\overline{)600} \end{array}$$

Exercise 6, page 14

1. 540 is x% of 720.
2. 81 is 54% of x.
3. 6 is x% of 10.
4. x is 30% of 320.
5. x is 40% of 30.

Exercise 7, page 15

1. Let x be the per cent comparing the number of tickets Martin sold to the number of tickets June sold.

   12 is x% of 8.

   $$\frac{12}{8} = \boxed{\frac{x}{100}} \longleftarrow \text{Ratio of Martin's sales to June's}$$

2. Let x be the number of center ice players on the Toronto Maple Leafs' team.

   x is 25% of 16.

   $$\frac{x}{16} = \boxed{\frac{25}{100}} \longleftarrow \text{Ratio of center icemen to total number}$$

3. Let n be the total number of students enrolled at Long School.

   92 is 20% of n.

   $$\frac{92}{n} = \boxed{\frac{20}{100}} \longleftarrow \text{Ratio of } 7^{th} \text{ graders to total enrollment}$$

End of Lesson 16

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ______

Time Spent on Lesson ______

(If label is missing
or incorrect)

File Number ______

Lesson Number ______

FOR SCHOOL USE ONLY

Assigned
Teacher: ______

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: ______

Graded by: ______

Assignment Code: ______

Date Lesson Received: ______

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ______
Address ______
______
______
Postal Code ______

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

______
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Formulas

Topic One: Formulas

During your lifetime you will find it convenient to know a few basic formulas. In the next three lessons you will be given a few formulas that you will be expected to learn and use.

Besides memorizing formulas you must learn how to make replacements in them and calculate the values of different variables. Once you learn how to use a few formulas, you will be able to use formulas which you have not previously studied.

Formulas save time and work. There is less chance of error if you use a formula though often if you think through a problem situation carefully, you are able to calculate the answer without using a formula.

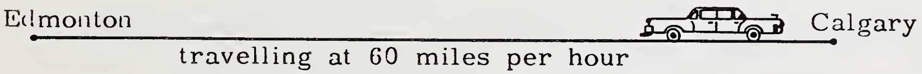
Topic Two: Distance formula

When you decide to go on a trip, you often consider three special factors that will affect your journey. They are:

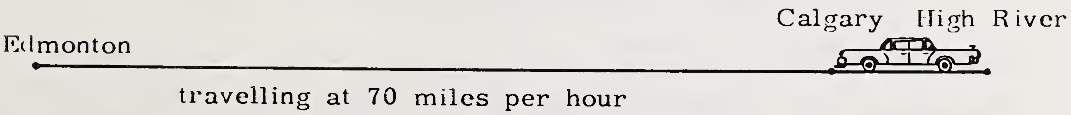
1. how far you are travelling.
2. how much time it will take.
3. the speed of the vehicle in which you are travelling.

These three things are related in special ways.

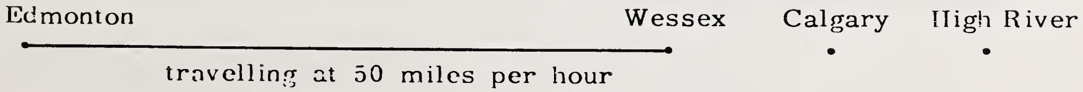
A. Let's consider a 3-hour trip.



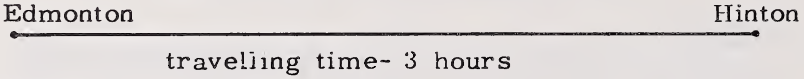
If in this time limit (3 hours) you increase your speed, you will also increase the distance you travel.



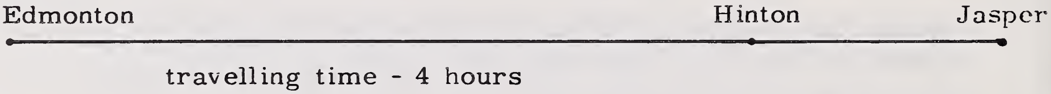
If in this time limit (3 hours) you decrease your speed, you will also decrease the distance you travel.



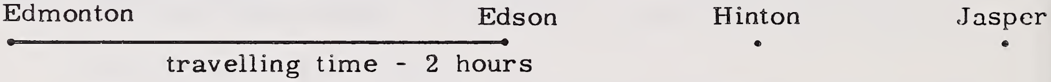
B. Let's consider a car travelling at 60 miles per hour.



If you increase the time during which you are travelling, you increase the distance you travel.



If you decrease the time during which you are travelling, you decrease the distance you travel.



C. Let's consider a trip of 400 miles.
e.g. At 50 miles per hour it would take 8 hours.

If you increase the speed at which you are travelling, you decrease the time it takes to travel this distance.
e.g. At 80 miles per hour it would take 5 hours.

If you decrease the speed at which you are travelling, you increase the time it takes to travel this distance.
e.g. At 40 miles per hour it would take 10 hours.

From this you can see that a change in time, distance or rate will affect one of the other variables. We can use a formula to show how these three variables are related. It is called the distance formula.

$\boxed{d = rt}$

d. distance travelled
r, rate of speed at which the distance was travelled
t, time required to travel the distance

rt is the same as r times t or $r \times t$.

The distance (d) travelled is measured in miles, feet, yards, meters or any other unit of length.
The rate of speed (r) is measured in miles per hour, feet per second. yards per minute, etc. Notice that it always contains a measure of distance, the word "per" and a measure of time.
The time (t) is measured in seconds. minutes, hours, days, years, etc.

We can use this formula (d = rt) to find solutions for problems about distance, speed and time.

EXAMPLE: How many miles does a freight train travel in 5 hr. at a rate of 20 mi. per hour?

Solution

Sort out the information given in the problem. Show which amounts will replace the variables in the formula.

r = 20 mi. per hour
t = 5 hr.
d = rt Replace r by 20 because the rate is 20 mi. per hr.
d = 20(5) Replace t by 5 because the time is 5 hr.
d = 100

The distance travelled by the freight train is 100 miles.
(Note: The distance is in miles because the rate is in miles per hour.)

Self-correcting Exercise 1

For each question:
- write the formula,
- make replacements in the formula and
- find the answer.

1. r = 7.5 mi. per minute; t = 15 minutes; d = ?.

Formula: ____________________

Replacements: ____________________ Computation

Answer: ____________________

2. t = 3 hours; r = 400 miles per hour; d = ?.

Formula: ____________________

Replacements: ____________________ Computation

Answer: ____________________

Corrections are on page 18.

Sometimes the information required in a distance problem is not the distance covered but rather the rate of speed or the time. We can use the same formula to find this information.

EXAMPLE: How many miles per hour is a car travelling if it travels 156 mi. in 3 hr.?

Solution

We must find the number of miles per hour (the rate of speed).
r = ? miles per hour
t = 3 hr.
d = 156 mi.

$d = rt$ Replace d by 156 because the distance is 156 miles.
$156 = r(3)$ Replace t by 3 because the time is 3 hours.
$156 \div 3 = r$
$r = 52$

(156) = (r) × (3)
Remember product = factor × factor, thus
(156) ÷ (3) = (r)
product ÷ factor = factor.

The car was travelling 52 miles per hour.

Assignment 1

For each problem:
- relate the given information to the correct variables,
- write the formula,
- make replacements in the formula,
- find the answer and
- write a statement answering the question asked.

1. How many hours does it take an airplane to fly 1500 mi. at a rate of 600 mi. per hour?

d = 1500 mi.

r = 600 mi. per hr.

t = ? hr.

Formula: $d = rt$

Replacements: $1500 = 600t$

$1500 \div 600 = t$

Computation

$$\begin{array}{r} 2\frac{1}{2} \\ 600\overline{)1500} \\ \underline{1200} \\ \cancel{300} \end{array}$$

Answer: $t = 2\frac{1}{2}$

Statement: It takes $2\frac{1}{2}$ hours.

2. How long does it take a car to travel 450 km. if its speed is 95 km. per hour?

d =

r =

t =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

3. A certain species of whale was observed swimming 10 nautical miles in $\frac{1}{3}$ hr. How many nautical miles per hour were they swimming?

d =

r =

t =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

4. From Edmonton to Wainwright, a passenger train travelled at a speed of 55 mi. per hour. The trip took 2.3 hr. How many miles is it from Edmonton to Wainwright?

d =

r =

t =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

5. Mr. David drove his car 250.2 miles in 4.5 hours. At what speed was he travelling?

d =

r =

t =

Formula: ____________________

Replacements ____________________ Computation

____

Answer: ____________________

Statement: ____

Topic Three: Average formula

You have probably encountered the term "average" many times. People say, "We travelled an average of 250 miles per day". This means that some days they travelled more than 250 miles; other days they travelled less than 250 miles. By taking miles from the longer trips and giving them to the shorter trips, it evens out to 250 miles per day. It is possible that they never travelled exactly 250 miles in one day.

When working with averages, we must consider three things:

1. the total of all the items.
2. the number of items.
3. the average or "evened-out" amount for 1 item.

We can use a formula to show how these three variables are related. It is called the average formula.

$$\boxed{a = \frac{t}{n}}$$

a, average for 1 item
t, total of all the items
n, number of items

$\frac{t}{n}$ means $t \div n$.

The units of measurement for a and t are limitless.

We can use this formula ($a = \frac{t}{n}$) to find solutions for problems about average.

EXAMPLE: In 8 baseball games, Ralph scored a total of 90 points. What was the average number of points scored per game by Ralph?

Solution

$t = 90$ (the total points of all the games)
$n = 8$ (the number of games)
$a = \frac{t}{n}$ Replace t by 90 because the total points scored are 90.
$a = \frac{90}{8}$ Replace n by 8 because the number of games is 8.
$a = 11\frac{1}{4}$

The average number of points scored per game is $11\frac{1}{4}$.
(As you know, it is impossible to score $11\frac{1}{4}$ points. This shows clearly what average means. It is not necessarily a true amount.)

Self-correcting Exercise 2

For each question:
- write the formula,
- make replacements in the formula and
- find the answer.

1. t = \$3549; n = 7 paintings; a = ? .

Formula: ____________________

Replacements: ____________________ Computation

Answer: ____

2. n = 12 clocks; t = \$91.80; a = ? .

Formula: ____________________

Replacements: ____________________ Computation

Answer: ____

Corrections are on page 18.

Sometimes the information required in a problem is not the average but rather the total of all the items or the number of items. We can use the same formula to find this information.

EXAMPLE: Clarence caught 5 fish. The average weight per fish was $2\frac{1}{3}$ lb. What was the total weight of the 5 fish in pounds?

Solution

$a = 2\frac{1}{3}$

$t = ?$

$n = 5$

Solution (continued)

$$a = \frac{t}{n}$$

$$2\frac{1}{3} = \frac{t}{5}$$ From our work with fractions you know that $2\frac{1}{3} = \frac{2\frac{1}{3}}{1}$.

$$\frac{2\frac{1}{3}}{1} = \frac{t}{5}$$ Now you can apply the ratio test.

$$2\frac{1}{3} \times 5 = 1 \times t$$

$$\frac{7}{3} \times 5 = t$$

$$\frac{35}{3} = t$$

$$11\frac{2}{3} = t$$

The 5 fish weighed $11\frac{2}{3}$ lb.

Assignment 2

For each problem
- relate the given information to the correct variables,
- write the formula,
- make replacements,
- give the answer and
- write a statement answering the question asked.

1. The Ritchie High School football team played 6 home games this year. The average attendance per home game was 950 persons. What was the total attendance at the 6 home games?

a = 950 persons

t = ?

n = 6 games

Formula: $a = \frac{t}{n}$

Replacements: $950 = \frac{t}{6}$

$\frac{950}{1} = \frac{t}{6}$

$950 \times 6 = t$

Computation

$$\begin{array}{r} 950 \\ 6 \\ \hline 5700 \end{array}$$

Answer: $5700 = t$

Statement: Total attendance was 5700 persons.

2. The Wallace Company bought 32 new typewriters of a certain model. The total cost was $4064. What was the price of each typewriter?

a =

t =

n =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

3. Last month Clyde worked a total of 24 days. He worked an average of $2\frac{1}{2}$ hr. per day last month. How many hours did Clyde work?

a =

t =

n =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

4. The Garcia family uses an average of $\frac{3}{4}$ gal. of milk per day. How many gallons of milk does the Garcia family use in 30 days?

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

5. A certain airplane can fly 625 mi. per hour. At this rate, how many hours will it take the airplane to fly 1000 mi.?

Formula ____________________

Replacements ____________________ Computation

____

Answer: ____________________

Statement: ____

6. A bag contains 44 oz. of soap. Each bar of soap weighs 4 oz. How many bars of soap are in the bag?

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

Topic [illegible]: Interest formula

You will encounter interest frequently. If you want to buy a car and you haven't enough money. you will probably obtain a loan. You must pay to borrow money. It is similar to rent.

If you rent a house you do not own it. You must pay the owner some money for allowing you to use it. You must give the house back to the owner when he wishes you to leave. You do not own a loan; you must give it back. You must also pay some extra money for being allowed to use the money provided by the loan. This extra money is called INTEREST.

Three factors affect the amount of interest you will have to pay. They are

1. the amount of money you borrow (which is called the PRINCIPAL).
2. the length of time until you pay back the principal.
3. the rate at which the principal is being loaned to you (this is usually given in per cent).

We can use a formula to show how these factors are related. It is called the interest formula.

$\boxed{i = prt}$

i, the interest you must pay
p, the principal (amount you borrowed)
r, the rate at which the principal is being loaned
t, the time for which the principal is being loaned

prt means $p \times r \times t$.

The interest (i) is measured in dollars and cents.
The principal (p) is measured in dollars and cents.
The rate (r) is measured in per cent for a certain length of time such as per cent per year, per cent per month, per cent per half-year, etc. This per cent is always changed to a decimal or a fraction before you replace r with it.
The time (t) is measured in years, months, weeks, etc.

We can use this formula (i = prt) to find solutions for problems about interest, principal, rate and time.

EXAMPLE. Mr. Mack borrowed \$1500 from a bank. The interest on a loan is $5\frac{1}{2}\%$ per year. Mr. Mack will repay this loan at the end of 3 years. How many dollars should he pay in interest?

Solution

$p = \$1500$

$r = 5\frac{1}{2}\% = 5.5\% = .055$ Remember: when changing a per cent to a decimal move the decimal 2 places to the left

$t = 3$ yr.

$i = prt$ Replace p by 1500 because the principal is \$1500.

$i = 1500(.055)(3)$ Replace r by .055 because the rate is $5\frac{1}{2}\% = .055$.

$i = 4500(.055)$ Replace t by 3 because the time is 3 years.

$i = 247.50$

Mr. Mack must pay \$247.50 in interest.

Computation

$$\begin{array}{r} 1500 \\ 3 \\ \hline 4500 \end{array} \qquad \begin{array}{r} 4500 \\ .055 \\ \hline 22500 \\ 225000 \\ \hline 247.500 \end{array}$$

Self-correcting Exercise 3

For each question:
- write the formula,
- make replacements in the formula and
- find the answer.

1. t = 1 year; p - \$500; r = $6\frac{1}{2}\%$; i = ? .

Formula ____________________

Replacements ____________________ Computation

____

Answer ____________________

2. $r = 5\frac{1}{2}\%$; $p = \$2400$; $t = 2$ years; $i = \underline{\ ?\ }$.

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____

Corrections are on page 19.

Sometimes the information required in a problem is not the interest but rather the principal, the rate of interest or the time. We can use this same formula to find this information.

EXAMPLE: Mr. Weaver borrowed \$6000 from the bank. At the end of 4 years, he paid the bank \$6000 plus \$1260 in interest. What was the rate of interest per year?

<u>Solution</u>

$i = \$1260$
$p = \$6000$
$r = ?$
$t = 4$ yr.
$i = prt$

$1260 = 6000r(4)$

$1260 = 24000\ r$

$1260 \div 24000 = r$

$.0525 = r$

Replace i by 1260 because the interest is \$1260.
Replace p by 6000 because the principal is \$6000.
Replace t by 4 because the time is 4 years.

```
          .0525
24000 ) 1260.0000
        1200 00
          60 000
          48 000
          12 0000
          12 0000
               0
```

Mr. Weaver has to pay <u>5.25%</u> interest.

Assignment 3

For each problem:

- relate the given information to the correct variables,
- write the formula,
- make replacements in the formula,
- find the answer and
- write a statement answering the question asked.

1. The interest charges at 6% on a $500 loan are $75. Over what length of time was this loan obtained?

$i = \$75$

$p = \$500$

$r = 6\% = .06$

$t = ?$

Formula: $i = prt$

Replacements: $75 = 500(.06)t$

$75 = 30t$

$75 \div 30 = t$

Answer: $2\frac{1}{2} = t$

Statement: The length of time is $2\frac{1}{2}$ years.

Computation

$$\begin{array}{r} 500 \\ \times .06 \\ \hline 30.00 \end{array} \qquad \begin{array}{r} 2\frac{1}{2} \\ 30\overline{)75} \\ 60 \\ \hline 15 \end{array}$$

2. A loan for 3 years cost $1365. The rate is 7%. What is the principal of the loan?

i =

p =

r =

t =

Formula: ______

Replacements: ______

______

______

Computation

Answer: ______________________

Statement: ____

3. Miss Newcomb borrowed $1500 for a period of 3 years. At the end of this time she repaid the loan and paid $270 interest. What rate of interest did Miss Newcomb pay?

i =

p =

r =

t =

Formula: ______________________

Replacements: ______________________ Computation

____

____

Answer: ______________________

Statement: ____

4. It is 420 miles from Edmonton to Prince Albert. Travelling by car, this trip takes 6 hours. At what speed is the car travelling?

Formula: ______________________

Replacements: ______________________ Computation

____

____

Answer: ______________________

Statement: ____

5. What are the interest charges on a $4500 loan at $5\frac{1}{2}\%$ for $1\frac{1}{2}$ years?

Formula: ____________________

Replacements: ____________________ Computation

____

____

Answer: ____________________

Statement: ____

6. Jack borrowed $900 from the bank at 6%. The interest charges were $27. What was the term of the loan?

Formula: ____________________

Replacements: ____________________ Computation

____

____

Answer: ____________________

Statement: ____

7. Mary reads an average of 8 books per month. How many books does she read in a year?

Formula: ____________________

Replacements: ____________________ Computation

____

____

Answer: ____________________

Statement: ____

Key to Self-correcting Exercises in Lesson 17

Exercise 1, page 3

1. $d = rt$

$d = 7.5(15)$

$d = 112.5$ miles

Distance is in miles because rate is in miles per minute.

$$\begin{array}{r} 7.5 \\ \underline{15} \\ 375 \\ \underline{750} \\ 112.5 \end{array}$$

2. $d = rt$

$d = 400(3)$

$d = 1200$ miles

$$\begin{array}{r} 400 \\ \underline{3} \\ 1200 \end{array}$$

Exercise 2, page 8

1. $a = \frac{t}{n}$

$$\begin{array}{r} 507 \\ 7\overline{)3549} \\ 4\ \end{array}$$

$a = \frac{3549}{7}$

$a = \$507$

2. $a = \frac{t}{n}$

$a = \frac{91.80}{12}$

$a = \$7.65$

$$\begin{array}{r}
7.65 \\
12\;\overline{)\;91.80} \\
\underline{84}^{\times\times} \\
7\;8 \\
\underline{7\;2} \\
60 \\
\underline{60} \\
0
\end{array}$$

Exercise 3, page 13

1. $i = prt$
$i = 500(.065)(1)$
$i = \$32.50$

$$\begin{array}{r}
.065 \\
\underline{500} \\
32.500
\end{array}$$

2. $i = prt$
$i = 2400(.055)(2)$
$i = 2400(.110)$
$i = \$264$

$$\begin{array}{r}
.055 \\
\underline{2} \\
.110
\end{array}
\qquad
\begin{array}{r}
2400 \\
\underline{.110} \\
24000 \\
\underline{240000} \\
264.000
\end{array}$$

End of Lesson 17

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________	(If label is missing or incorrect) File Number ____________________
Time Spent on Lesson ____________________	Lesson Number __________

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned Teacher: __________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received:

____

Lesson Recorded ______

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

MORE FORMULAS

TOPIC ONE: COST FORMULA

Many costs are involved in owning a small business or a farming business. For a farm, some of the things that must be bought and paid for are machinery, seed and fertilizer, and, for family needs, groceries, household goods, and clothing.

For example, the cost of floor wax is \$4 per can. If five cans are bought, the cost is 5 × \$4 or \$20.

The total cost to be paid depends on the price for one of the items and it also depends on how many items we buy. There are two variables:

- price for one item, and
- number of items we buy.

A man is buying four car tires at \$39.95 each. His cost may be worked out either this way:

$$\begin{array}{r} \$\ 39.95 \\ 39.95 \\ 39.95 \\ \underline{\quad 39.95} \\ \$159.80 \end{array}$$

or the shorter way of multiplication:

$$\begin{array}{rl} \$\ 39.95 & \leftarrow \text{price of one car tire} \\ \underline{\quad \times 4} & \\ \$159.80 & \end{array}$$

The bill that the man must pay is his total cost. Clearly, the cost increases if he buys more and more tires at \$39.95 per tire. Four tires will cost \$159.80.

We use the word "cost" for total cost. ← Let "c" be the symbol.

We use the word "number" for the number of items. ← Let "n" be the symbo

We use the word "price" for the price. ← Let "p" be the symbol.

$$\overset{\text{(Total)}}{\text{The cost}} = \text{number} \underset{\text{times}}{\times} \text{price}$$

$$c = n \times p$$

In short form, a formula for cost is $\boxed{c = np}$.

The "np" is "n times p".
The "np" is the product.

EXAMPLE: Using the formula, find the cost of 2 snow tires, each priced at $53.50.

Solution

Use the formula. $c = np$

$$\begin{array}{ccc} c = & n & p \\ & \downarrow & \searrow \\ c = & 2 \times & 53.50 \end{array}$$

Make replacements in the formula. $c = 2 \times 53.50$

The "p" is replaced by $53.5

The "n" is replaced by 2.

Computation

$$\begin{array}{r} 53.50 \\ 2 \\ \hline 107.00 \end{array}$$

$c = \$107.00$

Write a statement. The cost of the tires is $107.00.

Multiplication is indicated in different ways in mathematics:

- by the words "product" or "find the product"
- by brackets, $c = np$
  $c = (2)(53.50)$
- by the times symbol, 5×4
- by a placeholder and digit with no space between, $4x$
- by a placeholder and another placeholder, np

SELF-CORRECTING EXERCISE 1

Find the product. Multiplication of decimals by 10 and powers of 10 may be reviewed in Lesson 13, page 5. Corrections are on page 17.

1. $n \times 8 =$ ____________________
2. $(15)(4) =$ ____________________
3. $9 \times b =$ ____________________
4. $(63)(2) =$ ____________________
5. $(10)(11) =$ ____________________
6. $(35)(100) =$ ____________________
7. $8.5 \times 10 =$ ____________________
8. $4.75 \times 10 =$ ____________________
9. $(22)(p) =$ ____________________
10. $n \times 50 =$ ____________________
11. $9.45 \times 1000 =$ ____________________
12. $c = 1000 \times \$32$ $c =$ ____________________
13. $c = 100 \times \$1.85$ $c =$ ____________________
14. $c = n \times p$ $c =$ ____________________
15. $i = p \times r \times t$ $i =$ ____________________

Sometimes in a cost problem, the information required is not the cost but the number of items or the price per item. That is, using the formula c = np, we may wish to find the solution of "n" or of "p".

EXAMPLE: Brown's Grocery bought some new shelves at a cost of \$105. Using the formula c = np, find the value of p if the number of shelves is 7.

Solution

Sort out the information. We are to find p. p = ?

$c = 105$

$n = 7$

$p = ?$

Use the formula.

$$c = n\,p$$

$$\downarrow \quad \downarrow\ \downarrow$$

Make replacements in the formula.

$$105 = 7(p)$$

The c is replaced by 105.

The n is replaced by 7.

The p is p, because it is unknown

Remember, product = factor × factor.

$$105 = 7p$$
$$7p = 105$$
$$p = \frac{105}{7}$$
$$p = 15$$

Computation

$$\begin{array}{r} 15 \\ 7\overline{)105} \end{array}$$

Write a statement. The value of p, the price of each shelf, is \$15.

SELF-CORRECTING EXERCISE 2

For each question:
- write the formula,
- make replacements in the formula,
- find the answer, and
- write the statement.

1. $c = \$16.50$; $n = ?$; $p = \$2.75$

Formula:

Replacements:

Computation

Answer:

Statement:

Corrections are on page 18.

ASSIGNMENT 1

For each problem:
- relate the given information to the correct variables,
- write the formula,
- make replacements in the formula,
- find the answer, and
- write a statement answering the question asked.

1. How many 29-cent pens can Anne buy for 87¢ ?

$c = 87$
$n = ?$
$p = 29$

Formula: $c = np$

Replacements: $87 = n(29)$

$87 = 29n$

Answer: $n = 3$

Computation

$$\begin{array}{r} 3 \\ 29\overline{)87} \\ 87 \\ \hline 0 \end{array}$$

Statement: Anne can buy 3 pens.

ASSIGNMENT 1 (continued)

2. Bright Grocery bought 10 light fixtures at $12.95 each. What was the cost of the 10 light fixtures?

c =

n =

p =

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

3. How many one-metre-long Canada flags at $13 each can the Town Mart buy for their $16,900 purchase?

c =

n =

p =

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

4. Standard Company bought 110 typewriters for their office building. The price was the same for each typewriter. Altogether the typewriters cost $16,995. What was the price of each typewriter?

c =

n =

p =

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

5. How much will the Robins family pay for 325 wall tiles priced at $1.15 per tile?

c =

n =

p =

Formula: ______________________

Replacement: ______________________ Computation

____

Answer: ______________________

Statement: ____

TOPIC TWO: SELLING PRICE FORMULA

A storekeeper sets a selling price for the items he sells. Perhaps he buys cameras costing fifty dollars each, and then for a higher selling price he sells them in the store. Dollars and cents are important to him. The selling price should be fair. It should not be too high, but high enough. That is, his records, carefully kept, will show a profit for his store if it is well-managed. After he pays all bills for renting the store, electricity, heating and telephone bills, salaries, equipment and repairs, he will have money left over, and that is his gain, his profit.

Selling price is determined mainly by the cost per item the storekeeper pays. The original cost should be as low as possible. A supermarket, for example, aims to buy canned goods, fresh fruit, vegetables, flour, tea, and so on, at a low cost to the supermarket in the first place, and then to sell for a higher price which should be a fair price.

EXAMPLE: Melons are bought at 80 cents each by a supermarket and re-sold for 99 cents.

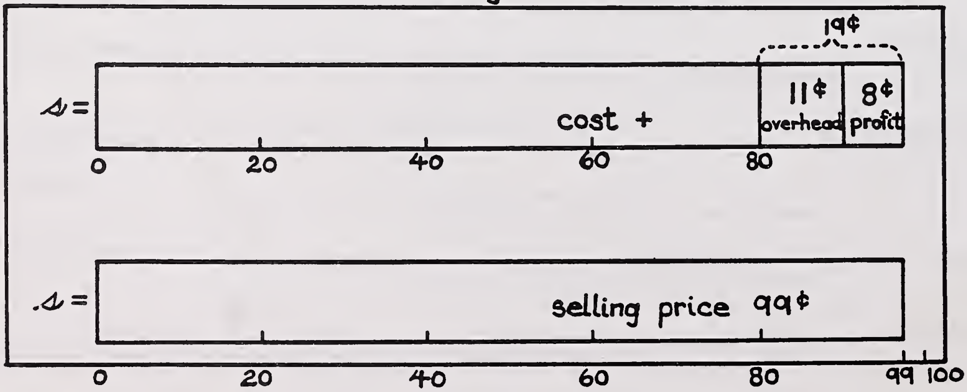
$$\begin{aligned} \text{Selling price} &= 80¢ + 19¢ \\ &= \text{cost plus } 19. \end{aligned}$$

$$\text{Selling price} = 99¢.$$

Nineteen cents is added to the original 80-cent cost. Above the 80-cent cost is the mark-up of 19 cents, which is partly "profit" and partly "overhead", the name given to expenses involved in operating the store. As the expenses go up, the selling price will go up also. Overhead and profit are 19 cents per melon.

A bar graph will show us what the selling price is. The overall length of the bar is the total selling price. Overhead and profit are clearly shown added to the cost per item. The variable "s" is the selling price.

How Selling Price is Set



Selling price $= \underbrace{80 + 11 + 8}$

$= 99$. The selling price is 99¢.

(You can see that the sum is the selling price. In earlier pages of this lesson, the operation was multiplication, but now addition and subtraction are the operations.)

The selling price is set by:

1. cost per item	80	
2. overhead (expenses)	11	← ← the merchant's expenses
3. profit	8	
The selling price is the sum of these.	99	

Using the variable s for selling price, c for cost to the merchant, o for overhead, and p for profit, we can write the selling price formula as:

$$\boxed{s = c + o + p}$$

We can use this formula to find solutions to certain problems about business.

EXAMPLE: What is the profit made by the Holt Hardware Store selling lawn movers if the store paid \$1800 for them? The total sales were \$2700 and \$640 was the overhead.

Solution

Sort out the information given in the problem. Show which amounts will replace the variables in the formula.

$s = \$2700$
$c = \$1800$
$o = \$640$
$p = ?$

Write the formula. $s = \underset{\downarrow}{c} + \underset{\downarrow}{o} + \underset{\downarrow}{p}$

Write the replacements. $2700 = 1800 + 640 + p$

The variable "p" is not known. We are finding p.

Work out the equation. $2700 = 2440 + p$

$$\begin{array}{r} 1800 \\ 640 \\ \hline 2440 \end{array}$$

$$2440 + p = 2700$$

$$\begin{array}{r} 2700 \\ -\ 2440 \\ \hline 260 \end{array}$$

$$p = 260$$

Statement: The profit made by the Holt Hardware Store is \$260.

In some problems, both cents and dollars have to be added or subtracted, in which case we add (or subtract) with the decimal points in a straight line.

EXAMPLE: Subtract \$3.50 from \$10.50.

$$\begin{array}{r} \$10.50 \\ -3.50 \\ \hline \$\ 7.00 \end{array}$$

Finally, the decimal is placed.

SELF-CORRECTING EXERCISE 3

For each question:
- write the formula,
- make replacements in the formula,
- find the answer, and
- write the statement.

1. s = ?; c = \$19.50; o = \$4.70; p = \$5.75

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

2. s = \$11.50; c = \$8.98; o = \$1.28; p = ?

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

Corrections are on page 18.

ASSIGNMENT 2

For each problem:

- relate the given information to the correct variables,
- write the formula,
- make replacements in the formula,
- find the answer, and
- write a statement answering the question asked.

1. What is the selling price of the table tennis sets that cost $11.55 when the operating cost plus profit is $6.95?

s = ?
c = $11.55
o = } $6.95
p = }

Formula: $s = c + o + p$

Replacements: $s = 11.55 + 6.95$

We replace o and p with 6.95.

Computation

$$\begin{array}{r} 11.55 \\ \underline{6.95} \\ 18.50 \end{array}$$

Answer: $s = 18.50$

Statement: The selling price of table tennis sets is $18.50.

2. Find the selling price of gloves that cost $4.85, when the overhead is $1.40 and the store's profit is $1.74.

s =
c =
o =
p =

Formula: ____________________

Replacements: ____________________

Computation

____

Answer: ____________________

Statement: ____________________

ASSIGNMENT 2 (continued)

3. What is the selling price of one hockey stick if the cost to the store is \$5.14 and the overhead is \$0.79 and the profit \$1.06?

s =

c =

o =

p =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

4. Apples are purchased at \$4.15 per box and sold for \$5.50 per box. The overhead is \$0.90 per box. What is the profit?

s =

c =

o =

p =

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statements: ____

5. At West Sports Company a nylon tent sells for \$109.50. The cost was \$84.25. The overhead was \$9.65. What is the profit?

s =
c =
o =
p =

Formula: ______________________

Replacements: ______________________ Computation

____

Answer: ______________________

Statement: ____

TOPIC THREE: MAKING REPLACEMENTS IN FORMULAS

In Lessons 17 and 18, you learned how to use the formula method. For a given problem, you have written the standard formula. You have re-read the problem to know what replacements in the formula can be made. Then you have "written into" the formula those replacements. (The method is called "plugging in" the replacements.) After all replacements were made, you were able to write an equation containing only one placeholder. Then you worked out the value of the placeholder.

Because you have used several formulas, you should be able to use the same method for other formulas that may be new to you.

EXAMPLE: If $A = bh$ state the value of b if $A = 15$ and $h = 5$.

Solution

$A = bh$

$15 = b\ (5)$

$5b = 15$

$b = 15 \div 5$

$b = 3$

The value of b is 3.

ASSIGNMENT 3

For each problem:

- relate the given information to the correct variables,
- write the formula,
- make replacements,
- find the answer, and
- write a statement answering the question asked.

1. Find the average number of hours worked per day in 20 days if 150 hours of work were reported. (Lesson 17, page 7)

a = ?
t = 150
n = 20

Formula: $a = \frac{t}{n}$

Replacements: $a = \frac{150}{20}$

$= 150 \div 20$

Answer: $a = 7\frac{1}{2}$

Statement: The average is $7\frac{1}{2}$ hours.

Computation

$$\begin{array}{r} 7 \\ 20\overline{)150} \\ 140 \\ \hline 10 \text{ R} \end{array}$$

$7\frac{10}{20} = 7\frac{1}{2}$

2. If a given formula is $p = br$, state the value of p if $b = 800$ and $r = \frac{1}{10}$. (Self-correcting Exercise 1)

p =
b =
r =

Formula: $p = br$

Replacements:

Answer:

Statement:

Computation

3. In the formula A = bh, find A if b = $7\frac{1}{3}$ and h = $5\frac{1}{2}$.

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

4. On 5 different dates in the month of May, Ed bought gas for his car. He bought a total of 175.5 litres for the month. Find the average number of litres he bought.

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

5. Mrs. Cook spent $15.50, $8.25, and $5.50 on house plants. What was the average amount spent per plant?

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

6. Mr. Brown's family travelled by car to a cabin 580 kilometres away. They travelled at the rate of 90 kilometres per hour. What was the time needed? (Hint: Use the distance formula, Lesson 17, page 2.)

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

7. Bob Jasper borrowed $1650 for one year at 9%. How many dollars in interest did he have to pay at the end of the year? (Lesson 17, page 12.)

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

8. What is the number of bicycles the department store will buy from the factory if each one is priced at $120 and if the store pays $12,000 for the cost of these bicycles?

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

9. Mr. and Mrs. Hardy borrowed \$6300 at $9\frac{1}{2}\%$ for 2 years. How much interest in dollars did they pay?

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ____

KEY TO SELF-CORRECTING EXERCISES IN LESSON 18

EXERCISE 1, PAGE 3

1. $n \times 8 = 8n$
2. $(15)(4) = 60$
3. $9 \times b = 9b$
4. $(63)(2) = 126$
5. $(10)(11) = 110$
6. $(35)(100) = 3500$
7. $8.5 \times 10 = 85$
8. $4.75 \times 10 = 47.5$
9. $(22)(p) = 22p$
10. $n \times 50 = 50n$
11. $9.45 \times 1000 = 9450$
12. $c = \$32,000$
13. $c = \$185$
14. $c = np$
15. $i = prt$

EXERCISE 2, PAGE 5

$c = n \times p$

$16.50 = n \times 2.75$

$2.75n = 16.50$

$$\frac{\overset{1}{\cancel{2.75}}n}{\underset{1}{\cancel{2.75}}} = \frac{16.50}{2.75}$$

$$2.75 \overline{)16.50}^{\,6.}$$

1650

0

$1n = 6$

The value of n is 6.

EXERCISE 3, PAGE 10

1. $s = c + o + p$

$s = 19.50 + 4.70 + 5.75$

$s = 29.95$

The selling price is $29.95.

$$\begin{array}{r} \$\,19.50 \\ 4.70 \\ 5.75 \\ \hline \$\,29.95 \end{array}$$

2. $s = c + o + p$

$11.50 = 8.98 + 1.28 + p$

$11.50 = 10.26 + p$

$11.50 - 10.26 = p$

$1.24 = p$

The profit is $1.24.

$$\begin{array}{r} \$\,8.98 \\ +\ 1.28 \\ \hline \$\,10.26 \end{array}$$

$$\begin{array}{r} \$\,11.50 \\ -\ 10.26 \\ \hline \$\ \ 1.24 \end{array}$$

END OF LESSON 18

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

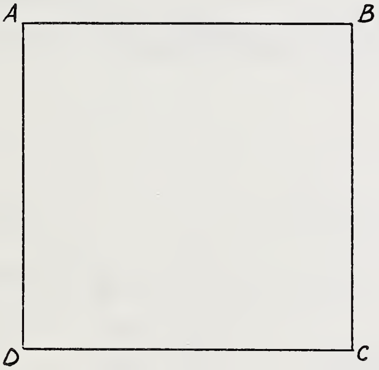
Perimeter, Area and Volume

Topic One: Perimeter of a square and a rectangle

Perimeter is the measure of the distance around an object. Some units of measure for perimeter are inches, feet, yards, miles, rods, meters, kilometers, etc. The perimeter of an object is obtained by finding the sum of the measures of the sides of an object. Perimeter is a linear measure because if we take the sides and place them one after another we would have a straight line. A line has only one dimension - length.

A. Perimeter of a square

A square is a quadrilateral (four-sided figure). Its sides are congruent. The angles determined by the sides are right angles (measure 90′).



$\overline{AD} \cong \overline{DC} \cong \overline{BC} \cong \overline{AB}$

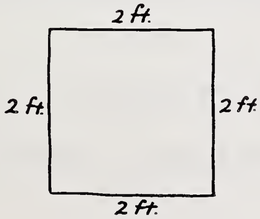
∠A, ∠B, ∠C and ∠D are right angles.

The perimeter of a square can be found by adding the measures of the sides together.

EXAMPLE: Find the perimeter of a square whose sides each measure 2 ft.

Solution

Make a sketch of a square. Place the measures of the sides on it.



The perimeter is the sum of the measures of the sides.
The perimeter of this square is 2 ft. + 2 ft. + 2 ft. + 2 ft. = 8 ft.

We can develop a formula for the perimeter of a square. A square has 4 congruent sides. If we multiply the measure of one side by 4, we will have the same result as adding the measures of the four sides.

e.g. $3 + 3 + 3 + 3 = 12$ is the same as
$4 \times 3 = 12$.

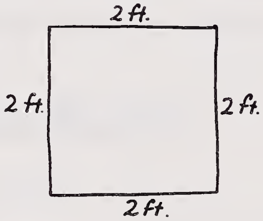
If we use P for the perimeter and
s for the measure of one side we can write a formula for the perimeter of a square.

$$\boxed{P = 4s}$$

EXAMPLE: Find the perimeter of a square whose sides each measure 2 ft.

Solution

Make a sketch of a square. Place the measures of the sides on it.



$P = ?$
$s = 2$ ft.

Use the formula. $P = 4s$

Make replacements in the formula. $P = 4(2)$

$P = 8$

Write a statement. The perimeter of a square whose sides each measure 2 ft. is 8 ft.

Assignment 1

For each problem:
- relate the given information to the correct variables,
- make a sketch,
- give the formula,
- make replacements in the formula,
- find the answer and
- write a statement answering the question asked.

1. The shape of a baseball diamond is a square. The measure of each side is 90 ft. What is the perimeter of the baseball diamond in feet?

$P = ?$

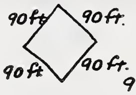
$s = 90$ ft.

Formula $P = 4s$

Replacements: $P = 4(90)$

Answer: $P = 360$

Sketch



$$\begin{array}{r}90\\ \underline{4}\\ 360\end{array}$$

Statement: The perimeter of a baseball diamond is 360 ft.

2. The measure of a side of a square is $9\frac{1}{2}$ ft. What is the perimeter of the square in feet?

$P =$

$s =$

Formula: ______________________

Sketch

Replacements: ______________________

Answer: ______________________

Statement: ____

3. The perimeter of a square is 13.6 millimeters. What is the measure of each side of the square in millimeters?

$P =$

$s =$

Formula: ______________________

Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ____

4. Betty hemmed a square napkin that had a perimeter of 62.5 in. What is the length of a side of the napkin in inches?

P =

s =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ____

5. In the city where Andy lives, the length of a side of a square city block is 360 ft. How many feet does Andy walk when he walks around the block?

P =

s =

Formula: ______________________ Sketch

Replacements: ______________________

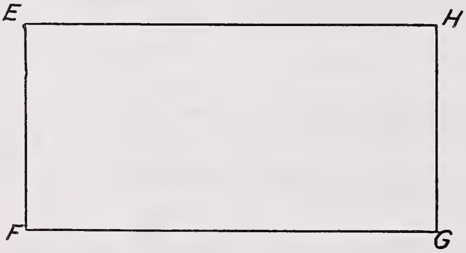
____

Answer: ______________________

Statement: ____

B. Perimeter of a rectangle

A rectangle is a quadrilateral. The opposite sides of a rectangle are congruent. The angles determined by the sides are right angles.



$\overline{EH} \cong \overline{FG}$
$\overline{EF} \cong \overline{HG}$

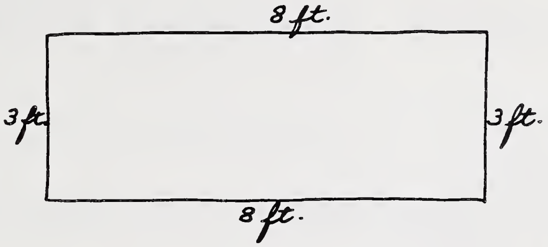
∠E, ∠F, ∠G and ∠H are right angles.

The perimeter can be found by adding the measures of the sides together.

EXAMPLE: Find the perimeter of a rectangle which measures 8 ft. by 3 ft.

Solution

Make a sketch of a rectangle. Place the measures of the sides on it.



The perimeter is the sum of the measures of the sides.
The perimeter of this rectangle is 8 ft. + 3 ft. + 8 ft. + 3 ft. = 22 ft.

We can develop a formula for the perimeter of a rectangle.
Since a rectangle has 2 pair of congruent sides, we can use the sum of two products.

e.g. 13 + 4 + 13 + 4 is the same as

$$\begin{aligned} & 2(13) + 2(4) \\ = & \ 26 + 8 \\ = & \ 34. \end{aligned}$$

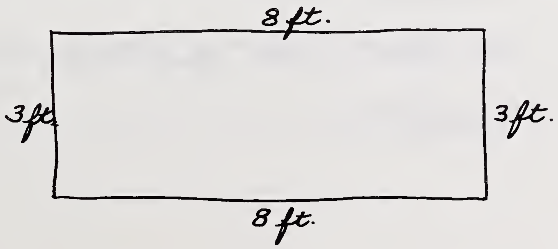
If we use P for the perimeter,
ℓ for the length (longer measure) and
w for the width (shorter measure)
we can write a formula for the perimeter of a rectangle.

$$\boxed{P = 2\ell + 2w}$$

EXAMPLE: Find the perimeter of a rectangle which measures 8 ft. by 3 ft.

Solution

Make a sketch of a rectangle. Place the measure of the sides on it.



$P = ?$

$\ell = 8$ ft.

$w = 3$ ft.

Use the formula. $P = 2\ell + 2w$

Make replacements in the formula. $P = 2(8) + 2(3)$

$P = 16 + 6$

$P = 22$

Write a statement. The perimeter of a rectangle measuring 8 ft. by 3 ft. is 22 ft.

Assignment 2

For each problem
- relate the given information to the correct variables.
- make a sketch,
- give the formula,
- make replacements in the formula.
- give the answer and
- write a statement answering the question asked.

1. Pat is going to put a fence around a rectangular flower garden. The length of the rectangle is $8\frac{1}{2}$ ft. The width of the rectangle is 4 ft. How many feet of fencing does Pat need for enclosing the garden?

$P = ?$

$\ell = 8\frac{1}{2}$ ft.

$w = 4$ ft.

Formula: $P = 2\ell + 2w$

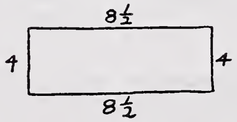
Replacements: $P = 2(8\frac{1}{2}) + 2(4)$

$P = 17 + 8$

Answer: $P = 25$

Statement: Pat needs 25 feet of fencing to enclose the garden.

Sketch



$8\frac{1}{2} \times 2 = \frac{17}{2} \times \frac{2}{1} = \frac{17 \times \overset{1}{\cancel{2}}}{\underset{1}{\cancel{2}} \times 1} = 17$

2. Sue is going to put lace edging on a rectangular tablecloth that is 70 in. long and 54 in. wide. How many inches of edging does she need?

P =

ℓ =

w =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ____

3. The length of a rectangle is $3\frac{1}{2}$ ft. The width of the rectangle is $5\frac{1}{3}$ ft. What is the perimeter of the rectangle in feet?

P =

ℓ =

w =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ____

4. The shape of a field is a square. The length of the side is 60 ft. What is the perimeter of this field?

Formula: ____________________ Sketch

Replacements ____________________

____

Answer: ____________________

Statement: ______________________________________

5. The length of a rectangle is 18 ft. The width is 15 ft. What is the perimeter of the rectangle in feet?

Formula: ____________________ Sketch

Replacements: ____________________

____

Answer. ____________________

Statement: ______________________________________

Topic Two: Area of a square and a rectangle

Area is a measure of the surface covered by an object. The area of an object is obtained by finding the number of square units it occupies. Area has two dimensions-length and width. Because of this, it is a square measure. Some units of measure for area are square inches, square feet, square rods, square meters, etc.

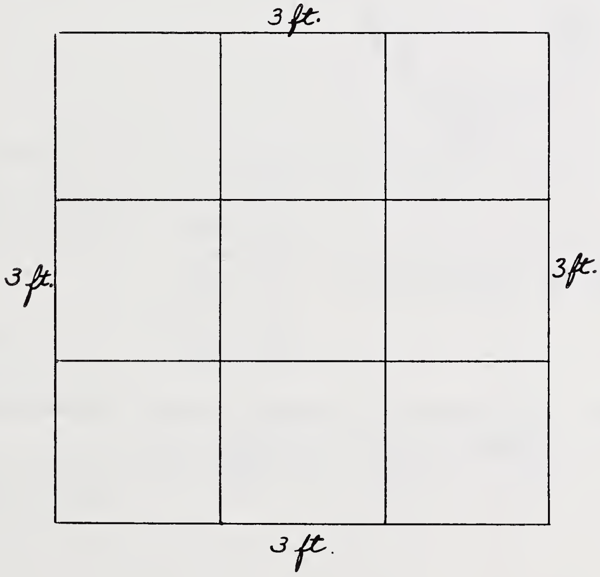
A. Area of a square

The area of a square can be found by using a diagram.

EXAMPLE: Find the area of a square whose sides each measure 3 ft.

Solution

Make a diagram drawn to scale. 1 inch on the diagram equals 1 foot of the actual object.



Each square shown is 1 ft. by 1 ft.
Each square shown is 1 sq. ft. ("sq." is the abbreviation for "square").

If you count the squares you will find that there are 9 of them.

The area of a square whose sides each measure 3 ft. is 9 sq. ft.

We can develop a formula for the area of a square. Both the length and the width of a square are the same. If we multiply the measure of the side by the measure of the side we will obtain the area. (In the example above, note that the sides are 3 ft. and 3 ft. and the area is 3 x 3 = 9 sq. ft.)

If we use A for area and
s for the measure of one side,
we can write the formula for the area of a square.

$$A = s \times s$$

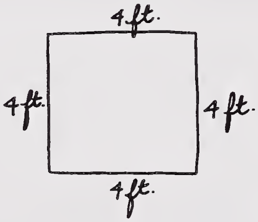
Here we can apply what we know about exponential notation.
Remember: $10 \times 10 = 10^2$; thus $s \times s = s^2$.

$$\boxed{A = s^2}$$

EXAMPLE: Find the area of a square whose sides each measure 4 ft.

Solution

Make a sketch of a square. Place the measures of the sides on it.



$A = ?$

$s = 4$ ft.

Use the formula. $A = s^2$

Make replacements in the formula. $A = (4)^2$

$A = 4 \times 4$

$A = 16$

Write a statement. The area of a square whose sides each measure 4 ft. is 16 sq. ft.

Assignment 3

For each problem:
- relate the given information to the correct variables,
- make a sketch,
- give the formula,
- make replacements in the formula,
- give the answer and
- write a statement answering the question asked.

1. Mrs. Jackson's kitchen is a square. The length of a side is 8 ft. What is the area of the kitchen floor in square feet?

A = ?

s = 8 ft.

Formula: $A = s^2$

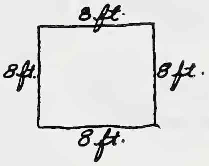
Replacements. $A = (8)^2$

$A = 8 \times 8$

Answer: $A = 64$

Statement: The area of the kitchen floor is 64 sq. ft.

Sketch



2. The measure of a side of a square is $3\frac{1}{4}$ in. What is the area of the square in square inches?

A =

s =

Formula:

Sketch

Replacements:

Answer:

Statement:

3. The measure of a side of a square is $1\frac{1}{3}$ ft. What is its area in square feet?

A =

s =

Formula:

Sketch

Replacements:

Answer:

Statement:

4. Mrs. Harvey bought a square tablecloth. The length of the side of the tablecloth is 52 in. What is the area of the tablecloth in square inches?

A =

s =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

5. Sam used a square piece of leather to cover the top of a table. The length of a side of the piece of leather is $39\frac{1}{2}$ in. What was the perimeter of the piece of leather in inches?

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

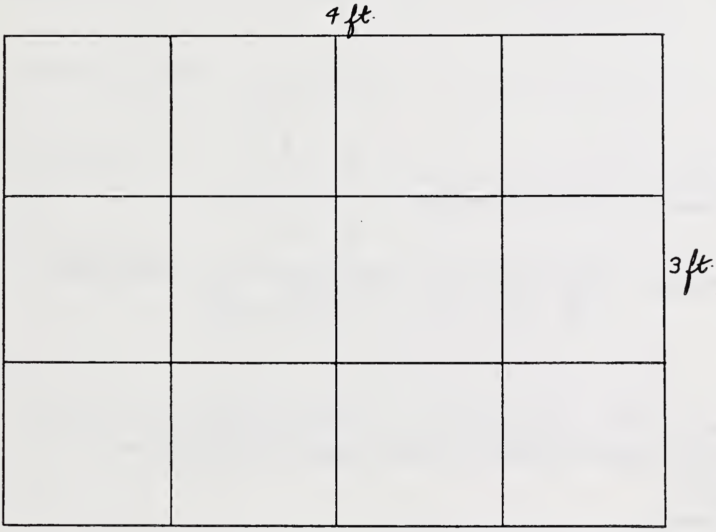
B. Area of a rectangle

The area of a rectangle can be found by using a diagram.

EXAMPLE: Find the area of a rectangle that measures 3 ft. by 4 ft.

Solution

Make a diagram drawn to scale. 1 inch on the diagram equals 1 foot of the actual object.



Each square shown is 1 ft. by 1 ft.

Each square shown is 1 sq. ft.

By counting the squares you will find that there are 12 of them.

The area of a rectangle whose sides measure 3 ft. by 4 ft. is 12 sq. ft.

We can develop a formula for the area of a rectangle. By multiplying the length by the width, we can obtain the area. (In the example above, note that the sides are 3 ft. and 4 ft. and the area is $3 \times 4 = 12$ sq. ft.)

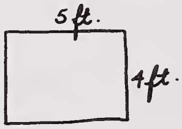
If we use A for area,
ℓ for the measure of the longer side (length) and
w for the measure of the shorter side (width)
we can write the formula for the area of a rectangle.

$\boxed{A = \ell w}$ (Remember ℓw means $\ell \times w$.)

EXAMPLE. Find the area of a rectangle that measures 5 ft. by 4 ft.

Solution

Make a sketch of a rectangle. Place the measures of the sides on it.



A = ?
ℓ = 5 ft.
w = 4 ft.
Use the formula. A = ℓw
Make replacements in the formula. A = 5 × 4
A = 20

Write a statement. The area of a rectangle that measures 5 ft. by 4 ft. is 20 sq. ft.

Assignment 4

For each problem:
- relate the given information to the correct variables,
- make a sketch,
- give the formula,
- make replacements in the formula,
- give the answer and
- write a statement answering the question asked.

1. If the length of a rectangle is 12" and its width is 6", state its area in sq. in.

A = ?

ℓ = 12 in.

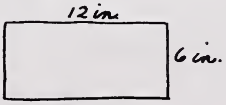
w = 6 in.

Formula: A = lw

Replacements: A = 12×6

Answer: A = 72

Sketch



$$\begin{array}{r} 12 \\ \underline{6} \\ 72 \end{array}$$

Statement: The area of the rectangle is 72 sq. in.

2. If the length of a rectangle is 2.5 cm. and its width is 2.0 cm., state its area in sq. cm.

$A =$

$\ell =$

$w =$

Formula: ______________________ Sketch

Replacements: ______________________

Answer: ______________________

Statement: ______________________________________

3. A floor is in the shape of a rectangle. The area of the floor is 197.6 sq. ft. The length is 15.5 ft. What is the width in feet?

$A =$

$\ell =$

$w =$

Formula: ______________________ Sketch

Replacements: ______________________

Answer: ______________________

Statement: ______________________________________

4. Find in square feet the area of a rectangular border 30 ft. long and 4 ft. wide.

Formula: ______________________ Sketch

Replacements: ______________________

Answer: ______________________

Statement: ______________________________________

5. A rectangle is 6 inches long and 3 inches wide. What is its perimeter?

Formula: ______________________ Sketch

Replacements: ______________________

Answer: ______________________

Statement: ______________________________________

Topic Three. Volume of a cube and a prism

Volume is the quantity or amount of space occupied by an object.

e.g. Suppose you are looking at a building. It occupies space. The amount of space it occupies is its volume. Think of your deep freezer. It occupies space. It has volume. The sizes of deep freezers are determined by calculating their volumes.

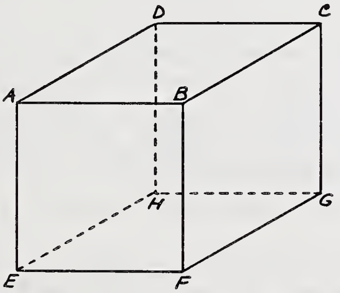
Volume is sometimes referred to as capacity.

e.g. A barrel occupies space. It has volume. The space inside the barrel can contain gas. The amount of gas it can hold is called its capacity. Capacity is the same as volume.

Volume has three dimensions - length, width and height. Because of this it is a cubic measure. Volume is obtained by finding the number of cubic units contained in an object. Some units of measure for volume are cubic inches, cubic feet, cubic meters and cubic kilometers.

A. Volume of a cube

A cube is a geometric figure which has six faces. Each face is a square having two identical dimensions. All six faces are congruent.



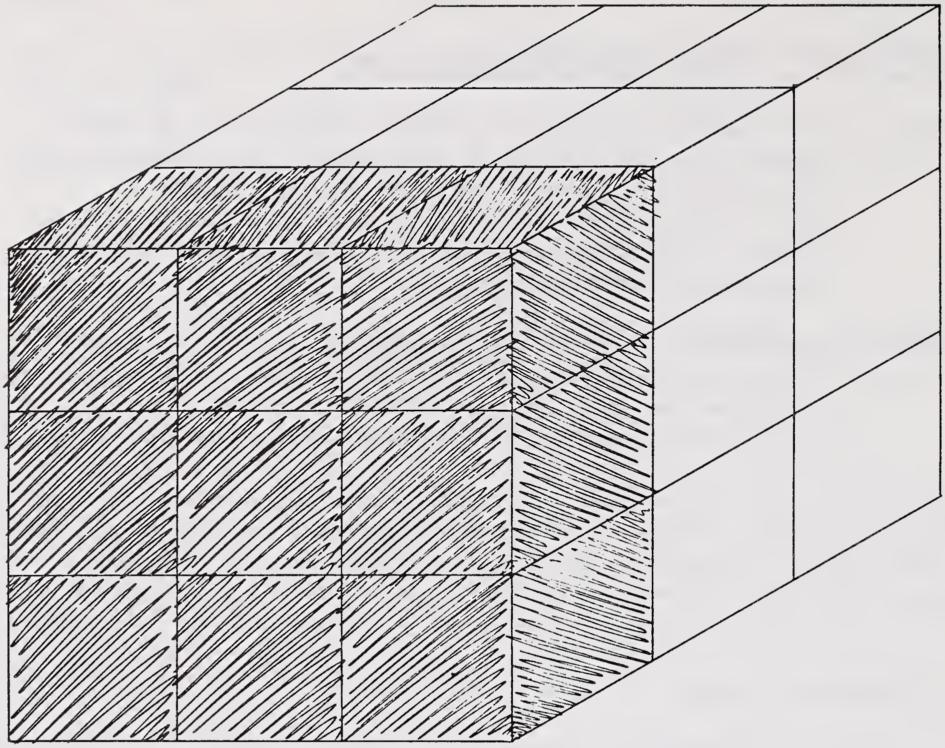
Faces AEFB, BFGC, CGHD, DHEA, ABCD and EFGH are squares that are congruent to each other.

The volume of a cube can be shown by a diagram.

EXAMPLE: Find the volume of a cube whose sides each measure 3 yards.

Solution

Make a diagram drawn to scale. 1 inch on the diagram equals 1 yard of the actual object.



Each of the smaller cubes measures 1 yd. in length by 1 yd. in width by 1 yd. in height.
The volume of each small cube is 1 cu. yd. ("cu." is the abbreviation for "cubic").

The shaded area contains 1 layer of smaller cubes (or 9 smaller cubes). There are 3 layers of smaller cubes in the diagram.
Therefore there are 3 × 9 or 27 smaller cubes in the large cube.
Remember that each smaller cube has a volume of 1 cubic yard.
Thus 27 of these cubes would have a volume of 27 cu. yd.
The volume of a cube whose sides each measure 3 yards is 27 cu. yd.

We can develop a formula for the volume of a cube. The length, width and height of a cube are the same. If we multiply the measure of a side by the measure of a side by the measure of a side, we will obtain the volume. (In the example above, note that the length, width and height are each 3 ft. and the volume is $3 \times 3 \times 3 = 27$ cu. ft.)

If we use V for the volume and
s for the measure of one side
we can write the formula for the volume of a cube.

$$V = s \times s \times s$$

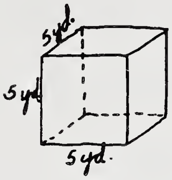
Here we can apply what we know about exponential notation.
Remember: $10 \times 10 \times 10 = 10^3$; thus $s \times s \times s = s^3$.

$$\boxed{V = s^3}$$

EXAMPLE: Find the volume of a cube whose sides each measure 5 yards.

Solution

Make a sketch of a cube. Place the measures of the sides on it.



$V = ?$

$s = 5$ yd.

Use the formula. $V = s^3$

Make replacements in the formula. $V = (5)^3$

$V = 5 \times 5 \times 5$

$V = 125$

Write a statement. The volume of a cube whose sides each measure 5 yd. is 125 cu. yd.

Assignment 5

For each problem:
- relate the given information to the correct variables,
- make a sketch,
- give the formula,
- make replacements in the formula,
- give the answer and
- write a statement answering the question asked.

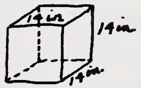
1. An aquarium is in the shape of a cube. The length of a side is 14 in. What is the volume of the aquarium in cubic inches?

$V = ?$

$s = 14$ in.

Formula: $V = S^3$

Sketch



Replacements: $V = (14)^3$

$V = 14 \times 14 \times 14$

$$\begin{array}{r} 14 \\ 14 \\ \hline 56 \\ 140 \\ \hline 196 \end{array} \qquad \begin{array}{r} 196 \\ 14 \\ \hline 784 \\ 1960 \\ \hline 2744 \end{array}$$

Answer: $V = 2744$

Statement: The volume of the aquarium is 2744 cu in.

2. A glass block is in the shape of a cube. The measure of a side of the block is $\frac{7}{8}$ ft. What is the volume of the block in cubic feet?

V =

s =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

3. A block of wood is in the shape of a cube. The length of a side of the block is 4.5 in. What is the volume of the block of wood in cubic inches?

V =

s =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

4. What is the volume of a cube with side measurements of 2 in.?

V =

s =

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

5. What is the volume of a cube with side measurements of 6 mm.?

V =

s =

Formula: ______________________ Sketch

Replacements: ______________________

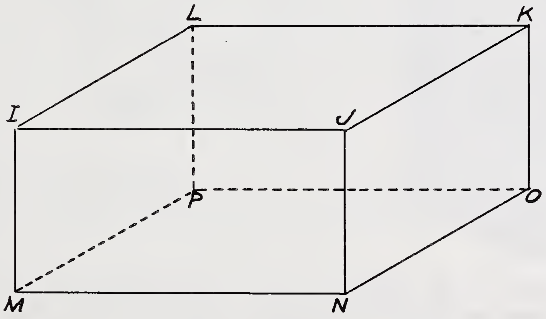
____

Answer: ______________________

Statement: ____

B. Volume of a rectangular prism

A rectangular prism is a geometric figure which has six faces. Each face is a rectangle. Opposite faces are congruent.



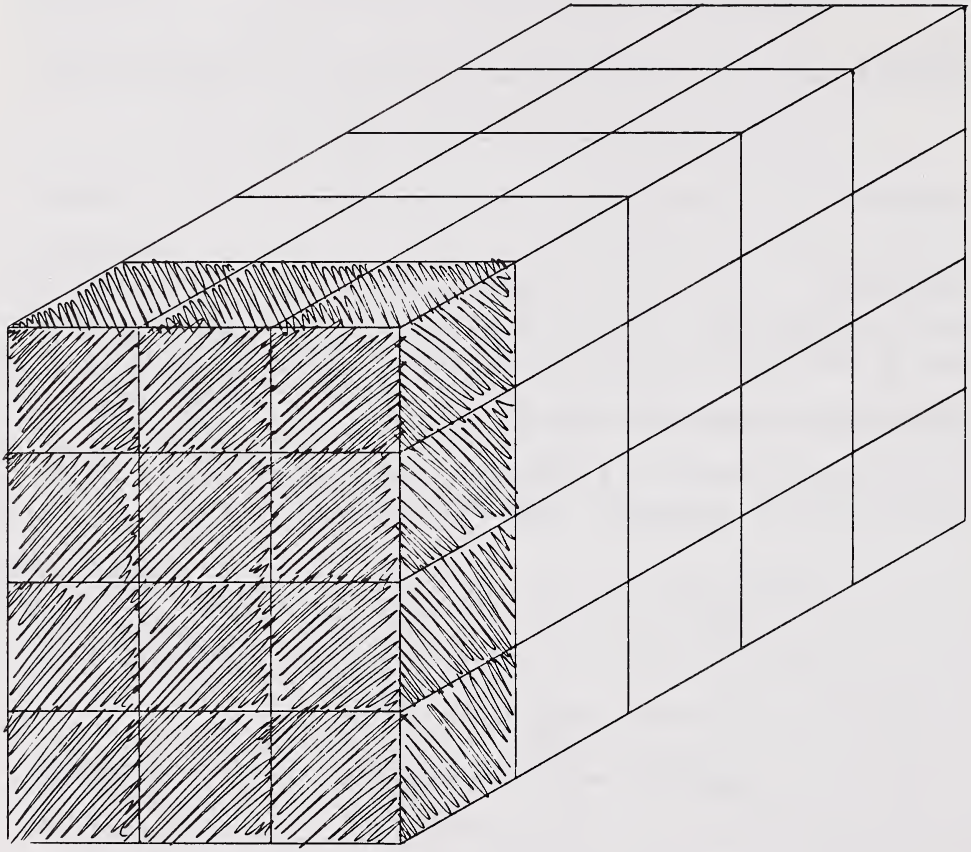
Faces IMNJ, JNOK, KOPL, LPMI, IJKL and MNOP are rectangles.

The volume of a rectangular prism can be shown by a diagram.

EXAMPLE: Find the volume of a rectangular prism which measures 3 ft. by 4 ft. by 5 ft.

Solution

Make a diagram drawn to scale. 1 inch on the diagram equals 1 foot of the actual object.



Each of the small cubes measures 1 ft. in length by 1 ft. in width by 1 ft. in height.
Each small cube is 1 cu. ft.

The shaded area contains 1 layer of cubes (or 12 cubes).
There are 5 layers of cubes in the diagram. Therefore there are 5 × 12 or 60 cubes in the rectangular prism. Remember that each cube has a volume of 1 cu. ft. Thus, 60 of these cubes would have a volume of 60 cu. ft.
The volume of a rectangular prism which measures 3 ft. by 4 ft. by 5 ft. is 60 cu. ft.

We can develop a formula for the volume of a rectangular prism. By multiplying the length by the width by the height, we can obtain the volume of a rectangular prism. (In the example above, note the length is 5 ft., the width is 3 ft., and the height is 4 ft. and the volume is 5 × 3 × 4 = 60 cu. ft.)

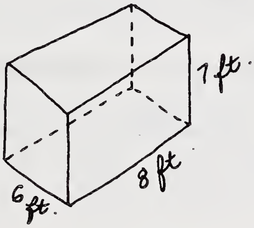
If we use V for the volume,
ℓ for the length,
w for the width and
h for the height,
we can write the formula for the volume of a rectangular prism.

$\boxed{V = \ell wh}$ (ℓwh means $\ell \times w \times h$.)

EXAMPLE: Find the volume of a rectangular prism which measures 6 ft. by 7 ft. by 8 ft.

Solution

Make a sketch of a rectangular prism. Place the measures of the sides on it.



$V = ?$
$\ell = 8$ ft.
$w = 6$ ft.
$h = 7$ ft.

Use the formula. $V = \ell wh$
Make replacements in the formula. $V = 8 \times 6 \times 7$
$V = 336$

Write a statement. The volume of a rectangular prism which measures 6 ft. by 7 ft. by 8 ft. is 336 cu. ft.

Assignment 6

For each problem:
- relate the given information to the correct variables,
- make a sketch,
- give the formula,
- make replacements in the formula,
- give the answer and
- write a statement answering the question asked.

1. The excavation for the foundation of a building is in the shape of a rectangular prism. The depth of the excavation is $4\frac{1}{3}$ yd., the length is 30 yd. and the width is 12 yd. How many cubic yards of dirt must be removed in digging the excavation?

V = ?

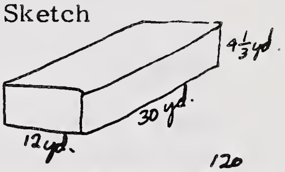
l = 30 yd.

w = 12 yd.

h = $4\frac{1}{3}$ yd.

Formula: $V = lwh$

Sketch



Replacements $V = 30 \times 12 \times 4\frac{1}{3}$

$V = 360 \times \frac{13}{3}$

Answer: $V = 1560$

$$\begin{array}{r} 12 \\ 30 \\ \hline 360 \end{array}$$

$$\frac{360}{1} \times \frac{13}{3} = \frac{\overset{120}{\cancel{360}} \times 13}{1 \times \underset{1}{\cancel{3}}} = \frac{1560}{1} = 15$$

$$\begin{array}{r} 120 \\ 13 \\ \hline 360 \\ 1200 \\ \hline 1500 \end{array}$$

Statement: 1560 cu. yd. of dirt must be removed in digging the excavation.

2. A food locker is in the shape of a rectangular prism. The locker is $4\frac{1}{4}$ ft. long, $1\frac{3}{4}$ ft. wide and 1 ft. high. What is the volume of the food locker in cubic feet?

Formula: ____________________ Sketch

Replacements: ____________________

____

Answer: ____________________

Statement: ____________________

3. A certain bar magnet is in the shape of a rectangular prism. It is 15 cm. long, 3 cm. wide and 1.25 cm. high. What is the volume of the magnet in cubic centimeters?

Formula: ____________________ Sketch

Replacements: ____________________

____

Answer: ____________________

Statement: ____

4. A brick is in the shape of a rectangular prism. The length of the brick is 10 in., the height is $2\frac{1}{2}$ in. and the width is 4 inches. What is the volume of this brick in cubic inches?

Formula: ____________________ Sketch

Replacements: ____________________

____

Answer: ____________________

Statement: ____

5. A box of chalk is 6 inches long, $4\frac{1}{4}$ inches wide and $3\frac{3}{4}$ inches deep. What is the volume of the box?

Formula: ______________________ Sketch

Replacements: ______________________

____

Answer: ______________________

Statement: ______________________________________

6. Perimeter is ________________ measure.

Area is ________________ measure.

Volume is ________________ measure.

Complete the following table.

7.

Measures of sides (Each figure has 4 sides.)	Perimeter	Area	Type of geometric figure
1. 16 inches			
2. 4 yd. by 6 yd.			
3. 3.5 mi. by 13 mi.			
4. $1\frac{1}{4}$ cm.			
5. 1.43 rd.			
6. $2\frac{1}{6}$ in. by $\frac{5}{6}$ in.			
7. 1 inch			
8. 2.25 ft. by $1\frac{1}{3}$ ft.			

Show Your Work.

End of Lesson 19

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ______________________

Time Spent on Lesson ______________________

(If label is missing
or incorrect)

File Number ______________________

Lesson Number __________

FOR SCHOOL USE ONLY

Assigned Teacher: __________

Lesson Grading: __________

Additional Grading E/R/P Code: __________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received: ______________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ______________________
Address ______________________
____
____
Postal Code ______________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Review

Following each question, reference pages where help may be found are given.

Marks

5 1. Find the complete factorization of each number. (Lesson 5, page 3)

a. 15 ____________________

b. 16 ____________________

c. 24 ____________________

d. 30 ____________________

e. 35 ____________________

5 2. Using the complete factorization of each number from #1, find the least common multiple of each group of numbers. (Lesson 6, page 9)

a. 15 and 35 ____________________

____

b. 16 and 30 ____________________

____

c. 24 and 30 ____________________

____

d. 16 and 15 ____________________

____

e. 35 and 24 ____________________

____

Marks

5 3. Using the complete factorization of each number from #1, find the greatest common factor of each group of numbers. (Lesson 5, page 8)

a. 15 and 30 ______________________

____

b. 16 and 24 ______________________

____

c. 15 and 24 ______________________

____

d. 16 and 35 ______________________

____

e. 30 and 35 ______________________

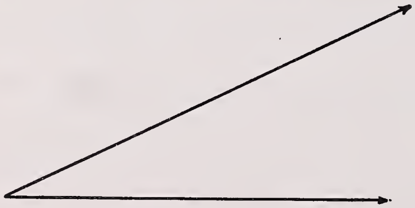
____

10 4. Find the complete factorization of each number. Then decide whether it is a prime number or a composite number. (Lesson 4, pages 11-15)

		Complete factorization	Prime or composite
a.	43	______________	______________
b.	39	______________	______________
c.	27	______________	______________
d.	19	______________	______________
e.	12	______________	______________

10 5. Measure each angle. Tell which type of angle it is (obtuse, acute or right). (Lesson 7, pages 9 and 14)

a.

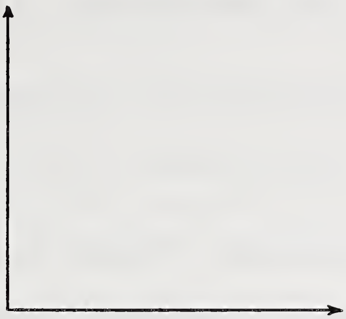


Measure ______________

Type ______________

Marks

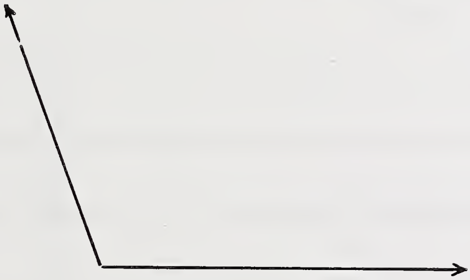
b.



Measure ____________________

Type ____________________

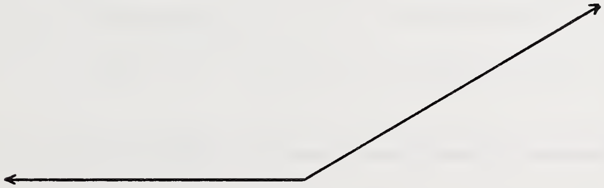
c.



Measure ____________________

Type ____________________

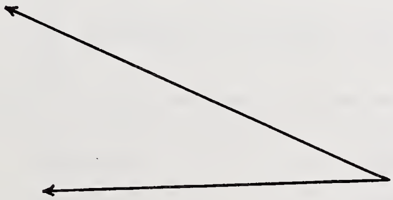
d.



Measure ____________________

Type ____________________

e.



Measure ____________________

Type ____________________

Marks

5 6. Name 5 geometric figures which are quadrilaterals. (Lesson 8, pages 27 and 28)

__________ __________ __________

__________ __________

5 7. Change each fraction to a mixed numeral. (Lesson 11, page 8)

a. $\frac{32}{5}$ __________

b. $\frac{41}{4}$ __________

c. $\frac{17}{6}$ __________

d. $\frac{13}{9}$ __________

e. $\frac{22}{3}$ __________

5 8. Write the following decimals using words instead of numbers. (Lesson 12, page 2)

a. 32,401

____

b. 26.7

____

c. .09

____

d. 1.807

____

e. 6.0071

____

Marks

5 9. What place is held by each digit in 3.5279? (Lesson 12, page 5)

3 is in the ____________________ place.

5 is in the ____________________ place.

2 is in the ____________________ place.

7 is in the ____________________ place.

9 is in the ____________________ place.

10 10. Write each fraction in its simplest form. Then name its reciprocal. (Lesson 11, pages 16 and 21)

		Fraction in its simplest form	Reciprocal
a.	$\frac{13}{39}$	____________________	______________
b.	$\frac{14}{49}$	____________________	______________
c.	$\frac{24}{32}$	____________________	______________
d.	$\frac{16}{18}$	____________________	______________
e.	$\frac{21}{27}$	____________________	______________

11. Perform the given operations. Then round each answer to the nearer one.

16 a. Products

$3\frac{7}{8} \times 4 =$ ____________________ Rounded answer ______
(Lesson 11, page 14)

$5 \times 2\frac{1}{10} =$ ____________________ Rounded answer ______

$6\frac{1}{4} \times 4\frac{3=}{8} =$ ____________________ Rounded answer ______

$1\frac{1}{9} \times \frac{3}{10} =$ ____________________ Rounded answer ______

73 (Lesson 13, pages 1-3)	96	1.09	.3
$\underline{.8}$	$\underline{2.4}$	$\underline{3.4}$	$\underline{.3}$
Rounded answer ____	Rounded answer ____	Rounded answer ____	Rounded answer ____

Marks
16

b. Quotients

$5\frac{1}{2} \div 5 =$ ______________________ Rounded answer ______
(Lesson 11, pages 22 and 23)

$7 \div 3\frac{1}{4} =$ ______________________ Rounded answer ______

$12\frac{5}{6} \div 2\frac{1}{3} =$ ______________________ Rounded answer ______

$4\frac{1}{5} \div \frac{6}{7} =$ ______________________ Rounded answer ______

(Lesson 14)

$25\overline{)\,.96}$ $.8\overline{)\,15}$

$.96 \div 25 =$ __ remainder __

Rounded answer ______

$15 \div .8 =$ ___ remainder ___

Rounded answer ______

$2.7\overline{)\,12.96}$ $3.5\overline{)\,2.8}$

$12.96 \div 2.7 =$ __ remainder __

Rounded answer ______

$2.8 \div 3.5 =$ __ remainder __

Rounded answer ______

Marks

8 12. Change each number to a fraction in its simplest form.

a. $4\frac{7}{8}$ ____________________ (Lesson 11, page 16)

b. $3\frac{1}{4}$ ____________________

c. .061 ____________________ (Lesson 12, page 11)

d. 53.6 ____________________

e. 9.80 ____________________

f. 11% ____________________ (Lesson 16, page 12)

g. 95% ____________________

h. 216% ____________________

6 13. Change each number to a decimal.

a. $\frac{9}{10}$ ____________________ (Lesson 15, page 4)

b. $4\frac{1}{20}$ ____________________

c. $\frac{1}{6}$ ____________________ (Lesson 15, page 7)

d. 8% ____________________ (Lesson 16, page 13)

e. 25% ____________________

f. 703% ____________________

6 14. Change each number to a per cent.

a. $\frac{3}{10}$ ____________________ (Lesson 16, page 9)

b. $16\frac{21}{25}$ ____________________

c. $\frac{2}{3}$ ____________________

d. .12 ____________________ (Lesson 16, page 10

e. .042 ____________________

f. 3.17 ____________________

Marks

8 15. Perform the following calculations.

a. $87.65 + .1 =$ ____________________

b. $345.98 \times 10^3 =$ ____________________

c. $9.49 \div 10^2 =$ ____________________

d. $52.78 - .01 =$ ____________________

e. $31625 \div 10^4 =$ ____________________

f. $35927 + 100 =$ ____________________

g. $74342 \times 10^0 =$ ____________________

h. $348215 - 10 =$ ____________________

8 16. Find the missing component of the rate pairs. Use the ratio test. (Lesson 16, page 4)

a. $\frac{3}{4} = \frac{n}{16}$

b. $\frac{3}{8} = \frac{9}{n}$

c. $\frac{n}{18} = \frac{5}{6}$

d. $\frac{14}{n} = \frac{35}{15}$

Marks

5 17. A flock of sheep has 15 black sheep. If there are 2 black sheep for each 100 sheep, how many sheep are there in the flock? (Lesson 16, page 4)

Placeholder: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

5 18. When grapefruit sell at 3 for 19¢, what is the cost of 18 of them? (Lesson 16, page 4)

Placeholder: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

6 19. A man sold a used car at 60% of the price he paid for it when new. If he received $1350 for the car, how much had he originally paid for it? (Lesson 16, page 15)

Placeholder: ____________________

Simple sentence: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ____________________

Marks

6 20. In one year, 6,049,000 people were employed in Canada. About 11.1% of these people were employed in agriculture. How many people were employed in agriculture? (Lesson 16, page 15)

Placeholder: ______________________________

Simple sentence: ______________________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

6 21. The area of the surface of Lake Superior is 80,000 sq. mi. The surface area of Lake Ontario is 35,000 sq. mi. The surface area of Lake Superior is what per cent of the surface area of Lake Ontario? (Express your answer to the nearest per cent.) (Lesson 16, page 15)

Placeholder: ______________________________

Simple sentence: ______________________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

5 22. John works an average of 7 hours per day Monday through Friday. How many hours does John work in one week? (Lesson 17, page 7)

Formula: ____________________

Replacements: ____________________ Computation

____

Answer: ____________________

Statement: ______________________________

Marks

5 23. At a speed of 96 km per hour, how far will a car travel in 3 hours? (Lesson 17, pages 2 and 3)

Formula: ______________________________

Replacements: ______________________________ Computation

____

Answer: ______________________________

Statement: ____

5 24. Suppose you have $100 in a savings account in a bank which pays 8% interest per year. How much interest does the bank pay into your savings account at the end of one year? (Lesson 17, page 12)

Formula: ______________________________

Replacements: ______________________________ Computation

____

Answer: ______________________________

Statement: ____

5 25. Find the cost of 48 wall tiles if the price of one wall tile is listed at $1.09. (Lesson 18, page 2)

Formula: ______________________________

Replacements: ______________________________ Computation

____

Answer: ______________________________

Statement: ____

Marks

5 26. The Art Shop is buying ten wall-hangings each one priced at $69.75. What is the cost of these ten wall-hangings? (Lesson 18, page 2)

Formula: ______________________________

Replacements: ______________________________ Computation

____

Answer: ______________________________

Statement: ____

5 27. A playground is 250 metres long and 125 metres wide. What is the area of this playground? (Lesson 19, page 13)

Formula: ______________________________

Replacements: ______________________________ Sketch and Computation

____

Answer: ______________________________

Statement: ____

5 28. The perimeter of a square is 89 cm. What is the measure of a side of the square? (Lesson 19, pages 1 and 2)

Formula: ______________________________

Replacements: ______________________________ Sketch and Computation

____

Answer: ______________________________

Statement: ____

larks

5 29. The side of a square room is 15 ft. in length. What is the area of the floor? (Lesson 19, pages 9 and 10)

Formula: ______________________________

Replacements: ______________________________ Sketch

____

Answer: ______________________________

Statement: ____

5 30. The length of a rectangle is 4 feet and its width is 3 feet. Find its perimeter. (Lesson 19, pages 4-6)

Formula: ______________________________

Replacements: ______________________________ Sketch

____

Answer: ______________________________

Statement: ____

5 31. A cube measures 4" by 4" by 4". How many cubic inches does it contain? (Lesson 19, pages 17-19)

Formula: ______________________________

Replacements: ______________________________ Sketch

____

Answer: ______________________________

Statement: ____

Marks

5 32. A box measures 30.4 cm. long, 15.2 cm. wide and 15.2 cm. high. Find its volume in cubic centimeters. (Lesson 19, pages 21-23)

Formula: ______________________________

Replacements: ______________________________ Sketch

____

Answer: ______________________________

Statement: ____

61 33. Complete this chart. Tell what each variable represents. Give one set of units of measure that could be used for the variables of each formula. Name the formula.

Formula	Variables	Units of measure	Name
$d = rt$	d, r, t,	d, r, t,	
$a = \frac{t}{n}$	a, t, n,	a, t, n,	
$i = prt$	i, p, r, t,	i, p, r, t,	
$P = 4s$	P, s,	P, s,	

Marks

Formula	Variables	Units of measure	Name
$P = 2\ell + 2w$	P, ℓ, w,	P, ℓ, w,	
$A = s^2$	A, s,	A, s,	
$A = \ell w$	A, ℓ, w,	A, ℓ, w,	
$V = s^3$	V, s,	V, s,	
$V = \ell wh$	V, ℓ, w, h,	V, ℓ, w, h,	

Total Marks
271

End of Lesson 20

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ________

Time Spent on Lesson ________

(If label is missing
or incorrect)

File Number ________

Lesson Number ________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ________

Date Lesson Received: ________

Lesson Recorded ________

**Student's Questions
and Comments**

Apply Lesson Label Here

Name

Address

Postal Code

*Please verify that preprinted label is for
correct course and lesson.*

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Conversion of Measures

Topic One: Conversion of linear measures

For some time you have probably, mentally, changed one linear unit of measure to another linear unit of measure.
e.g. You can say without hesitation that 12 inches equal 1 foot, and 24 inches equal 2 feet, and so on.

You will now learn a formal method of changing familiar units of measure so that you can deal with units with which you are not so familiar. You can use what you learned in Lesson 16 about ratio to work with these equivalent measures. You can find the equivalent measure by applying the ratio test to two equal ratios, one of which has a missing component.

EXAMPLE: The length of the Lions Gate Bridge at Vancouver is about 516 yards. What is the length of the bridge in feet?

Solution

We know that 3 ft. = 1 yd.

We can write the ratio $\frac{3}{1}$ to compare feet to yards.

If we let n be the length of the bridge in feet, we can write another ratio $\frac{n}{516}$ which compares the length of the bridge in feet to the length of the bridge in yards. These two ratios are equal.

$$\text{Ratio of ft. to yd.} \rightarrow \left(\frac{3}{1}\right) = \frac{n \leftarrow \text{ft.}}{516 \leftarrow \text{yd.}}$$

Use the ratio test to find the value of n.

$$\begin{aligned} 3 \times 516 &= 1n \\ 1548 &= 1n \longleftarrow (1n \text{ is the same as } n.) \\ 1548 &= n \end{aligned}$$

The length of the bridge is 1548 ft.

To be able to convert units of measure you must know the basic relationships between these units of measure. Following is a chart containing the more commonly used units of linear measure. If you do not already know these conversion facts, be sure to learn them. They are very necessary for your future work.

Conversion Facts — English Linear Units

12 inches = 1 foot
36 inches = 1 yard
3 feet = 1 yard
$5\frac{1}{2}$ yards = 1 rod

5280 feet = 1 mile
1760 yards = 1 mile

PLEASE LEARN.

Self-correcting Exercise 1

In each case tell what the placeholder represents and state the conversion fact you must use. Then, write a pair of equal ratios. One ratio will state the basic relationship between the units of measure. The other ratio will contain the missing component and the given information. Use the ratio test to find the missing component.

1. 1 inch = ___?___ feet

2. 1 foot = ___?___ yards

3. 18480 ft. = ___?___ miles

4. 6 miles = ___?___ yards

Corrections are on page 18.

Assignment 1

For each problem:
- tell what the placeholder represents,
- state the conversion fact you must use,
- write an equation made up of two ratios,
- find the answer and
- write a statement answering the question asked.

1. The surface of the Dead Sea is 1293 feet below sea level. The surface of the Dead Sea is how many miles below sea level? (hundredths)

Placeholder: Let x be the number of miles the surface of the Dead Sea is below sea level.

Conversion fact: 5280 ft. = 1 mi.

Computation

$$\begin{array}{r} .24 \\ 5280\overline{)1293.00} \\ \underline{10560} \\ 23700 \\ \underline{21120} \\ 2580 \end{array}$$

Equation: $\frac{5280}{1} = \frac{1293}{x}$

$5280x = 1293 \times 1$

$5280x = 1293$

$x = 1293 \div 5280$

Answer: $x = .24$

Statement: The surface of the Dead Sea is .24 miles below sea level.

2. The height of Angel Falls in Venezuela is .625 of a mile. What is the height in feet of Angel Falls?

Placeholder: ______________________

Conversion fact: ______________________ Computation

Equation: ______________________

____

____

____

Answer: ______________________

Statement: ______________________

____

3. One of the largest sharks ever caught was 163 in. long. What was the length of the shark in feet? (tenths)

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

4. The highest mountain in the world is Mount Everest in the Himalayas. The summit of Mount Everest is 29,028 feet above sea level. How many yards above sea level is the summit of Mount Everest?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

5. The average depth of the Pacific Ocean is $2\frac{2}{3}$ miles. What is its average depth in yards?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

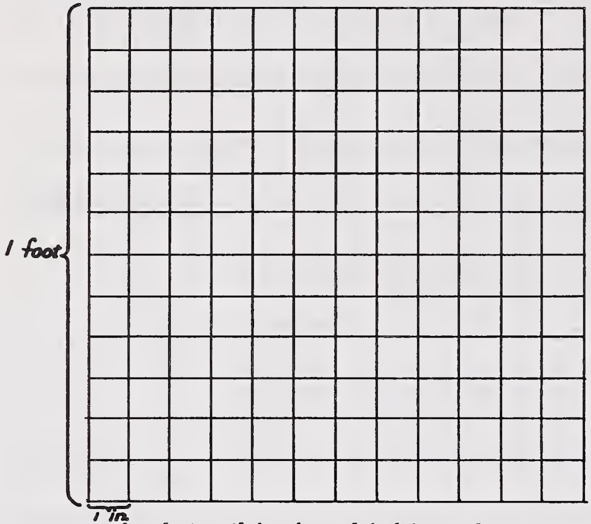
Statement: ______________________________

Topic Two: Conversion of square measures

It is possible to calculate a conversion table for square units of measure whenever it is needed. However, it is more convenient to know a few basic relationships between square measures.

e.g. Suppose you want to buy a carpet for your front room. We usually measure the sides of a room in feet. Therefore the area is a number of square feet. Carpeting is sold by the square yard. It is useful to know the basic relationship between square feet and square yards so that you can quickly change the units of measure.

Following is a diagram showing the relationship between square inches and square feet.



Scale: 1 inch = 4 inches

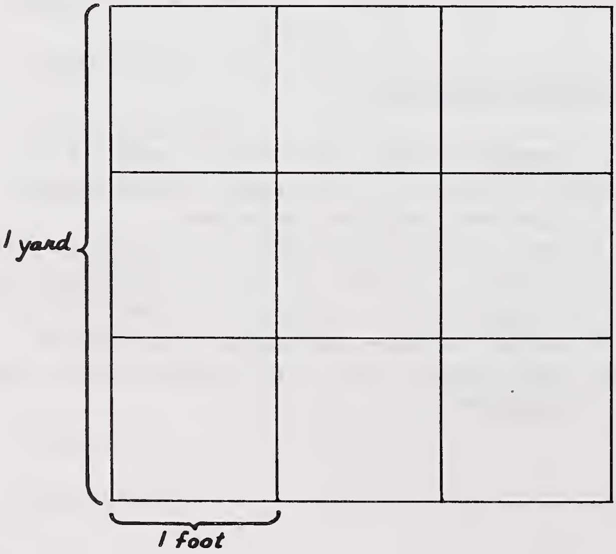
Each side measures 1 foot or 12 inches.

Each small square is 1 sq. in. The large square is 1 sq. ft.

Therefore, by counting the squares, you can see that:
1 square foot = 144 square inches.

You can calculate this by thinking the square is 1 foot by 1 foot or 12 inches by 12 inches. 1 ft. × 1 ft. = 1 sq. ft.
12 inches × 12 inches = 144 sq. in. 1 sq. ft. = 144 sq. in.

Following is a diagram showing the relationship between square feet and square yards.



Scale: 1 inch = 1 foot

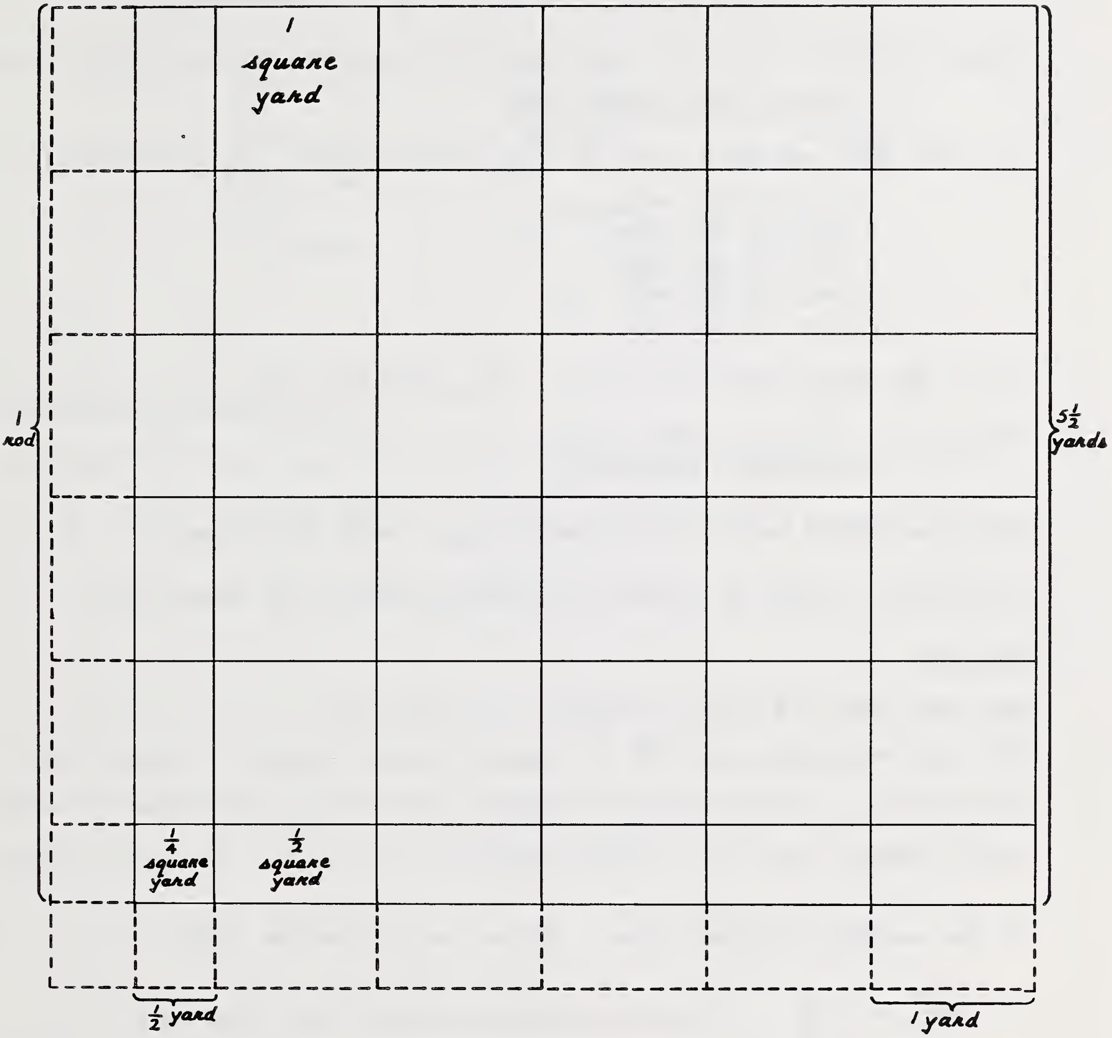
Each side measures 1 yard or 3 feet.

Each small square is 1 sq. ft. The large square is 1 sq. yd.

Therefore, by counting the squares, you can see that:
1 square yard = 9 square feet.

You can calculate this by thinking the square is 1 yard by 1 yard or 3 feet by 3 feet. 1 yard × 1 yard = 1 sq. yd.
3 feet × 3 feet = 9 sq. ft. 1 sq. yd. = 9 sq. ft.

Following is a diagram showing the relationship between square yards and square rods.



Scale: 1 inch = 1 yard

Each side measures $5\frac{1}{2}$ yards or 1 rod.

There are 25 squares the size of 1 square yard for a total of 25 sq. yd.

There are 10 squares the size of $\frac{1}{2}$ square yard for a total of 5 sq. yd.

There is 1 square the size of $\frac{1}{4}$ square yard for a total of $\frac{1}{4}$ sq. yd.

The area of the square shown by solid lines is $30\frac{1}{4}$ sq. yd.

1 square rod = $30\frac{1}{4}$ square yards.

By multiplying we can obtain the same area.

$$5\frac{1}{2} \times 5\frac{1}{2} = \frac{11}{2} \times \frac{11}{2} = \frac{121}{4} = \underline{30\frac{1}{4}}$$

Square rods and acres are units that are used to measure areas of land.

1 acre = 160 square rods

An acre does not have to be in any special shape. It could measure

1 rod by 160 rods,
2 rods by 80 rods,
4 rods by 40 rods,
5 rods by 32 rods or
8 rods by 20 rods.

A. is the abbreviation for acre. 1A. = 160 sq. rd.

Also: 1 section = 640 acres
1 section = 1 sq. mi.

Now you should be able to convert square units of measure.

EXAMPLE: State the number of square inches in $\frac{1}{2}$ square foot.

Solution

We know that 144 square inches = 1 square foot.
We can write the ratio $\frac{144}{1}$ to compare square inches to square feet.
If we let n be the number of square inches in $\frac{1}{2}$ square foot, we can write another ratio $\frac{n}{\frac{1}{2}}$ which compares the number of square inches to the number of square feet. These two ratios are equal.

$$\frac{144}{1} = \frac{n}{\frac{1}{2}}$$

Use the ratio test to find the value of n.

$$144 \times \frac{1}{2} = 1n$$

$$72 = 1n$$

$$72 = n$$

$\frac{1}{2}$ square foot is the same as 72 square inches.

Below is a chart showing relationships between square units of measure.

Conversion Facts - English Square Units

144 sq. in. = 1 sq. ft.
9 sq. ft. = 1 sq. yd.
$30\frac{1}{4}$ sq. yd. = 1 sq. rd.
160 sq. rd. = 1 A.
640 A. = 1 section
1 section = 1 sq. mi.

Self-correcting Exercise 2

Use a pair of ratios and the ratio test to find the required measure.

1. ? sq. in. = 2 sq. ft.

2. ? sq. yd. = 189 sq. ft.

3. ? sq. yd. = 2 sq. rd.

4. ? sq. rd. = 5A.

Corrections are on page 20.

Assignment 2

For each problem:
- tell what the placeholder represents,
- state the conversion fact you must use,
- write an equation made up of two ratios,
- find the answer and
- write a statement answering the question asked.

1. Mr. Hansen is going to pave his driveway. The area of the driveway is 306 sq. ft. What is the area of the driveway in square yards?

Placeholder: Let n be the area of the driveway in square yards.

Conversion fact: 9 sq. ft. = 1 sq. yd.

Equation: $\frac{9}{1} = \frac{306}{n}$

$9n = 306 \times 1$

$9n = 306$

$n = 306 \div 9$

Computation

$$\begin{array}{r} 34 \\ 9\overline{)306} \\ 3 \end{array}$$

Answer: $n = 34$

Statement: The area of the driveway is 34 square yards.

2. The Johnson Realty Company bought $\frac{1}{8}$ sq. mi. of land. How many acres did the company buy?

Placeholder: ______

Conversion fact: ______

Equation: ______ Computation

______

______

______

Answer: ______

Statement: ______

3. Mr. Horton's field covers 51280 sq. rd. How many acres are included in Mr. Horton's field?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

4. The area of a rectangle is $27\frac{1}{2}$ sq. ft. What is the area of the rectangle in square inches?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

5. A $\frac{1}{4}$ section of land is how many acres?

Placeholder: ______________________________

Conversion fact: ______________________________

Equation: ______________________________ Computation

____

____

____

Answer: ______________________________

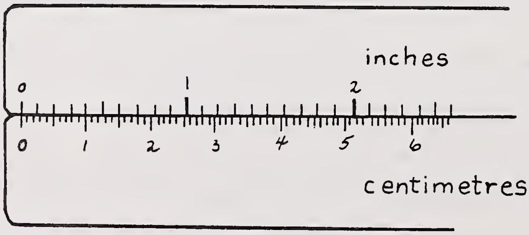
Statement: ______________________________

Topic Three: Conversion of English units of measure to metric units of measure

In recent years, Canada has decided to change over to the Metric system. It is a gradual change, which will become complete eventually. The reason for Canada's change is that the Metric system is used by so many other countries around the world. Also the Metric system is relatively simple. Naturally, Canadians are very interested in the names of the units, called SI (Système International d'Unités).

Some everyday metric units are the gram, the litre (pronounced lē′tər), the metre (pronounced mē′tər), the kilometre (pronounced kil′əmē′tər), and the degree Celsius (°C).

A unit of measure in the metric system that is similar to the size of an inch is a centimetre.



(actual size)

From this you can see that 1 inch is a little more than $2\frac{1}{2}$ centimetres.

1 inch = 2.54 cm.

You can also see that 1 centimetre is very near $\frac{4}{10}$ inches.

1 centimetre = .394 in.

Since this course was first written, Canada's use of SI units of measurement has been approved. The names, spellings, and symbols are given correctly on this colored page. Please note that the officially - approved spelling has "-re" at the end.

Spelling (SI units)	Symbol with no period
millimetre	mm
centimetre	cm
decimetre	dm
metre	m
kilometre	km
gram	g
kilogram	kg
litre	L
degree Celsius	°C

A hint for remembering the correct symbol is that it is the first letter in the prefix and the first letter in the root word. It is a symbol only and therefore it has no period. It is not an abbreviation. (An abbreviation would have had a period, but a symbol has no period at the end.)

SI symbols for area in square units

Because area is two-dimensional, use the "two" as an exponent.

cm^2 for square centimetre

m^2 for square metre

Example: The door of a kitchen refrigerator has an area of 1 m^2 approximately.

SI symbols for volume in cubic units

Because volume is three-dimensional, use the "three" as an exponent.

cm^3 for cubic centimetre

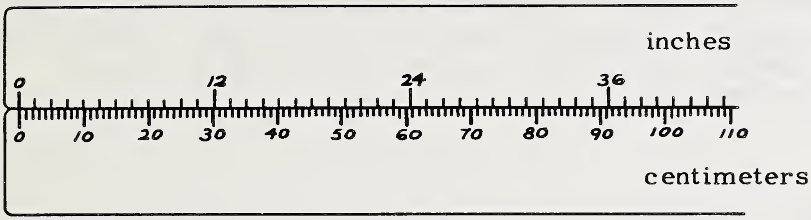
m^3 for cubic metre

Example: There is one cubic metre, 1 m^3 in the amount of space occupied by a box that is 1 metre long, 1 metre wide, and 1 metre high.

I feel that the easier one to remember of this pair of relationships between inches and centimeters is 1 inch = 2.54 cm. You need to know only one of them in order to do any problems you encounter.

There is no unit of measure in the metric system that is similar in size to the foot.

A unit of measure in the metric system that is similar to the size of a yard is a meter.



Scale: 1 inch = 10 inches

From this diagram, you can see that 1 meter (100 centimeters) is a little more than 1 yard (36 inches).

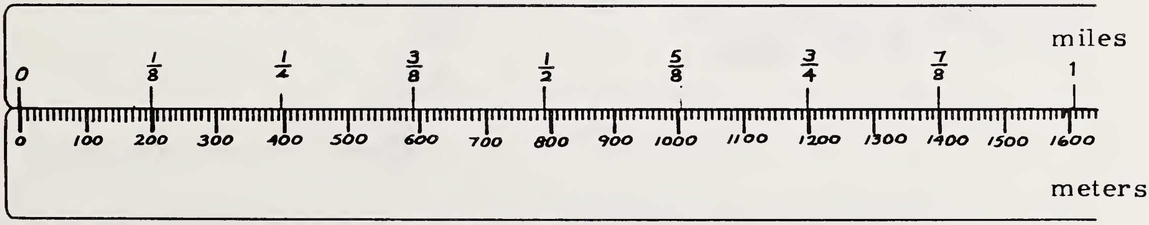
1 meter = 39.37 inches or
1 meter = 1.094 yards

You can also see that 1 yard is a little less than 1 meter.

1 yard = .9144 meters

I feel that of this group of relationships either 1 meter = 39.37 inches or 1 meter = 1.094 yards should be remembered. 1 meter = 39.37 inches appears frequently but the actual relationship between yards and meters is shown by 1 meter = 1.094 yards.

The next larger pair of units of measure that are at all similar in size is miles and kilometers.



Scale: 1 cm. = 100 m.

From this diagram you can see that 1 mile is more than 1 kilometer (1000 meters).

1 mile = 1.609 km. (The abbreviation for kilometers is km.)

You can also see that 1 kilometer is less than 1 mile.

1 kilometer = .621 miles

I feel that 1 kilometer = .621 miles is the easier of the two miles - kilometer relationships to remember.

Now we can convert English units of measure to metric units of measure or vice versa.

EXAMPLE: Standard atmospheric pressure is the pressure at sea level that will support a column of mercury 76 cm. high. What is the approximate height of the column of mercury in inches? (thousandths)

Solution

We know that 1 inch = 2.54 cm.

We can write the ratio $\frac{1}{2.54}$ to compare inches and centimeters. If we let n be the height of the column of mercury in inches, we can write another ratio $\frac{n}{76}$ which compares the height of the column of mercury in inches to the height of the column of mercury in centimeters. These two ratios are equal.

$$\frac{1}{2.54} = \frac{n}{76}$$

Use the ratio test to find the value of n.

$$1 \times 76 = 2.54n$$

$$76 = 2.54n$$

$$76 \div 2.54 = n$$

$$29.921 = n$$

$$\begin{array}{r}
29.921 \\
2.54\overline{)\,76.00000} \\
\underline{50\ 8} \\
25\ 20 \\
\underline{22\ 86} \\
2\ 340 \\
\underline{2\ 286} \\
540 \\
\underline{508} \\
320 \\
\underline{254} \\
66
\end{array}$$

The height of the column of mercury is 29.921 inches.

Below is a chart comparing English and metric units of measure.

Comparison of English and Metric Units of Linear Measure

1 inch = 2.54 cm.
39.37 inches = 1 meter
1.094 yards = 1 meter
.621 miles = 1 kilometer

Self-correcting Exercise 3

Use a pair of ratios and the ratio test to find the required measures.

1. 52 cm. = $\frac{?}{\text{(tenths)}}$ in.

2. 45 yd. = $\frac{?}{\text{(tenths)}}$ m.

3. 350 km. = ___?___ mi.

4. 45 in. = ___?___ cm.

Corrections are on page 21.

Assignment 3

For each problem:

- tell what the placeholder represents,
- state the conversion fact you must use,
- write an equation made up of two ratios,
- find the answer and
- write a statement answering the question asked.

1. While Sarah was in France, she bought some silk cloth. The cloth is 1.3 m. wide. What is the approximate width of the cloth in inches? (Round your answer to the nearest inch.)

Placeholder: Let w be the width of the cloth in inches.

Conversion fact: 39.37 in. = 1 m.

Equation: $\frac{39.37}{1} = \frac{w}{1.3}$

$39.37 \times 1.3 = 1 \times w$

Answer: $51 = w$

Computation

$$\begin{array}{r} 39.37 \\ \underline{1.3} \\ 11811 \\ \underline{39370} \\ \cancel{51.181} \quad 51 \end{array}$$

Statement: The width of the cloth is approximately 51 inches.

2. In some countries, a person's height is measured in centimeters. How many centimeters tall is a person who is 72 inches tall? (ones)

Placeholder: ____________________

Conversion fact: ____________________

Equation: ____________________

Computation

____

____

____

Answer: ____________________

Statement: ____________________

3. A dragonfly can fly 38.616 kilometers per hour. What is its speed in miles per hour?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

4. What is the length in yards of a building 100 meters long?

Placeholder: ______________________________

Conversion fact: ____________________

Equation: ____________________ Computation

____

____

____

Answer: ____________________

Statement: ______________________________

5. The distance from New York to Boston is 200 miles. What is the distance in kilometers?

Placeholder: ____

Conversion fact: ________________________

Equation: ____________________________ Computation

____

____

____

Answer: ____________________________

Statement: ____

6. Arrange these units of measure in order of size. Begin with the smallest unit of measure.

centimeters, feet, inches, kilometers, meters, miles, rods, yards

____

____

Key to Self-correcting Exercises in Lesson 21

Exercise 1, page 2

1. 1 inch = $\underline{\ ?\ }$ feet

Let n be the number of feet in 1 inch. ← (Tell what the placeholder represents.)

12 inches = 1 foot ← (State conversion fact you must use.)

$$\text{Ratio of in. to ft.} \quad \boxed{\frac{12}{1}} = \frac{1}{n} \begin{matrix}\leftarrow \text{inches} \\ \leftarrow \text{feet}\end{matrix}$$

← (Set up two equal ratios.)

$$12 \times n = 1 \times 1$$

← (Apply ratio test.)

$$12n = 1$$

$$n = 1 \div 12$$

$$n = \frac{1}{12}$$

Thus, 1 inch = $\underline{\frac{1}{12}}$ foot.

2. 1 foot = ___?___ yards
Let n be the number of yd. in 1 ft.

3 feet = 1 yard

Ratio of ft. to yd. $\boxed{\frac{3}{1}} = \frac{1 \leftarrow ft.}{n \leftarrow yd.}$

$$3 \times n = 1 \times 1$$
$$3n = 1$$
$$n = 1 \div 3$$
$$n = \frac{1}{3}$$

Thus, 1 foot = $\underline{\frac{1}{3}}$ yd.

3. 18,480 ft. = ___?___ miles
Let n be the number of miles in 18,480 ft.

5280 ft. = 1 mile

Ratio of ft. to mi. $\boxed{\frac{5280}{1}} = \frac{18,480 \leftarrow ft.}{n \leftarrow mi.}$

$$\begin{array}{r} 3.5 \\ 5280 \,\overline{)\,18480.\underset{\times}{0}} \\ \underline{15840} \\ 2640\;0 \\ \underline{2640\;0} \\ 0 \end{array}$$

$$5280 \times n = 1 \times 18,480$$
$$5280n = 18,480$$
$$n = 18,480 \div 5280$$
$$n = 3.5$$

Thus, 18,480 ft. = $\underline{3.5}$ mi.

4. 6 miles = ___?___ yards
Let n be the no. of yd. in 6 mi.

1760 yd. = 1 mi.

Ratio of yd. to mi. $\boxed{\frac{1760}{1}} = \frac{n \leftarrow yd.}{6 \leftarrow mi.}$

$$\begin{array}{r} 1760 \\ \underline{6} \\ 10,560 \end{array}$$

$$1760 \times 6 = 1 \times n$$
$$10,560 = n$$

Thus, 6 miles = $\underline{10,560}$ yards.

Exercise 2, page 9

1. $\underline{\ ?\ }$ sq. in. = 2 sq. ft.

Let n be the no. of sq. in. in 2 sq. ft.

144 sq. in. = 1 sq. ft.

Ratio of sq. in. to sq. ft. $\boxed{\frac{144}{1}} = \frac{n \leftarrow \text{sq. in.}}{2 \leftarrow \text{sq. ft.}}$

$144 \times 2 = 1 \times n$

$288 = n$

Thus, $\underline{288}$ sq. in. = 2 sq. ft.

2. $\underline{\ ?\ }$ sq. yd. = 189 sq. ft.

Let n be the no. of sq. yd. in 189 sq. ft.

9 sq. ft. = 1 sq. yd.

$$\frac{9}{1} = \frac{189}{n}$$

$9 \times n = 1 \times 189$

$9n = 189$

$n = 189 \div 9$

$n = 21$

$$\begin{array}{r} 21 \\ 9\overline{)189} \end{array}$$

Thus, $\underline{21}$ sq. yd. = 189 sq. ft.

3. $\underline{\ ?\ }$ sq. yd. = 2 sq. rd.

Let n be the no. of sq. yd. in 2 sq. rd.

$30\frac{1}{4}$ sq. yd. = 1 sq. rd.

$$\frac{30\frac{1}{4}}{1} = \frac{n}{2}$$

$30\frac{1}{4} \times 2 = 1 \times n$

$\frac{121}{4} \times 2 = n$

$\frac{121 \times \overset{1}{\cancel{2}}}{\cancel{4}_2} = n$

$$\begin{array}{r} 60\frac{1}{2} \\ 2\overline{)121} \end{array}$$

$60\frac{1}{2} = n$

Thus, $\underline{60\frac{1}{2}}$ sq. yd. = 2 sq. rd.

4. ? sq. rd. = 5 A.

Let n be the no. of sq. rd. in 5 A.

160 sq. rd. = 1 A.

$$\frac{160}{1} = \frac{n}{5}$$

$160 \times 5 = 1 \times n$

$800 = n$

Thus, 800 sq. rd. = 5 A.

Exercise 3, page 15

1. 52 cm. = $\frac{?}{\text{(tenths)}}$ in.

Let n be the no. of in. in 52 cm.

1 inch = 2.54 cm.

Ratio of in. to cm.

$$\boxed{\frac{1}{2.54}} = \frac{n \leftarrow \text{in.}}{52 \leftarrow \text{cm.}}$$

$1 \times 52 = 2.54 \times n$

$52 = 2.54n$

$52 \div 2.54 = n$

$20.5 = n$

Thus, 52 cm. = 20.5 in.

```
          20.5
         ~~20.4~~
2.54.) 52.00.0
       50 8
        1 20
           0
        1 200
        1 016
        ~~184~~
```

2. 45 yd. = $\frac{?}{\text{(tenths)}}$ m.

Let n be the number of m. in 45 yd.

1.094 yd. = 1 m.

$$\frac{1.094}{1} = \frac{45}{n}$$

$1.094 \times n = 1 \times 45$

$1.094n = 45$

$n = 45 \div 1.094$

$n = 41.1$

Thus, 45 yd. = 41.1 m.

```
              41.1
1.094.)  45.000.0
         43 76
          12 40
          10 94
           1 460
           1 094
             366
```

3. 350 km. = $\underline{\;?\;}$ mi.
Let n be the no. of mi. in 350 km.

.621 mi. = 1 km.

$$\frac{.621}{1} = \frac{n}{350}$$

$$.621 \times 350 = 1 \times n$$

$$217.350 = n$$

$$\begin{array}{r} .621 \\ \underline{350} \\ 31050 \\ \underline{186300} \\ 217.350 \end{array}$$

Thus, 350 km. = 217.35 mi.

4. 45 in. = $\underline{\;?\;}$ cm.
Let n be the no. of cm. in 45 in.

1 inch = 2.54 cm.

$$\frac{1}{2.54} = \frac{45}{n}$$

$$1 \times n = 2.54 \times 45$$

$$n = 114.3$$

$$\begin{array}{r} 2.54 \\ \underline{45} \\ 1270 \\ \underline{10160} \\ 114.30 \end{array}$$

Thus, 45 in. = 114.3 cm.

End of Lesson 21

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned Teacher: ________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

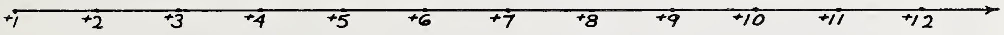
Integers

Topic One: Positive integers

In this course we have already dealt with part of the system of integers. Positive integers are the same as the set of natural numbers. Remember - the set of natural numbers contains the numbers 1, 2, 3 and so on. The set of positive integers is also made up of the numbers 1, 2, 3 and so on. Positive integers can be indicated by placing a small plus sign (+) to the upper left of the number.

e.g. $^{+}4$ is a positive integer. It is read "positive four".

Thus the set of positive integers contains the numbers $^{+}1$, $^{+}2$, $^{+}3$ and so on. These numbers can be placed on a number line. A number line is a line divided into congruent segments. An integer is associated with the endpoint of each segment.



This is the number line of the set of positive integers. Notice that the integers are positioned in their correct order.

$^{+}1$ is associated with the beginning of the number line.
$^{+}2$ is associated with the endpoint of the first segment.
$^{+}3$ is associated with the endpoint of the second segment and so on.

The arrow on the right of the line indicates that this number line goes on and on to the right. The arrow indicates that the numbers $^{+}13$, $^{+}14$, $^{+}15$ and all those greater than $^{+}15$ are included in this line.

By using the number line we can decide whether numbers are greater than or less than each other.

If a number is to the right of another number it is greater than that number.

e.g. Consider $^{+}9$ and $^{+}5$ on the number line.
$^{+}9$ is to the right of $^{+}5$. Therefore $^{+}9$ is greater than $^{+}5$.

If a number is to the left of another number it is less than that number.

e.g. Consider $^{+}2$ and $^{+}7$ on the number line.
$^{+}2$ is to the left of $^{+}7$. Therefore $^{+}2$ is less than $^{+}7$.

We can use symbols to indicate "is greater than" and "is less than".

The symbol for "is greater than" is $>$.
e.g. $^{+}9 > {}^{+}5$ is read "$^{+}9$ is greater than $^{+}5$".

The symbol for "is less than" is $<$.
e.g. $^{+}2 < {}^{+}7$ is read "$^{+}2$ is less than $^{+}7$".

Assignment 1

Use "yes" to indicate which numbers are positive integers and use "no" to indicate which numbers are not positive integers. Remember—fractions and decimals are not integers. $^+1$, $^+2$, $^+3$, and so on are positive integers.

1. 3 yes
2. $^+4$ ________
3. $^+2.1$ ________
4. 6 ________
5. $\frac{2}{3}$ ________

Between each pair of numbers place < or > to make the sentence true.

6. $^+8$ > $^+5$
7. $^+7$ ____ $^+3$
8. $^+2$ ____ 6
9. $^+4$ ____ $^+11$
10. 8 ____ $^+4$

If possible, find a positive integer that makes each statement true. If no positive integer will make the sentence true, write "impossible".

11. 8 - 5 = 3
12. 5 - 8 = ________
13. 8 - 4 = ________
14. 18 - 13 = ________
15. 7 - 3 = ________
16. 3 - 7 = ________
17. 4 - 8 = ________
18. 12 - 16 = ________

Topic Two: Negative integers

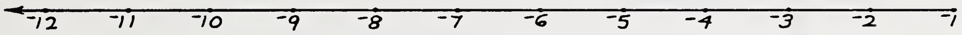
Negative integers use the same numerals as positive integers do. To indicate that a number is a negative integer a dash is placed in front (to the upper left) of the number.

e.g. $^{-}3$ is a negative integer. It is read "negative three".

Thus the set of negative integers is $^{-}1$, $^{-}2$, $^{-}3$ and so on. These integers are needed to express debts, drops, below zero temperatures, etc.

e.g. A drop of 10 in the price of stocks can be expressed by $^{-}10$.

Negative integers can also be placed on a number line.



$^{-}1$ is associated with the beginning of the number line.
$^{-}2$ is associated with the endpoint of the first segment.
$^{-}3$ is associated with the endpoint of the second segment and so on.

The arrow on the left of the line shows that this number line goes on and on to the left. It indicates that $^{-}13$, $^{-}14$, $^{-}15$ and all those numbers less than $^{-}15$ are included in this line.

The number line can again be used to indicate whether numbers are less than or greater than each other. The same rules apply to negative integers as to positive integers.

If a number is to the right of another number it is greater ~~than~~ ~~that~~ number.

e.g. Consider $^{-}5$ and $^{-}9$ on the number line.
$^{-}5$ is to the right of $^{-}9$. Therefore $^{-}5 > ^{-}9$.

If a number is to the left of another number it is less than that number.

e.g. Consider $^{-}7$ and $^{-}2$ on the number line.
$^{-}7$ is to the left of $^{-}2$. Therefore $^{-}7 < ^{-}2$.

It may seem strange that $^{-}5 > ^{-}9$ and $^{-}7 < ^{-}2$ in comparison to $^{+}5 < ^{+}9$ and $^{+}7 > ^{+}2$. $^{-}5$ and $^{-}9$ can represent debts of \$5 and \$9. Suppose you and your friend have no money today but there is a sale on at Eaton's. You and your friend charge your purchases. You charged a sum of \$5; your friend charged a sum of \$9. You are richer than your friend because your debt is smaller. To be lacking \$5 is better than to be lacking \$9. You have more than your friend has (in terms of money). $^{-}5 > ^{-}9$
Your friend has less than you have (in terms of money).
$^{-}9 < ^{-}5$

Later on we will use these negative integers to find the answer when you subtract a greater number from a lesser one.

e.g. 4 - 8 will have an answer.

Self-correcting Exercise 1

Use a positive integer or a negative integer to represent each of these situations.

1. loss of 20 yards ____________
2. gain of 5 yards ____________
3. rise of 10 degrees ____________
4. drop of 15 degrees ____________
5. increase of $30 ____________
6. deficit of $150 ____________
7. asset of $1500 ____________
8. debt of $500 ____________

Decide whether each of the following statements is true or false. (Think of how these pairs of numbers lie in relation to each other on the number line.)

9. ${}^{-}2 < {}^{-}6$ ____________________
10. ${}^{-}5 < {}^{-}4$ ____________________
11. ${}^{-}13 > {}^{-}17$ ____________________
12. ${}^{-}11 > {}^{-}4$ ____________________
13. ${}^{-}4 < {}^{-}11$ ____________________
14. ${}^{-}4 < {}^{-}5$ ____________________
15. ${}^{-}10 > {}^{-}15$ ____________________

Corrections are on page 11.

Assignment 2

Use "yes" to indicate which numbers are negative integers and use "no" to indicate which numbers are not negative integers.

1. $^{-}21$ yes
2. 18 ________
3. $^{-}17.5$ ________
4. $^{-}101$ ________
5. $\frac{^{-}5}{8}$ ________

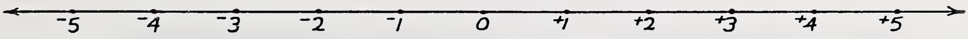
Between each pair of numbers place > or < to make the sentence true.

6. $^{-}1 \underline{\ >\ } {}^{-}2$
7. $^{-}1$ _____ $^{-}5$
8. $^{-}5$ _____ $^{-}3$
9. $^{-}2$ _____ $^{-}6$
10. $^{-}2$ _____ $^{-}1$

Topic Three: Integers

The set of integers is made up of the set of positive integers, zero, and the set of negative integers.

A number line can be used to show the set of integers.



Notice that the number line for the set of integers has no endpoints. Both ends have arrows to show that $^{+}6$, $^{+}7$, $^{+}8$ and all the numbers greater than $^{+}8$ are included and that $^{-}6$, $^{-}7$, $^{-}8$ and all the numbers less than $^{-}8$ are included.

Since the negative integers are to the left of the positive integers, each negative integers is less than each positive integer.

e.g. $^{-}7 < 6$
$^{-}2 < 3$
$^{-}5 < 1$

Pairs of integers like $^{+}4$ and $^{-}4$ are called opposites. $^{-}4$ is opposite of $^{+}4$. $^{+}4$ is opposite of $^{-}4$. Opposites have the same numeral but opposite signs. They lie the same distance from zero on the number line.

Assignment 3

Use "yes" to indicate which numbers are integers and use "no" to indicate which numbers are not integers.

1. 1432 yes
2. $^{-}1432$ ________
3. $\frac{15}{31}$ ________
4. $^{-}1.05$ ________
5. $^{-}33475$ ________

Name the opposite of each given number.

6. $^{-}260$ $^{+}260$(or 260)
7. $^{+}56$ ________
8. $^{-}786$ ________
9. $^{-}20$ ________
10. $^{+}597$ ________
11. 9 ________
12. $^{-}268$ ________

Give the negative measurement when each of the following situations is considered to be a positive measurement.

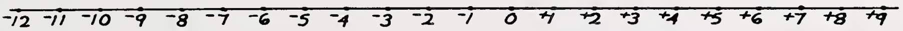
13. a rise in temperature a drop in temperature
14. a gain in a business transaction ________
15. a deposit in a bank account ________
16. a decrease in the price of a house ________
17. the distance a car travelled north ________
18. a loss in yards for your football team ________

19. a force pulling downward ______________________________

20. a force pulling upward ______________________________

21. On this number line which number is the least? ____________

Which number is the greatest? ____________



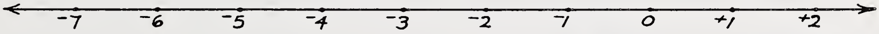
Topic Four: Graphing of integers

To graph a set of integers we must first draw a number line which has been extended to include the integers to be graphed. Then the required integers can be shown by circling the points associated with the integers.

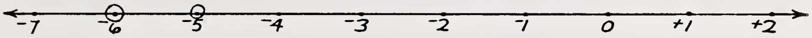
EXAMPLE: Draw a graph of $^{-}5$ and $^{-}6$.

Solution

Draw a number line of the set of integers. Be sure that it is extended far enough to the left to include $^{-}5$ and $^{-}6$.



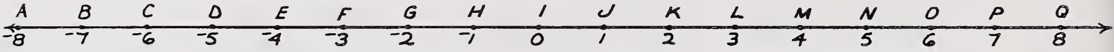
Look at the point above the integer $^{-}5$ and above the integer $^{-}6$. Put circles around these points.



This is the graph of $^{-}5$ and $^{-}6$.

Self-correcting Exercise 2

Both letters and numerals are used to label the points on this number line. What integers are between the given points?



1. N and Q ____________
2. M and O ____________
3. E and H ____________
4. M and Q ____________
5. J and P ____________

Arrange the numbers in order. Begin with the least number.

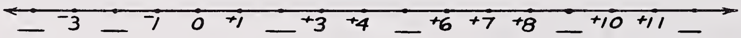
6. 2, 8, 1 ________________
7. $^{-}1$, $^{-}2$, $^{-}3$ ________________
8. $^{-}4$, 6, $^{-}12$ ________________
9. $^{+}6$, 0, $^{-}1$ ________________
10. 2, $^{-}3$, $^{+}1$, $^{-}4$ ________________

Corrections are on page 11.

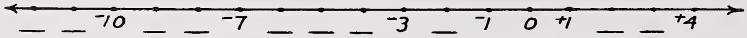
Assignment 4

Insert each of the missing integers under the number lines.

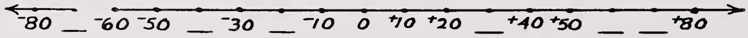
1.



2.



3.



Insert < or > in each sentence to make it true.

4. $^{-}15 \; \underline{\;<\;} \; ^{-}10$

5. $^{-}10$ ____ $^{-}5$

6. $^{-}5$ ____ 0

7. 0 ____ $^{+}5$

8. $^{+}5$ ____ 10

Complete each sentence by inserting "to the right of" or "to the left of" in the blank.

9. $^{+}8$ is to the right of $^{+}5$.

10. $^{-}5$ is ____________________ $^{-}2$.

11. $^{-}6$ is ____________________ $^{+}6$.

12. 7 is ____________________ $^{+}8$.

13. $^{-}5$ is ____________________ $^{-}4$.

14. $^{-}2$ is ____________________ 0.

Complete each of the following sentences correctly.

15. $^{+}6$ is to the right of $^{+}4$, so $^{+}6$ is greater than $^{+}4$.

16. $^{+}5$ is to the right of $^{+}2$, so _____ is greater than $^{+}2$.

17. $^{+}3$ is to the right of $^{-}5$, so ____ > ____.

18. $^{+}5$ is to the right of $^{-}5$, so ____ > ____.

19. $^{+}8$ is to the left of $^{+}14$, so $^{+}8$ is less than ______.

20. $^{+}1$ is to the left of $^{+}7$, so _____ is less than $^{+}7$.

21. $^{-}12$ is to the left of $^{-}11$, so ____ < ____.

22. $^{-}8$ is to the left of $^{+}11$, so ____ < ____.

Write these integers in order, with the least first. (Think of how they are placed in relation to each other on the number line.)

23. 0, $^{-}2$, 4, 6 -2, 0, 4, 6

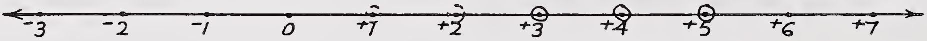
24. $^{+}10$, $^{-}10$, $^{+}5$, $^{-}5$, 0, $^{-}20$, $^{+}20$ ____________________

25. $^{+}3$, $^{+}6$, $^{-}9$, $^{+}12$, $^{-}15$, $^{+}5$, $^{+}11$, $^{-}7$ ____________________

26. 2, $^{-}4$, 6, $^{-}8$, 10, $^{-}12$, 14, $^{-}16$ ____________________

Draw a graph showing each set of integers. Encircle the answer.

27. Integers between 0 and $^{+}6$



28. Integers between $^{-}2$ and $^{+}2$

29. Integers between $^{-}8$ and $^{-}6$

30. Integers between $^{-}4$ and 0

Key to Self-correcting Exercises in Lesson 22

Exercise 1, page 4

1. $^{-}20$
2. $^{+}5$
3. $^{+}10$
4. $^{-}15$
5. $^{+}30$
6. $^{-}150$
7. $^{+}1500$
8. $^{-}500$

9. F
10. T
11. T
12. F
13. F
14. F
15. T

Exercise 2, page 8

1. $^{+}6$, $^{+}7$
2. $^{+}5$
3. $^{-}3$, $^{-}2$
4. $^{+}5$, $^{+}6$, $^{+}7$
5. $^{+}2$, $^{+}3$, $^{+}4$, $^{+}5$, $^{+}6$

6. 1, 2, 8
7. $^{-}3$, $^{-}2$, $^{-}1$
8. $^{-}12$, $^{-}4$, 6
9. $^{-}1$, 0, $^{+}6$
10. $^{-}4$, $^{-}3$, $^{+}1$, 2

End of Lesson 22

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ______________________

Time Spent on Lesson ______________________

(If label is missing
or incorrect)

File Number ______________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ______

Additional Grading
E/R/P Code: ______

Mark: __________

Graded by: ________

Assignment Code: _____

Date Lesson Received: ______________

Lesson Recorded _____

Student's Questions and Comments

Apply Lesson Label Here

Name ______________________
Address ______________________
____
____
Postal Code ______________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Addition of Integers

Topic One: Trips on a number line

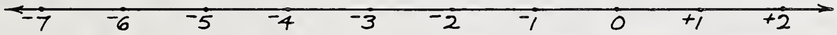
When working with integers, we must consider both the numeral involved in the integer and the sign in front of this numeral.

All integers can be thought of in terms of movements (or trips) along the number line. To represent an integer in this way, we can begin at zero and draw a line above the number line to the given integer. The numeral involved in the integer affects the length of line you draw and the sign affects the direction of the line. All positive integers are represented by trips to the right and all negative integers by trips to the left. An arrow is placed at the end of the trip to indicate whether the trip is to the right or left.

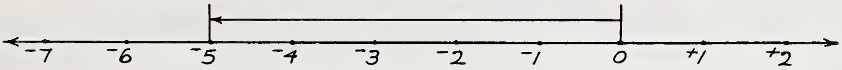
EXAMPLE. Show a trip of $^{-}5$ on the number line.

Solution

Draw a number line which includes the integer from $^{-}7$ to $^{+}2$.



Since $^{-}5$ is a negative number the trip is to the left. Draw a line above the number line from 0 to $^{-}5$. Place an arrow at the end of the line above $^{-}5$. Also draw a vertical line at each end of the above line to the points representing the integers $^{-}5$ and 0.



This is the trip expressed by $^{-}5$.

Self-correcting Exercise 1

Represent each of the following numbers as a trip on the number line.

1. $^{-}4$

2. $^{-}10$

3. $^{+}7$

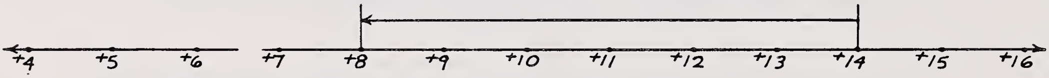
4. $^{+}15$

5. 2

Corrections are on page 13.

Some trips on the number line can begin at points other than zero. If the arrow at the end of the trip is to the right, the trip represents a positive number. If the arrow at the end of the trip is to the left, the trip represents a negative number. The number represented by the trip can be found by counting the congruent spaces that it passes over.

EXAMPLE: What number is represented by this trip?



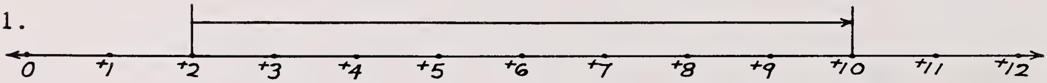
Solution

Since the arrow is to the left, it is a negative number. The number of spaces between $^{+}8$ and $^{+}14$ is 6. (Count them and see.) Therefore, this trip represents the integer $\underline{^{-}6}$.

Assignment 1

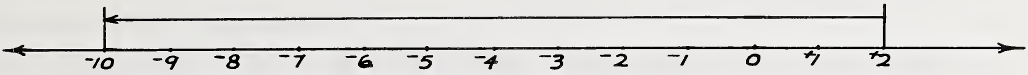
What number is represented by each trip?

1.



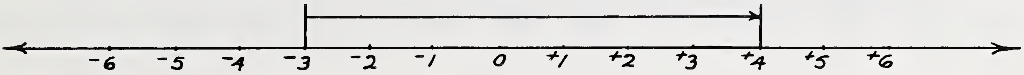
$\underline{\quad ^{+}8 \quad}$

2.



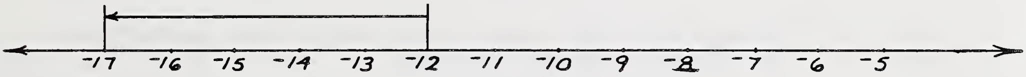
____

3.



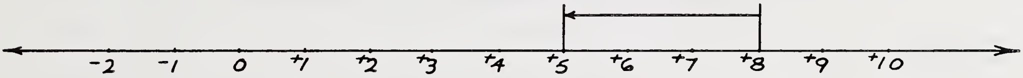
____

4.



____

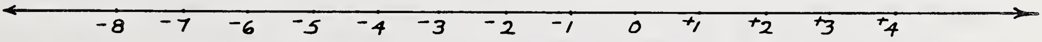
5.



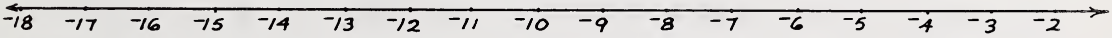
____

Use trips on a number line to represent each number. Use the given starting point.

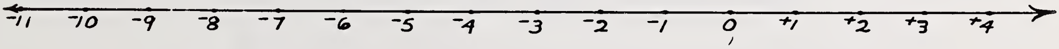
6. ⁻4, starting at ⁻4



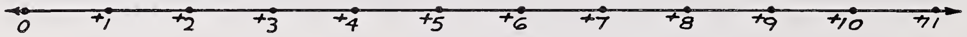
7. ⁺8, starting at ⁻10



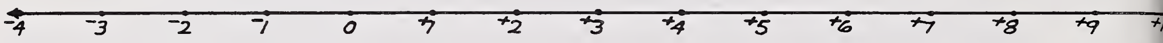
8. ⁻10, starting at 4



9. 4, starting at $^{+}6$



10. -6, starting at 3



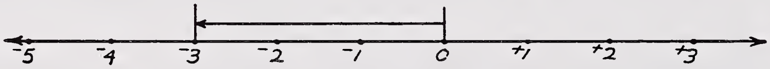
Computation involves at least two numbers. You cannot add, subtract, multiply or divide if you have only one number. Therefore, if we are going to use number lines to help us compute with integers, we need to know how to represent two numbers on one number line.

To show two consecutive trips, we represent the first number by beginning at 0 as you were shown on page 1. To represent the second number we begin at the end of the first number as you were shown on page 2.

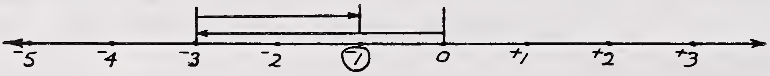
EXAMPLE: Use consecutive trips on a number line to represent a movement of $^{-}3$ followed by a movement of $^{+}2$.

Solution

Show $^{-}3$ on the number line. Remember since $^{-}3$ is a negative number, it is represented by a trip to the left. The trip begins at 0 and ends at $^{-}3$.



Show $^{+}2$ as a trip following the trip of $^{-}3$. Since $^{+}2$ is a positive number, it is represented by a trip to the right. Begin at $^{-}3$. Move 2 segments to the right.

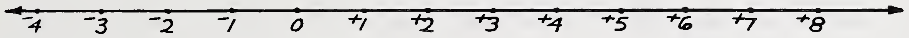


The integer which represents the end of the two trips is encircled.

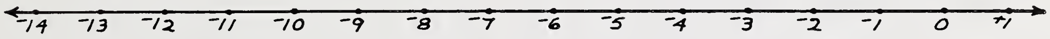
Self-correcting Exercise 2

Use consecutive trips on a number line to represent the given numbers. Encircle the integer which represents the end of the trips.

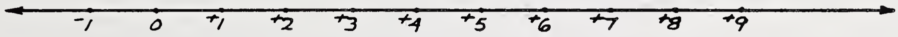
1. ⁻4, ⁺12



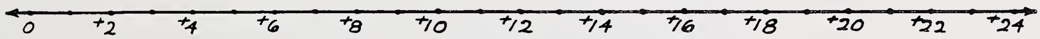
2. ⁻6, ⁻7



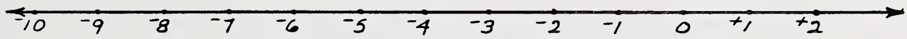
3. ⁺8, ⁻2



4. ⁺7, ⁺14



5. ⁻9, ⁺11



Corrections are on page 13.

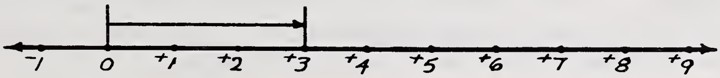
Topic Two: Sum of positive numbers

We have actually learned how to add positive numbers already by using the number line. To do this, we begin at zero and draw a line to the right for the first number (because it is positive). Then we draw a line from the endpoint of the first trip to the right to represent the second number (which is also positive). The sum of the two numbers is the integer below the end of the last trip.

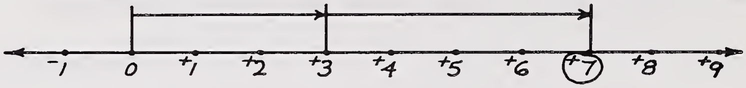
EXAMPLE: Use the number line to find the sum of ⁺3 and ⁺4.

Solution

Use a trip on the number line to represent ⁺3. Remember to begin at 0. Since ⁺3 is a positive number, the trip is to the right.



Use a second trip on the number line to show that $^{+}4$ is added to $^{+}3$. Begin at the end of the trip for $^{+}3$. Since $^{+}4$ is a positive number the trip is to the right.

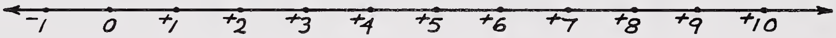


The integer at the end of the last trip is $^{+}7$. (It is encircled.) Thus $^{+}3 + {}^{+}4 = \underline{{}^{+}7}$.

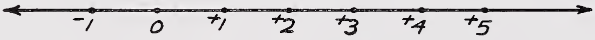
Self-correcting Exercise 3

Use a number line to find the sum of each pair of numbers.

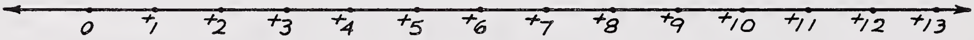
1. $^{+}4 + {}^{+}5 =$ ______



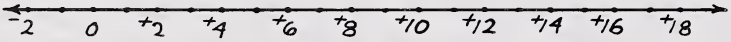
2. $^{+}2 + {}^{+}1 =$ ______



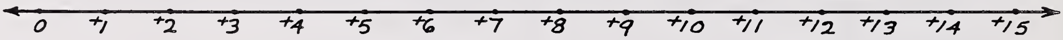
3. $^{+}5 + {}^{+}8 =$ ______



4. $^{+}8 + {}^{+}9 =$ ______



5. $^{+}6 + {}^{+}8 =$ ______



Corrections are on page 14.

You will notice that in each case the sum was a positive number. Therefore we can say that the sum of two or more positive numbers is a positive number. When adding two positive numbers we must first ignore the $^{+}$ sign and add the two numerals. Then, we attach the $^{+}$ sign to the sum to indicate that it is a positive number.

We add two positive numbers such as $^{+}1.8 + {}^{+}3.5$ in the same way.

Assignment 2

Find the sum of each group of numbers. Rewrite them vertically.

1. $^{+}2 + {}^{+}5 =$ ______ 2. $^{+}3 + {}^{+}2 =$ ______ 3. $^{+}56 + {}^{+}44 =$ ______

$^{+}2$
$\underline{^{+}5}$
$^{+}7$ ← *Sum of 2 and 5*
Attach + sign to sum.

4. $^{+}9 + {}^{+}4 =$ ______ 5. $^{+}6 + {}^{+}3 =$ ______ 6. $^{+}6.2 + {}^{+}3.4 =$ ______

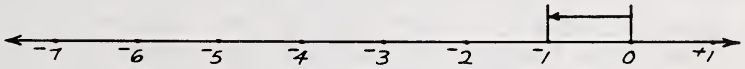
Topic Three: Sum of negative numbers

We can learn how to add negative numbers by using the number line. To do this, we begin at zero and draw a line to the left for the first number (because it is negative). Then we draw a line from the endpoint of the first trip to the left to represent the second number (which is also negative). The sum of the two numbers is the integer below the end of the last trip.

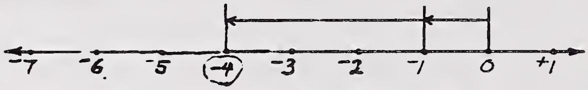
EXAMPLE. Use the number line to find the sum of $^{-}1$ and $^{-}3$.

Solution

Use a trip on the number line to represent $^{-}1$. Remember to begin at 0. Since $^{-}1$ is a negative number, the trip is to the left.



Use a second trip on the number line to show that $^{-}3$ is added to $^{-}1$. Begin at the end of the trip for $^{-}1$. Since $^{-}3$ is a negative number, the trip is to the left.

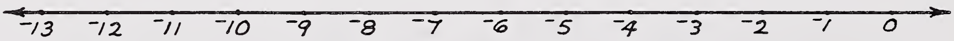


The integer at the end of the last trip is $^{-}4$. (It is encircled.)
Thus $^{-}1 + {}^{-}3 = \underline{^{-}4}$.

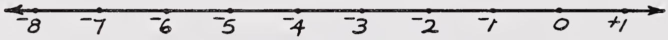
Self-correcting Exercise 4

Use a number line to find the sum of each pair of numbers.

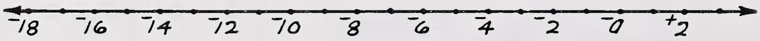
1. $^{-}6 + {}^{-}7 =$ ______



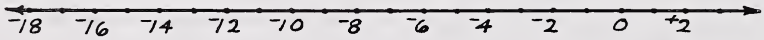
2. $^{-}2 + {}^{-}5 =$ ______



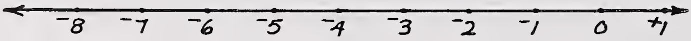
3. $^{-}7 + {}^{-}9 =$ ______



4. $^{-}8 + {}^{-}9 =$ ______



5. $^{-}4 + {}^{-}3 =$ ______



Corrections are on page 14.

You will notice that in each case the sum was a negative number. Therefore we can say that the sum of two or more negative numbers is a negative number. When adding two negative numbers, we must first ignore the $^{-}$ signs and add the two numerals. Then, we attach the $^{-}$ sign to the sum to indicate that it is a negative number.

Assignment 3

Find the sum of each group of numbers. Rewrite them vertically.

1. $^-8 + {}^-7 =$ $^-15$ 2. $^-4.5 + {}^-1.5 =$ ____ 3. $^-9 + {}^-5 =$ ____

$^-8$
$^-7$
$^-15$ ← Sum of 8 and 7
Attach $^-$ sign to the sum.

4. $^-6.4 + {}^-3.7 =$ ____ 5. $^-3 + {}^-1 =$ ____ 6. $^-3 + {}^-4 =$ ____

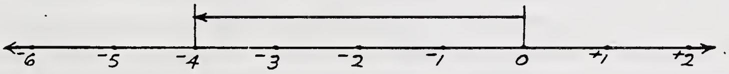
Topic Four. Sum of positive and negative numbers

The sum of positive and negative numbers can be shown by a number line. The positive number is shown by a trip to the right. A negative number is shown by a trip to the left. The sum of the two numbers is the integer below the end of the last trip.

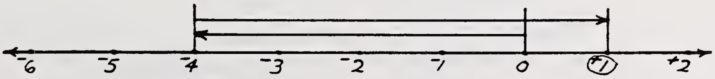
EXAMPLE. Use the number line to find the sum of $^-4$ and $^+5$.

Solution

Use a trip on the number line to represent $^-4$. Remember to begin at 0. Since $^-4$ is a negative number, the trip is to the left.



Use a second trip on the number line to show $^+5$. Begin at the end of the trip for $^-4$. Since $^+5$ is a positive number, the trip is to the right.

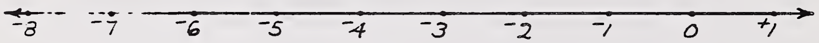


The integer at the end of the last trip is $^+1$. (It is encircled.)
Thus $^-4 + {}^+5 =$ $^+1$.

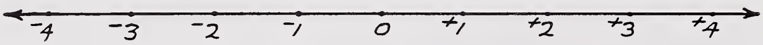
Self-correcting Exercise 5

Use a number line to find the sum of each pair of numbers.

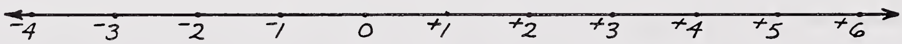
1. $^{-}7 + {}^{+}2 =$ ______



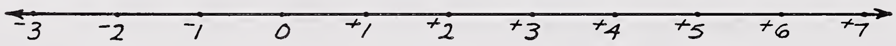
2. $^{-}3 + {}^{+}6 =$ ______



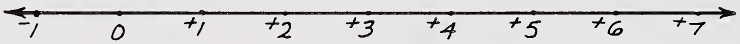
3. $^{+}5 + {}^{-}8 =$ ______



4. $^{+}6 + {}^{-}2 =$ ______



5. $^{+}6 + {}^{-}6 =$ ______



Corrections are on page 15.

You can see that the answer to the sum of a negative number and a positive number can not be found by adding. You must ignore the "+" and "-" symbols to the left of the numbers. Subtract the smaller number from the larger number. If the symbol in front of the larger number is "-", then the answer is negative. If the symbol in front of the larger number is "+", then the answer is positive.

e.g. $^{-}7 + {}^{+}3 = {}^{-}4$

Difference of 7 and 3

Attach ⁻sign to difference

(⁻sign is the sign of 7, the larger number).

Assignment 4

Find the sum of each group of numbers. Rewrite them vertically.

Sum is positive since 5 is positive.

1. $^{-}3 + {}^{+}5 =$ $^{+}2$

$$\begin{array}{r} ^{+}5 \\ ^{-}3 \\ \hline ^{+}2 \end{array}$$

2. $^{+}3 + {}^{-}1 =$ ____

3. $^{-}7 + {}^{+}4 =$ ____

4. $^{+}8 + {}^{-}9 =$ ____

5. $^{+}15 + {}^{-}22 =$ ____

6. $^{-}4 + {}^{+}12 =$ ____

7. $^{-}11 + {}^{+}11 =$ ____

8. $^{+}27 + 0 =$ ____

9. $^{-}37 + {}^{+}256 =$ ____

10. $^{+}6 + {}^{-}9 =$ ____

11. $^{+}18 + {}^{-}23 =$ ____

12. $^{-}9.2 + {}^{+}3.4 =$ ____

The rules for addition of integers are:

1. The sum of two positive numbers is a positive number.
2. The sum of two negative numbers is a negative number.
3. The sum of a positive number and a negative number is the difference of the two numbers with the sign of the larger number attached to this difference.

Assignment 5

For each problem:

- tell what the placeholder represents,
- write an equation,
- find the answer and
- write a statement answering the question asked.

1. On the first down, a football team gained 8 yd. On the second down, the team lost 5 yd. How many yards did the team gain on the two downs?

Placeholder: Let x be the number of yards gained on two downs

Equation: $^{+}8 + {}^{-}5 = x$

Computation

$$\begin{array}{r} ^{+}8 \\ + \ \underline{^{-}5} \\ ^{+}3 \end{array}$$

Answer: $^{+}3 = x$

Statement: The team gained 3 yards on the two downs.

2. Bill's football team first lost 5 yards and then gained 15 yards. What is the sum of $^{-}5$ yd. and $^{+}15$ yd.?

Placeholder: ______

Equation: ______ Computation

Answer: ______

Statement: ______

______

3. Another football team had a move of 8 yards backwards and then 36 yards forward. How many yards did this team gain as an overall effort?

Placeholder: ______

Equation: ______ Computation

Answer: ______

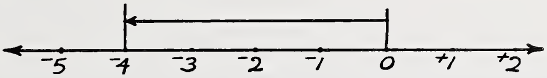
Statement: ______

______

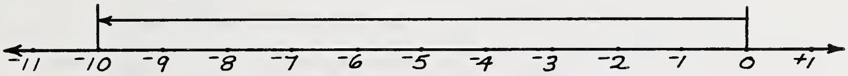
Key to Self-correcting Exercises in Lesson 23

Exercise 1, page 1

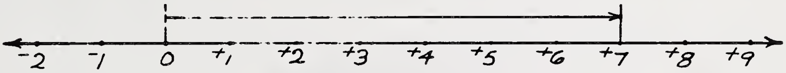
1.



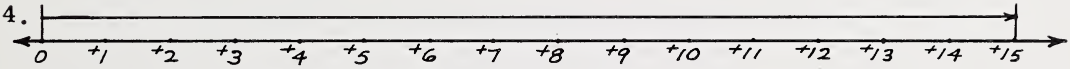
2.



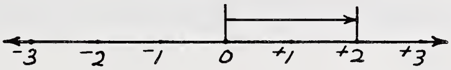
3.



4.

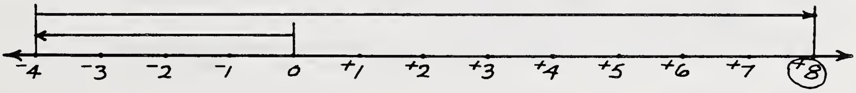


5.

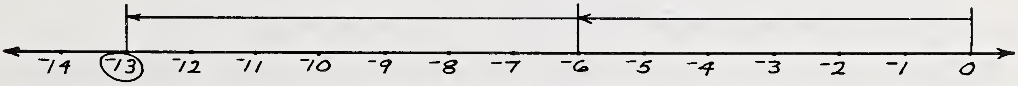


Exercise 2, page 5

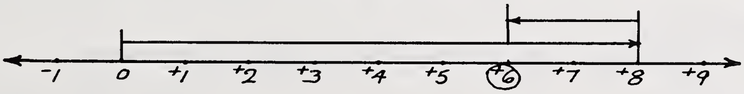
1.



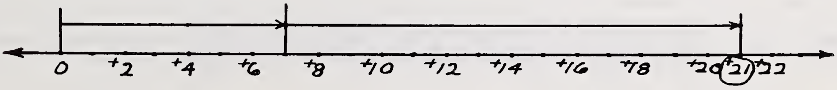
2.



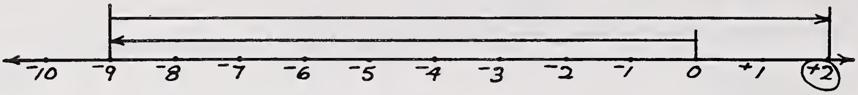
3.



4.

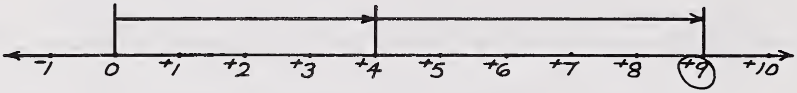


5.



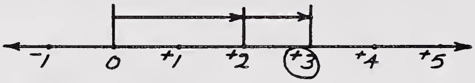
Exercise 3, page 6

1.



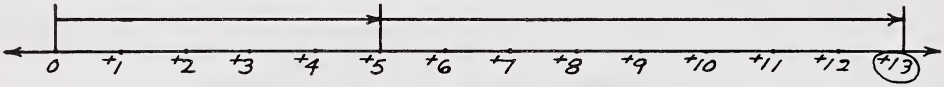
$^{+}4 + {}^{+}5 = \underline{^{+}9}$

2.



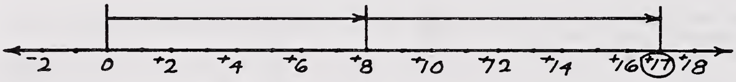
$^{+}2 + {}^{+}1 = \underline{^{+}3}$

3.



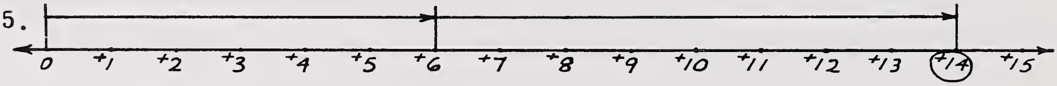
$^{+}5 + {}^{+}8 = \underline{^{+}13}$

4.



$^{+}8 + {}^{+}9 = \underline{^{+}17}$

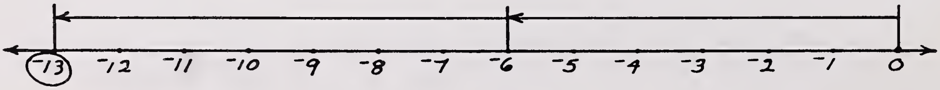
5.



$^{+}6 + {}^{+}8 = \underline{^{+}14}$

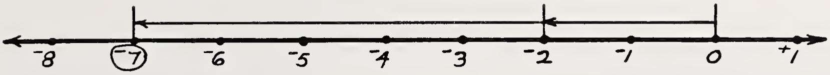
Exercise 4, page 8

1.



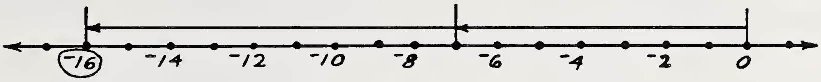
$^{-}6 + {}^{-}7 = \underline{^{-}13}$

2.



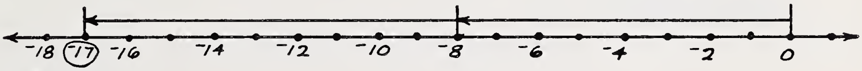
$^{-}2 + {}^{-}5 = \underline{^{-}7}$

3.



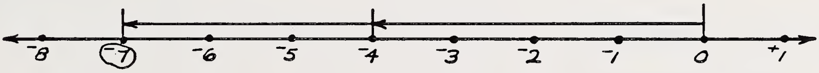
$^{-}7 + {}^{-}9 = \underline{^{-}16}$

4.



$^{-}8 + {}^{-}9 = \underline{^{-}17}$

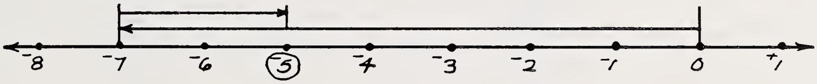
5.



$^{-}4 + {}^{-}3 = \underline{^{-}7}$

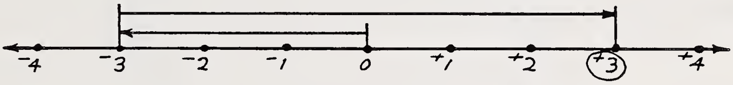
Exercise 5, page 10

1.



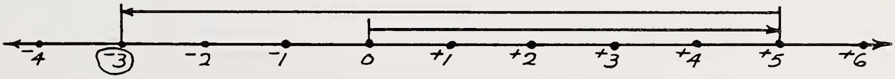
$^{-}7 + {}^{+}2 = \underline{^{-}5}$

2.



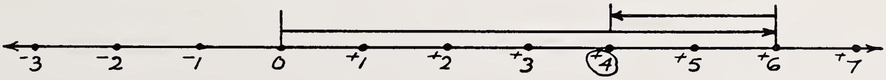
$^{-}3 + {}^{+}6 = \underline{^{+}3}$

3.



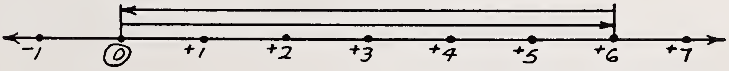
$^{+}5 + {}^{-}8 = \underline{^{-}3}$

4.



$^{+}6 + {}^{-}2 = \underline{^{+}4}$

5.



$^{+}6 + {}^{-}6 = \underline{0}$

End of Lesson 23

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________________

FOR SCHOOL USE ONLY

Assigned
Teacher: __________

Lesson Grading: __________

Additional Grading
E/R/P Code: __________

Mark: __________

Graded by: __________

Assignment Code: ______

Date Lesson Received:
____

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Multiplication of Integers

Topic One: Product of two positive numbers

Multiplication is similar to addition. Each multiplication problem can be thought of as a number of groups each containing the same number of objects.

e.g. 7 × 4 means 7 groups each containing 4 objects.

A multiplication problem can be rewritten as a sum.

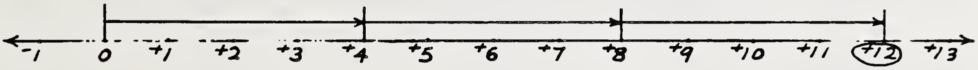
e.g. 7 × 4 is the same as 4 + 4 + 4 + 4 + 4 + 4 + 4.

We can use the number line to find the product of two positive numbers. The product can be shown by successive trips on the number line.

EXAMPLE: Use the number line to find the product of $^{+}3$ and $^{+}4$.

Solution

$^{+}3$ times $^{+}4$ means 3 groups of $^{+}4$. Since $^{+}4$ is a positive number the trip will be to the right. Since there are 3 groups of $^{+}4$, there will be 3 successive trips of $^{+}4$.



The integer below the end of the last trip is $^{+}12$. (It is encircled.) Thus $(^{+}3)(^{+}4)$ = $\underline{^{+}12}$. (Brackets around numbers mean multiply.)

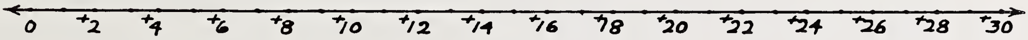
This can also be shown by addition.

$(^{+}3)(^{+}4) = {^{+}4} + {^{+}4} + {^{+}4} = \underline{^{+}12}$

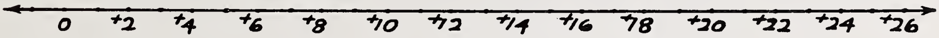
Self-correcting Exercise 1

Use a number line to find the product of each pair of numbers.

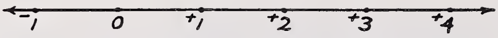
1. $(^{+}5)(^{+}6)$ = ______



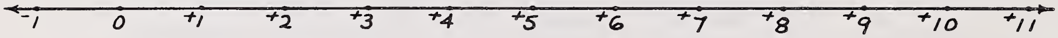
2. $(^{+}3)(^{+}8)$ = ______



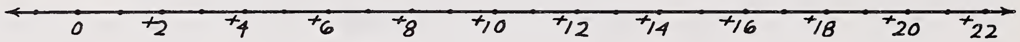
3. $(^+3)(^+1) =$ ______



4. $(^+1)(^+10) =$ ______



5. $(^+4)(^+5) =$ ______



Corrections are on page 10.

You will notice that in each case the product was a positive number. Therefore we can say that the product of two positive numbers is a positive number. When multiplying two positive integers, we must first ignore the $^+$ signs and multiply the two numerals. Then we attach the $^+$ sign to the product to indicate that it is a positive number.

Assignment 1

Find the product of each pair of numbers. Write them vertically.

1. $(^+3)(^+5) =$ $^+15$ 2. $(^+1.5)(^+2) =$ ______ 3. $(^+12)(^+8) =$ ______

$$\begin{array}{r} ^+3 \\ ^+5 \\ \hline ^+15 \end{array}$$

$^+15$ ← Product of 3 and 5

Attach $^+$ sign to the product.

4. $(^+11)(^+10) =$ ______ 5. $(^+3)(^+13) =$ ______ 6. $(^+16)(^+4) =$ ______

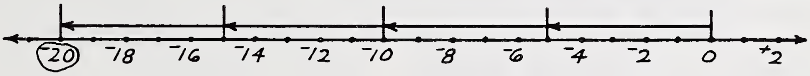
Topic Two: Product of positive and negative numbers

We can use the number line to find the product of a positive number and a negative number. The product can be shown by successive trips on the number line.

EXAMPLE: Use the number line to find the product of $^{+}4$ and $^{-}5$.

Solution

$^{+}4$ times $^{-}5$ means 4 groups of $^{-}5$. Since $^{-}5$ is a negative number, the trip will be to the left. Since there are 4 groups of $^{-}5$, there will be 4 successive trips of $^{-}5$.



The integer below the end of the last trip is $^{-}20$. (It is encircled.) Thus $(^{+}4)(^{-}5) = \underline{^{-}20}$.

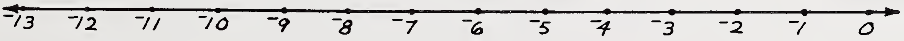
This can also be shown by addition.

$$\begin{aligned}(^{+}4)(^{-}5) &= \underbrace{^{-}5 + {}^{-}5}_{} + \underbrace{^{-}5 + {}^{-}5}_{} \\ &= \quad ^{-}10 \quad + \quad ^{-}10 \\ &= \underline{^{-}20}\end{aligned}$$

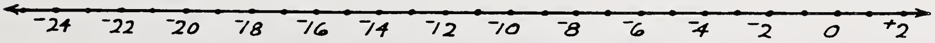
Self-correcting Exercise 2

Use a number line to find the product of each pair of numbers.

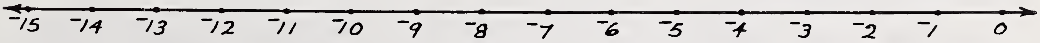
1. $(^{+}2)(^{-}6) =$ ______



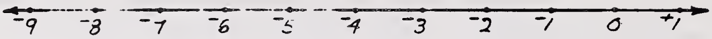
2. $(^{+}4)(^{-}6) =$ ______



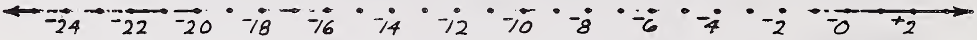
3. $(^{+}3)(^{-}5) =$ ______



4. $(^+4)(^-2) =$ ______



5. $(^+8)(^-3) =$ ______



Corrections are on page 10.

You will notice that in each case the product was a negative number. Therefore we can say that the product of a positive number and a negative number is a negative number. When multiplying a positive integer and a negative integer, we must first ignore the ⁺ and ⁻ signs and multiply the two numerals. Then we attach the ⁻ sign to the product to indicate that it is a negative number.

Assignment 2

Find the product of each pair of numbers. Write them vertically.

1. $(^+9)(^-5) =$ $^-45$ 2. $(^+7)(0) =$ ______ 3. $(^+4.5)(^-3) =$ ______

$$\begin{array}{r} ^+9 \\ \underline{^-5} \\ ^-45 \end{array}$$

⁻45 ← Product of 9 and 5

Attach ⁻ sign to the product.

4. $(^+7)(^-8) =$ ______ 5. $(^+2)(^-9) =$ ______ 6. $(^+7)(^-9) =$ ______

Before we can learn how to obtain the product of a negative number and a positive number we must study one of the properties of the system of integers.

You know that $(^+3)(^+6) = {}^+18$ and $(^+6)(^+3) = {}^+18$ are true statements. Therefore, we can say that it is true that $(^+3)(^+6) = (^+6)(^+3)$ because each side of the equation is equal to $^+18$. From this you can see that the order in which you multiply numbers does not affect the product. This is called the COMMUTATIVE PROPERTY OF MULTIPLICATION OF INTEGERS. (Commute means "to interchange".)

Thus, according to this property, $(^+3)(^-6) = (^-6)(^+3)$. Therefore, multiplying a negative number by a positive number will produce the same product as multiplying a positive number by a negative number.

e.g. $(^+3)(^-6) = {}^-18$ so
$(^-6)(^+3) = {}^-18$.

Thus when multiplying a negative integer and a positive integer (in either order) we must ignore the $^+$ and $^-$ signs and multiply the two numerals. Then we attach the $^-$ sign to the product to indicate that it is a negative number.

Self-correcting Exercise 3

Use the commutative property to rewrite each product.

1. $(^+7)(^-3) =$ ________
2. $(^-5)(5) =$ ________
3. $(^-1)(^-3) =$ ________
4. $(^-2)(^-7) =$ ________
5. $(^-3)(^+2) =$ ________
6. $(^+3)(^+5) =$ ________
7. $(^-3)(^-7) =$ ________
8. $(1)(^-4) =$ ________
9. $(^-3)(^-3) =$ ________
10. $(^-12)(^-6) =$ ________

Corrections are on page 11.

Assignment 3

Find the product of each pair of numbers. Write them vertically.

1. $(^-3)(^+5) = {}^-15$ 2. $(^-5)(0) =$ ______ 3. $(^-12)(^+4) =$ ______

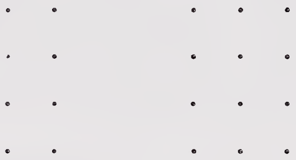
-3
$^+5$
$^-15$ ← Product of 3 and 5
Attach $^-$ sign to the product.

4. $(^-2.5)(^+2) =$ _____ 5. $(^-5)(^+6) =$ _____ 6. $(^-5)(^+12) =$ _____

Topic Three: Product of two negative numbers

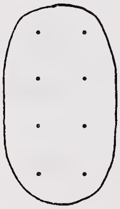
Before we learn how to obtain the product of two negative numbers, we must study another property of the system of integers.

This property will show that there are two ways of working with multiplication and addition when they both occur in the same problem.

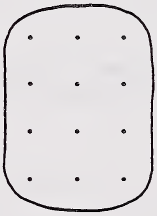


We can find the total number of dots in the above diagram in two different ways. Both ways involve multiplication and addition.

To find the total number of dots we could consider the groups of 2 dots and the groups of 3 dots.



4 × 2



4 × 3

In the first circle, there are 4 groups of 2 dots. This can be expressed by 4 × 2.

In the second circle, there are 4 groups of 3 dots. This can be expressed by 4 × 3.

To obtain the total we can add the dots in the two groups (that is, we must add the two products).

$(4 \times 2) + (4 \times 3)$

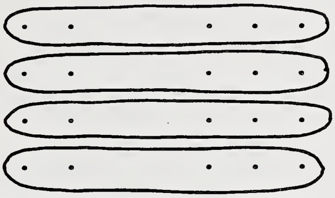
$= 8 + 12$

$= 20$

(The plus sign indicates that you must add the answers obtained from the products within the brackets.)
(4 × 2 = 8 and 4 × 3 = 12)

There are a total of 20 dots.

We can find the total number of dots in another way.



Each group contains 2 + 3 dots. There are 4 groups. Therefore there is a total of 4(2 + 3) dots. (When no symbol of operation appears between a number and brackets, you must multiply.)

To evaluate 4(2 + 3), we can multiply after finding the sum.

$$\begin{aligned}&4(2 + 3)\\ &= 4 \times 5 \qquad (2 + 3 = 5)\\ &= 20\end{aligned}$$

There are a total of 20 dots.

There two methods of finding the total number of dots produce the same answer. Therefore the statements must be equal to each other.

$$\begin{aligned}4(2 + 3) &= (4 \times 2) + (4 \times 3)\\ 4 \times 5 &= 8 + 12\\ 20 &= 20\end{aligned}$$

This is called the DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION OF INTEGERS.

This means that the multiplier distributes (spreads) over the addends of a sum.

$$4(2 + 3)$$

First 2(an addend) is multiplied by the multiplier 4; then 3(another addend) is also multiplied by the multiplier 4.

$$4(2 + 3) = (4 \times 2) + (4 \times 3)$$

Then the products are added.

$$\begin{aligned}4(2 + 3) &= (4 \times 2) + (4 \times 3)\\ &= 8 + 12\\ &= 20\end{aligned}$$

This property can be used to find the product of a one-digit number and a two or more digit number.

EXAMPLE: Use the distributive property to find the product of 6 and 59.

Solution

$$\begin{aligned} 6(59) = 6(50 + 9) &= (6 \times 50) + (6 \times 9) \\ &= 300 + 54 \\ &= 354 \end{aligned}$$

The product of 6 and 59 is 354.

Self-correcting Exercise 4

Use the distributive property to find the solution to each of the following products.

1. $9(30 + 7) =$
2. $9(40 + 4) =$
3. $2(20 + 1) =$
4. $8(70 + 7) =$
5. $7(60 + 9) =$
6. $3(80 + 9) =$

Corrections are on page 11.

Since it is only possible to show the product of numbers, on a number line when at least one of the numbers is positive, we must resort to a different method of finding the product of two negative numbers.

We can use the following example to show the product of two negative numbers.

$^{-}4(0) = 0 \longleftarrow$ (Any number times **zero** equals zero.)

$^{-}4(3 + {}^{-}3) = 0 \longleftarrow$ ($3 + {}^{-}3 = 0$ therefore we can replace 0 by $3 + {}^{-}3$.)

$({}^{-}4 \times 3) + ({}^{-}4 \times {}^{-}3) = 0 \longleftarrow$ (Apply the distributive property.)

$^{-}12 + ({}^{-}4 \times {}^{-}3) = 0 \longleftarrow$ (${}^{-}4 \times {}^{+}3 = {}^{-}12$ The product of a negative and positive integer is negative.)

$^{-}12 + \underline{\qquad} = 0$

Think of a trip of $^{-}12$ on the number line. To indicate a sum which has a result of zero there must be a second trip made from $^{-}12$ to 0. Since this trip is to the right it is a positive number. Since it is 12 units it is a trip of $^{+}12$.

$^{-}12 + {}^{+}12 = 0$

Thus $({}^{-}4)({}^{-}3) = {}^{+}12$.

From this we must deduce that the product of two negative numbers is a positive number. When multiplying two negative integers, we must first ignore the $^{-}$signs and multiply the two numerals. Then we attach the $^{+}$ sign to the product to indicate that it is a positive number.

Assignment 4

Find the product of each group of numbers. Rewrite them vertically.

1. $({}^{-}3)({}^{-}5) =$ $^{+}15$

$$\begin{array}{r} ^{-}3 \\ \underline{^{-}5} \\ ^{+}15 \end{array}$$

2. $({}^{-}10)({}^{-}10) =$ _____

3. $({}^{-}8)({}^{-}3) =$ _____

4. $({}^{-}3)({}^{-}4) =$ _____

5. $({}^{-}5)({}^{-}7) =$ _____

6. $({}^{-}3)({}^{-}3) =$ _____

7. $({}^{+}3)({}^{-}8) =$ _____

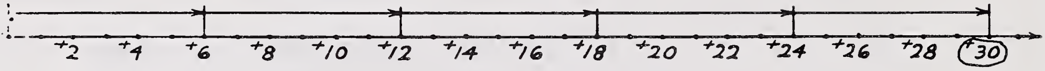
8. $({}^{-}3)({}^{+}9) =$ _____

9. $({}^{+}12)({}^{+}4) =$ _____

Key to Self-correcting Exercises in Lesson 24

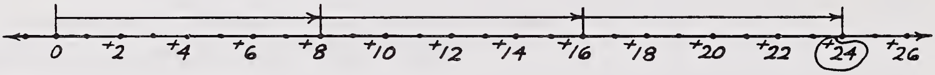
Exercise 1, page 1

1.



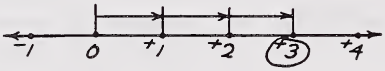
$(^{+}5)(^{+}6) = \underline{^{+}30}$

2.



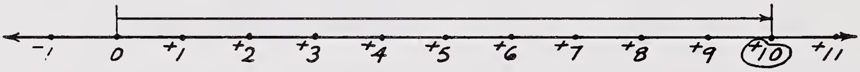
$(^{+}3)(^{+}8) = \underline{^{+}24}$

3.



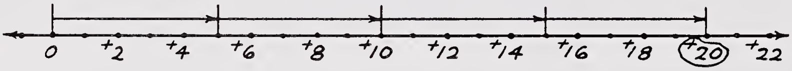
$(^{+}3)(^{+}1) = \underline{^{+}3}$

4.



$(^{+}1)(^{+}10) = \underline{^{+}10}$

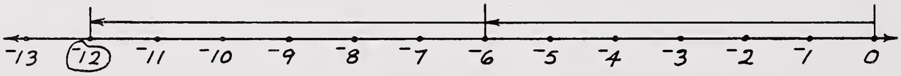
5.



$(^{+}4)(^{+}5) = \underline{^{+}20}$

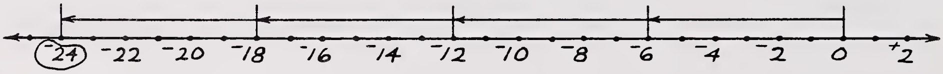
Exercise 2, page 3

1.



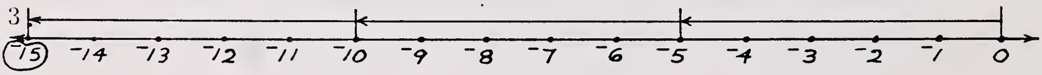
$(^{+}2)(^{-}6) = \underline{^{-}12}$

2.



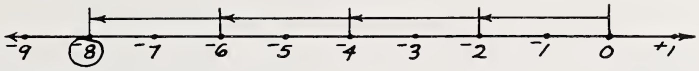
$(^{+}4)(^{-}6) = \underline{^{-}24}$

3.



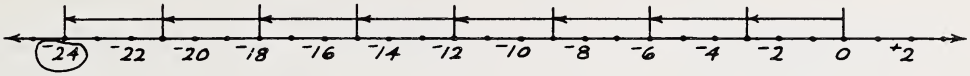
$(^{+}3)(^{-}5) = \underline{^{-}15}$

4.



$(^{+}4)(^{-}2) = \underline{^{-}8}$

5.



$(^{+}8)(^{-}3) = \underline{^{-}24}$

Exercise 3, page 5

1. $(^{+}7)(^{-}3) = (^{-}3)(^{+}7)$
2. $(^{-}5)(5) = (5)(^{-}5)$
3. $(^{-}1)(^{-}3) = (^{-}3)(^{-}1)$
4. $(^{+}2)(^{-}7) = (^{-}7)(^{+}2)$
5. $(^{-}3)(^{+}2) = (^{+}2)(^{-}3)$
6. $(^{+}3)(^{+}5) = (^{+}5)(^{+}3)$
7. $(^{-}3)(^{-}7) = (^{-}7)(^{-}3)$
8. $(1)(^{-}4) = (^{-}4)(1)$
9. $(^{-}3)(^{-}3) = (^{-}3)(^{-}3)$
10. $(^{-}12)(^{-}6) = (^{-}6)(^{-}12)$

Exercise 4, page 8

1. $$\begin{aligned} 9(30 + 7) &= 9(30) + 9(7) \\ &= 270 + 63 \\ &= 333 \end{aligned}$$

2. $$\begin{aligned} 9(40 + 4) &= 9(40) + 9(4) \\ &= 360 + 36 \\ &= 396 \end{aligned}$$

3. $$\begin{aligned} 2(20 + 1) &= 2(20) + 2(1) \\ &= 40 + 2 \\ &= 42 \end{aligned}$$

4. $$\begin{aligned} 8(70 + 7) &= 8(70) + 8(7) \\ &= 560 + 56 \\ &= 616 \end{aligned}$$

5. $$\begin{aligned} 7(60 + 9) &= 7(60) + 7(9) \\ &= 420 + 63 \\ &= 483 \end{aligned}$$

6. $$\begin{aligned} 3(80 + 9) &= 3(80) + 3(9) \\ &= 240 + 27 \\ &= 267 \end{aligned}$$

End of Lesson 24

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____

Time Spent on Lesson ____

(If label is missing
or incorrect)

File Number ____

Lesson Number ____

FOR SCHOOL USE ONLY

Assigned
Teacher: ____

Lesson Grading: ____

Additional Grading
E/R/P Code: ____

Mark: ____

Graded by: ____

Assignment Code: ____

Date Lesson Received:

____

Lesson Recorded ____

Student's Questions and Comments

Apply Lesson Label Here

Name ____
Address ____
____
____
Postal Code ____

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Subtraction and Division of Integers

Topic One: Subtraction of integers

To deal with subtraction of numbers we must again study another property of integers.

We mentioned opposites in Lesson 22. Numbers like $^+6$ and $^-6$ are opposites. These numbers can also be called additive inverses. $^+6$ is the additive inverse of $^-6$. $^-6$ is the additive inverse of $^+6$.

We can use the idea of additive inverses and the relationship between addition and subtraction to find the difference of two integers. You know that if $5 - 3 = 2$ then $2 + 3 = 5$. If we have

$^+5 - {}^-3 = x$ then $x + {}^-3 = {}^+5$. We know that $^+8 + {}^-3 = {}^+5$.

Therefore, $^+5 - {}^-3 = {}^+8$. Since $^+5 + {}^+3$ also equals 8, we can make subtraction questions simpler by writing them as addition questions. Change the operation to addition and use the additive inverse of the subtrahend (the number being taken away). Then apply the rules for addition of integers.

$$\begin{array}{l} {}^+5 \; - \; {}^-3 \; = \; {}^+8 \\ {}^+5 \; + \; {}^+3 \; = \; {}^+8 \end{array}$$

Change subtraction to addition.

Use the additive inverse of the subtrahend.

EXAMPLE: Rewrite $^+6 - {}^-2$ as a sum. Then add.

Solution

The additive inverse of $^-2$ is $^+2$.
Therefore $^+6 - {}^-2$ can be rewritten as a sum.

$$^+6 - {}^-2 = {}^+6 \oplus {}^+2 \leftarrow \text{Additive inverse of } {}^-2$$

$\uparrow$ Operation is changed to addition.

From our rules for adding integers, we know that the sum of $^+6$ and $^+2$ is $^+8$.

$$^+6 - {}^-2 = \underline{{}^+8}$$

Self-correcting Exercise 1

Decide whether each of the two numbers named are additive inverses of each other.

1. $\frac{^{-}7}{3}, \frac{^{-}3}{7}$ ________________

2. $^{+}4, {}^{-}4$ ________________

3. $0, 0$ ________________

4. $^{+}64, {}^{-}64$ ________________

5. $^{+}1, 0$ ________________

Rewrite each difference as a sum.

6. $^{-}15 - {}^{-}29 =$ ______________

7. $^{+}6 - {}^{-}11 =$ ______________

8. $^{-}6 - {}^{+}23 =$ ______________

9. $^{-}6 - {}^{+}49 =$ ______________

10. $0 - {}^{-}396 =$ ______________

Corrections are on page 11.

Assignment 1

First rewrite each difference as a sum. Then write the sum vertically and find the answer.

1. $^{+}6 - {}^{+}2 = {}^{+}6 + {}^{-}2 = {}^{+}4$ 2. $^{+}8 - {}^{-}9 =$ ____________ $=$ ________

$$\begin{array}{r} ^{+}6 \\ \underline{^{-}2} \\ ^{+}4 \end{array}$$

3. $^{-}6 - 0 =$ __________ = ________ 4. $^{-}3 - {}^{-}1 =$ __________ = ________

5. $^{-}5 - {}^{+}5 =$ __________ = ________ 6. $^{-}34 - {}^{+}23 =$ __________ = ________

7. $^{-}8 - {}^{-}2 =$ __________ = ________ 8. $^{-}2 - {}^{+}11 =$ __________ = ________

Topic Two: Division of integers

Just as subtraction is related to addition, division is related to multiplication. You know that if $35 \div 7 = 5$ then $5 \times 7 = 35$. If we have $^{+}36 \div {}^{-}9 = x$ then $(x)(^{-}9) = {}^{+}36$. We know that $(^{-}4)(^{-}9) = {}^{+}36$, therefore $^{+}36 \div {}^{-}9 = {}^{-}4$.

Remembering this, we can then apply the rules for multiplication of integers to division of integers.

Self-correcting Exercise 2

1. $(^{-}43)(^{-}7) =$ ____________

   A negative number times a negative number equals a ____________ number.

2. $(^{+}15)(^{+}8) =$ ____________

   A positive number times a positive number equals a ____________

3. $(^{-}11)(^{+}6) =$ ____________

   The product of a positive number and a negative number is a

   ____________________________ number.

   Corrections are on page 11.

The rules for multiplication of integers are:

1. The product of two positive numbers is a positive number.
2. The product of two negative numbers is a positive number.
3. The product of a positive number and a negative number is a negative number.

Since the rules for multiplication and division of integers are the same, the rules for division of integers are:

1. The quotient of two positive numbers is a positive number.
   e.g. $^{+}16 \div {}^{+}8 = {}^{+}2$

2. The quotient of two negative numbers is a positive number.
   e.g. $^{-}21 \div {}^{-}3 = {}^{+}7$

3. The quotient of a negative number and a positive number (or vice versa) is a negative number.
   e.g. $^{-}24 \div {}^{+}6 = {}^{-}4$
   $^{+}27 \div {}^{-}9 = {}^{-}3$

Assignment 2

Find the quotient of each group of numbers.

1. $^{-}9 \div {}^{+}3 =$ $^{-}3$

$$\begin{array}{r} ^{-}3 \\ ^{+}3\overline{)\,^{-}9} \end{array}$$

2. $^{+}12 \div {}^{-}4 =$ ______

3. $^{-}6 \div {}^{-}2 =$ ______

4. $^{-}4.2 \div {}^{+}6 =$ ______

5. $^{+}2.4 \div {}^{-}6 =$ ______

6. $^{+}25 \div {}^{+}5 =$ ______

REVIEW

Marks

Carefully restudy Lessons 21, 22, 23, 24 and the first part of Lesson 25. Notice where you made errors. Try to eliminate the errors (similar to those you made earlier) from this review.

Use a pair of ratios and the ratio test to find the required measure.

2 1. 68 in. = __?__ ft. 2. 3 mi. = __?__ yd.

Let n be the no. of ft. in 68 in.

$$\begin{aligned} 12 \text{ in.} &= 1 \text{ ft.} \\ \frac{12}{1} &= \frac{68}{n} \\ 12 \times n &= 1 \times 68 \\ 12n &= 68 \\ n &= 68 \div 12 \\ n &= 5\tfrac{2}{3} \\ 68 \text{ in.} &= 5\tfrac{2}{3} \text{ ft.} \end{aligned}$$

4 3. $2\frac{1}{2}$A. = __?__ sq. rd. 4. 45 sq. ft. = __?__ sq. yd.

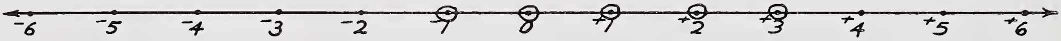
4 5. 8 sq. yd. = __?__ sq. in. 6. 36 in. = __?__ cm.

Marks

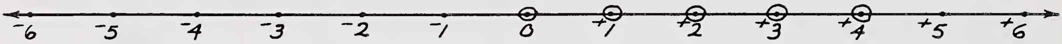
4 7. 40 mi. = ? km. 8. 6 m = ? yd.

Name the integers that are graphed on each number line.

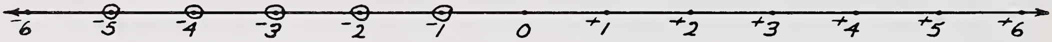
1 9.



1 10.



1 11.



Graph each of the groups of integers on a number line.

2 12. ⁻2, ⁻1, 0, 1, 2

2 13. ⁻4, ⁻3, ⁻2, ⁻1

Marks

2 14. 1, 2, 3, 4, 5

Write "T" if the statement is true; write "F" if the statement is false.

1 15. $^+3 < 0$ ____________

1 16. $^-4 > {}^-2$ ____________

1 17. $^-8 < 0$ ____________

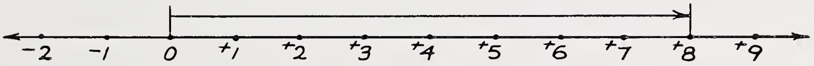
1 18. $^+2 < {}^-4$ ____________

1 19. $^-8 < {}^-2$ ____________

1 20. $0 > {}^+4$ ____________

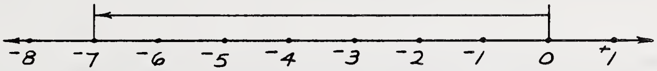
Name the number represented by each trip on the number line.

1 21.



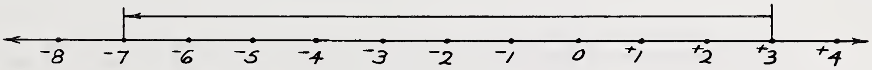
____

1 22.



____

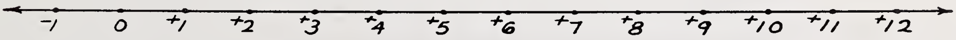
2 23.



____

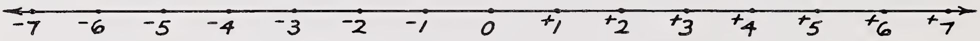
Use a number line to find the following sums.

2 24. $^+4 + {}^+7 =$ _____

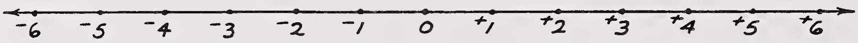


Marks

2 25. $^{-}4 + {}^{+}7 =$ ______



2 26. $^{-}6 + {}^{+}5 =$ ______



Name the additive inverse of each of the given integers.

1 27. 4 ______________

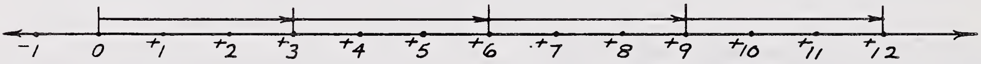
1 28. $^{-}10$ ______________

1 29. $^{-}5$ ______________

1 30. $^{+}11$ ______________

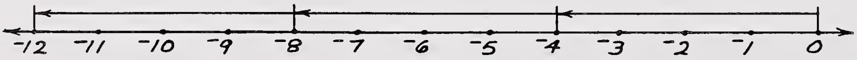
Write the multiplication sentence shown by the trips on the number line.

1 31.



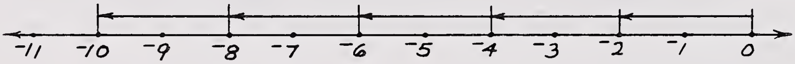
____

1 32.



____

1 33.



____

Name the kind of integer (positive or negative) that the product will be in each case.

1 34. The product of a positive and a negative factor is a __________ integer.

1 35. The product of two negative factors is a ______________ integer.

1 36. The product of two positive factors is a ______________ integer.

Marks

Find a solution for each of the given sentences.

2 37. $^{+}4 + {}^{-}3 =$ ______ 38. $^{-}6 + {}^{-}4 =$ ______

2 39. $^{-}10 + {}^{+}2 =$ ______ 40. $^{+}6 + {}^{-}6 =$ ______

2 41. $^{-}4 + {}^{+}8 =$ ______ 42. $^{-}10 + {}^{-}6 =$ ______

3 43. $^{+}6 - {}^{-}2 = \underline{{}^{+}6 + {}^{+}2} =$ ______ 44. $^{-}8 - {}^{-}2 =$ ________ = ______

4 45. $^{-}3 - {}^{-}4 =$ ________ = ______ 46. $^{-}7 - {}^{+}6 =$ ________ = ______

4 47. $^{+}2 - {}^{+}8 =$ ________ = ______ 48. $^{-}10 - 4 =$ ________ = ______

Marks

2 49. $(^+5)(^-6) =$ ______ 50. $(^-4)(^-4) =$ ______

2 51. $(^-6)(^+6) =$ ______ 52. $(^+6)(^+8) =$ ______

2 53. $^-40 \div {^-2} =$ ______ 54. $^+20 \div {^+5} =$ ______

2 55. $^-80 \div {^+1} =$ ______ 56. $^-80 \div {^-1} =$ ______

2 57. $^-1 \div {^+1} =$ ______ 58. $851 \div 37 =$ ______

5 59. A business man expressed the results of Monday's business as $^-\$215$ and Tuesday's results as $^+\$152$. What was the overall result of both days?

Placeholder:

Equation: Computation

Answer:

Statement:

Total Marks
81

Key to Self-correcting Exercises in Lesson 25

Exercise 1, page 2

1. no
2. yes
3. yes
4. yes
5. no

6. $^-15 - {}^-29 = {}^-15 + {}^+29$
7. $^+6 - {}^-11 = {}^+6 + {}^+11$
8. $^-6 - {}^+23 = {}^-6 + {}^-23$
9. $^-6 - {}^+49 = {}^-6 + {}^-49$
10. $0 - {}^-396 = 0 + {}^+396$

Exercise 2, page 3

1. $(^-43)(^-7) = \underline{^+301}$
   positive

2. $(^+15)(^+8) = \underline{^+120}$
   positive number

3. $(^-11)(^+6) = \underline{^-66}$
   negative

End of Lesson 25

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned Teacher: ________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name

Address

Postal Code

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____

Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Properties of Integers

Topic One: Commutative properties of integers

One of the commutative properties was introduced in Lesson 24 to help you understand how to obtain the product of a negative and a positive integer. You may have encountered these properties even earlier in your study of mathematics. They are sometimes called the "order properties".

A. Commutative property of addition of integers

You know that $4 + 7 = 11$. You also know that $7 + 4 = 11$. From this and many other examples you can think of, you can conclude that the sum of two numbers is not affected by the order in which you add them. Therefore we can say that $4 + 7 = 7 + 4$
because $11 = 11$.

We can write a general formula which describes this property.
If a represents one number and
b represents another number, then
$a + b = b + a$ (The sum of two integers added in one particular order is the same as the sum of the same two numbers added in the opposite order.)

Self-correcting Exercise 1

Fill in the blanks with the correct numbers.

1. Since ${}^{-}1 + {}^{+}1 =$ _____, we know that ${}^{+}1 + {}^{-}1 =$ _____.
2. Since ${}^{-}3 + {}^{-}10 =$ _____, we know that ${}^{-}10 + {}^{-}3 =$ _____.
3. Since ${}^{+}4 + {}^{+}1 =$ _____, we know that ${}^{+}1 + {}^{+}4 =$ _____.
4. Since ${}^{+}8 + {}^{-}6 =$ _____, we know that ${}^{-}6 + {}^{+}8 =$ _____.
5. ${}^{+}9 + {}^{+}7 =$ _____ $+ {}^{+}9$
6. _____ $+ {}^{-}93 = {}^{-}93 + {}^{+}68$
7. ${}^{-}53 +$ _____ $= {}^{-}86 + {}^{-}53$
8. ${}^{-}37 + {}^{+}13 = {}^{+}13 +$ _____

Corrections are on page 10.

B. Commutative property of multiplication of integers

You know that $(8)(3) = 24$. You also know that $(3)(8) = 24$. From this and many other examples you can think of, you can conclude that the product of two numbers is not affected by changing the order of the multiplier and multiplicand.
Therefore we can say that $(8)(3) = (3)(8)$
because $24 = 24$.

We can write a general formula which describes this property.
If a represents one number and
b represents another number, then
$(a)(b) = (b)(a)$. (The product of two integers multiplied in one particular order is the same as the product of the same two numbers multiplied in the opposite order.)

Assignment 1

Fill in the blanks with the correct numbers.

1. Since $(^{-}5)(^{+}1) =$ $^{-}5$, we know that $(^{+}1)(^{-}5) =$ $^{-}5$.
2. Since $(^{+}2)(^{-}3) =$ ______, we know that $(^{-}3)(^{+}2) =$ ______.
3. Since $(^{+}3)(^{+}4) =$ ______, we know that $(^{+}4)(^{+}3) =$ ______.
4. Since $(^{-}5)(^{-}3) =$ ______, we know that $(^{-}3)(^{-}5) =$ ______.

Decide whether each of the following sentences is true or false.

5. $119 + {^{-}45} = {^{-}45} + 119$ True
6. ${^{-}186} \div 3 = 3 \div {^{-}186}$ ______
7. $(^{-}42)(^{-}59) = (^{-}59)(^{-}42)$ ______
8. ${^{+}23} - {^{+}18} = {^{+}18} - {^{+}23}$ ______
9. $(^{-}74)(16) = (16)(74)$ ______
10. $(^{-}26)(0) = (0)(^{-}26)$ ______
11. ${^{+}116} + {^{+}9} < {^{+}9} + {^{+}116}$ ______
12. ${^{+}14} + 0 = 0 + {^{+}14}$ ______

13. $(95)({}^-20) < ({}^-20\,(95)$ $\underline{\hspace{2cm}}$

14. ${}^-14 + {}^-1001 = {}^-1001 + {}^-14$ $\underline{\hspace{2cm}}$

Fill in the blanks with the correct numbers.

15. ${}^-36 + \underline{{}^-15} = {}^-15 + {}^-36$

16. $(\underline{\hspace{1cm}})({}^+17) = ({}^+17)({}^-4)$

17. $41 + {}^-1 = \underline{\hspace{1cm}} + 41$

18. ${}^-9 + {}^-4 = {}^-4 + \underline{\hspace{1cm}}$

19. $({}^-2)({}^-3) = ({}^-3)(\underline{\hspace{1cm}})$

20. $(\underline{\hspace{1cm}})({}^+24) = ({}^+24)(2)$

Topic Two: Associative properties of integers

The associative properties involve a minimum of three numbers. You may have already encountered these properties in your study of mathematics. They are sometimes called the "grouping properties".

A. Associative property of addition of integers

Addition is a binary operation. This means that you can add only two numbers at one time. Therefore if you have a sum made up of three addends, you must add two of them first and then add the third addend to the sum of the other two addends. We can use parenthesis to show which numbers must be added first.

We know that $(2 + 3) + 4$
$= 5 + 4$
$= 9.$
In this case we added the first two numbers first $(2 + 3 = 5)$; then we added the last number to this sum $(5 + 4 = 9)$.

We also know that $2 + (3 + 4)$
$= 2 + 7$
$= 9.$
In this case we added the last two numbers first $(3 + 4 = 7)$; then we added the first number to this sum $(2 + 7 = 9)$.

From this and many other examples you can think of, you can conclude that the way in which you group the numbers before you add does not affect the sum. Therefore we can say that
$(2 + 3) + 4 = 2 + (3 + 4)$
and $5 + 4 = 2 + 7$
because $9 = 9$.

We can write a general formula which describes this property.
If a represents one number,
b represents a second number and
c represents a third number, then
$(a + b) + c = a + (b + c)$. (The sum of the first two integers and the third integer is the same as the sum of the last two integers and the first integer.)

Self-correcting Exercise 2

Fill in the blanks with the correct numbers. Write each sum shown in brackets as a single number. Then find the sum of that number and the other given number.

1. Since $(^{+}4 + {}^{+}3) + {}^{-}6 =$ ______ + ______ = ______, we know that

$^{+}4 + (^{+}3 + {}^{-}6) =$ ______ + ______ = ______.

2. Since $(^{-}14 + {}^{-}5) + {}^{+}8 =$ ______ + ______ = ______, we know that

$^{-}14 + (^{-}5 + {}^{+}8) =$ ______ + ______ = ______.

3. Since $(^{-}52 + {}^{+}20) + {}^{-}19 =$ ______ + ______ = ______, we know that

$^{-}52 + (^{+}20 + {}^{-}19) =$ ______ + ______ = ______.

4. Since $(^{+}112 + {}^{-}13) + 1 =$ ______ + ______ = ______, we know that

$^{+}112 + (^{-}13 + 1) =$ ______ + ______ = ______.

Fill in the blanks with the correct numbers.

5. $({}^{+}9 + {}^{-}8) + 16 = {}^{+}9 + (\underline{\qquad} + 16)$

6. $({}^{-}1 + {}^{-}45) + {}^{-}24 = \underline{\qquad} + ({}^{-}45 + {}^{-}24)$

7. $(13 + {}^{+}11) + {}^{-}19 = 13 + ({}^{+}11 + \underline{\qquad})$

8. $\underline{\qquad} + ({}^{-}72 + {}^{-}92) = (37 + {}^{-}72) + {}^{-}92$

Corrections are on page 10.

B. Associative property of multiplication of integers

Multiplication is a binary operation. Therefore it is possible to multiply only two numbers at one time. If you have a product made up of three numbers, you must find the product of two of the numbers and then multiply this result by the other number. Again we will use parenthesis to indicate which numbers must be multiplied first.

You know that $(3 \times 4)5$
$= (12)5$
$= 60.$

In this case we found the product of the first two numbers first ($3 \times 4 = 12$); then we multiplied this result by the third number ($12 \times 5 = 60$).

You also know that $3(4 \times 5)$
$= 3(20)$
$= 60.$

In this case we found the product of the last two numbers first ($4 \times 5 = 20$); then we multiplied this result by the first number ($3 \times 20 = 60$).

From this and many other examples you can think of, you can conclude that the way in which you group the numbers before you multiply does not affect the product. Therefore we can say that

$(3 \times 4)5 = 3(4 \times 5)$
and $(12)5 = 3(20)$
because $60 = 60.$

We can write a general formula which describes this property.
If a represents one number,
b represents a second number and
c represents a third number, then
$(a \times b)c = a(b \times c)$. (The product of the first two integers and the third integer is the same as the product of the first integer and the last two integers.)

Assignment 2

Decide which property (commutative or associative) of which operation (addition or multiplication) is used to write each statement.

1. $({}^{-}43 + {}^{+}16) + {}^{-}9 = {}^{-}43 + ({}^{+}16 + {}^{-}9)$ Associative property of addition
2. ${}^{+}34 + {}^{-}5 = {}^{-}5 + {}^{+}34$ ____________________
3. $({}^{-}67 \times 15)({}^{-}3) = ({}^{-}67)(15 \times {}^{-}3)$ ____________________
4. $(44)(11) = (11)(44)$ ____________________
5. $(13 + {}^{-}5) + {}^{+}7 = 13 + ({}^{-}5 + {}^{+}7)$ ____________________
6. $({}^{-}17 + {}^{-}4) + {}^{-}2 = {}^{-}17 + ({}^{-}4 + {}^{-}2)$ ____________________
7. ${}^{-}14 + {}^{+}6 = {}^{+}6 + {}^{-}14$ ____________________
8. $(12 \times {}^{-}2)({}^{-}2) = 12({}^{-}2 \times {}^{-}2)$ ____________________
9. $({}^{-}6)({}^{-}5) = ({}^{-}5)({}^{-}6)$ ____________________
10. $27 + (5 + 2) = (27 + 5) + 2$ ____________________

Fill in the blanks with the correct numbers.

11. $({}^{+}18 + {}^{+}14) + {}^{-}10 = {}^{+}18 + ({}^{+}14 + \underline{\;{}^{-}10\;})$
12. $(\underline{\qquad})({}^{+}85) = ({}^{+}85)({}^{-}12)$
13. ${}^{+}27 + \underline{\qquad} = {}^{+}19 + {}^{+}27$
14. $({}^{-}44 \times \underline{\qquad})3 = ({}^{-}6 \times {}^{-}44)3$
15. $(\underline{\qquad} \times {}^{+}12)({}^{+}15) = 1({}^{+}12 \times {}^{+}15)$
16. $(\underline{\qquad} + {}^{-}71) + 12 = {}^{-}3 + ({}^{-}71 + 12)$
17. $({}^{-}19 + {}^{-}21) + \underline{\qquad} = {}^{-}11 + ({}^{-}19 + {}^{-}21)$
18. $(37 + \underline{\qquad}) + {}^{-}14 = 37 + (13 + {}^{-}14)$
19. $({}^{-}17 \times {}^{+}23)0 = {}^{-}17({}^{+}23 \times \underline{\qquad})$
20. $(47 \times 4)4 = \underline{\qquad}(4 \times 4)$

Topic Three: Identity-element properties of integers

The identity-element properties provide, that after computing with two numbers, the answer you obtain will always be the same as one of the numbers used in the computation.

A. Identity-element property of addition of integers

You know that $8 + 0 = 8$,
$0 + 13 = 13$
and $37 + 0 = 37$.

From these and many other examples you can think of, you can conclude that the sum of a number and zero (or vice versa) is equal to that number. In other words, to add zero to a number does not affect the number.

We can write a general formula which describes this property. If a represents a number, then
$a + 0 = a$ and
$0 + a = a$. (The sum of any number and zero is the same as that number.)

Self-correcting Exercise 3

Fill in the blanks with the correct numbers.

1. $^{+}16 + 0 =$ ______
2. $3(^{-}42 + 0) = 3($___$) =$ ______
3. $14 + 0 + {}^{-}19 = 14 +$ ___ $=$ ______
4. $^{-}5 + 0 =$ ______
5. ______ $+ {}^{-}7 = {}^{-}7$
6. $^{+}6 +$ ___ $= {}^{+}6$
7. $0 +$ ___ $= {}^{-}4$

Corrections are on page 10.

B. Identity-element property of multiplication of integers

You know that $(6)(1) = 6$,
$(11)(1) = 11$
and $(1)(26) = 26$.

From these and many other examples you can think of, you can conclude that the product of a number and one (or vice versa) is equal to that number. In other words, to multiply a number by one does not affect the number.

We can write a general formula which describes this property. If a represents a number, then

$a(1) = a$ and
$1(a) = a$. (The product of any number and one is the same as that number.)

Assignment 3

Decide which property of which operation is shown by each statement.

1. $^-3 + 20 = 20 + {}^-3$ Commutative property of addition
2. $^+7 = (1)(^+7)$ ______
3. $^-2(^+9 \times {}^-3) = (^-2 \times {}^+9)(^-3)$ ______
4. $^-23 + 0 = {}^-23$ ______
5. $^-5(^-9 + {}^-2) = {}^-5(^-2 + {}^-9)$ ______
6. $(^-5 + 12) + {}^+1 = {}^-5 + (12 + {}^+1)$ ______

Fill in the blanks with the correct numbers.

7. $^-4 + {}^-6 = \underline{\ ^-6\ } + {}^-4$
8. $(^+7 \times {}^+9)(^+11) = {}^+7(^+9 \times \underline{\qquad})$
9. $(^-16)(^+8) = (\underline{\qquad})(^-16)$
10. $(^-196)(1) = \underline{\qquad}$
11. $(\underline{\qquad})(12) = 12$
12. $(^-18 + \underline{\qquad}) + {}^+2 = {}^-18 + (^-26 + {}^+2)$

Topic Four: Additive-inverse property of integers

You know that $^{+}23 + {}^{-}23 = 0$,
$^{-}5 + {}^{+}5 = 0$
and $^{+}14 + {}^{-}14 = 0$.

From these and many other examples you can think of, you can conclude that the sum of two numbers is zero when each addend is the additive inverse of the other.

e.g. $^{-}3 + {}^{+}3 = 0$
$^{-}3$ is the additive inverse of $^{+}3$.
$^{+}3$ is the additive inverse of $^{-}3$.

We can write a general formula which describes this property.
If a represents a number and
^{-}a represents the additive inverse of that number, then
$a + {}^{-}a = 0$. (The sum of two numbers which are additive inverses of each other is zero.)

Assignment 4

Fill in the blanks with the correct numbers.

1. The identity element for multiplication is ______________________.
2. The identity element for addition is ______________________.
3. $^{-}2 + 9 = 9 +$ ______
4. ______ $+ {}^{-}3 = {}^{-}3 + {}^{-}7$
5. $^{-}7 + 0 =$ ______
6. ______ $+ 6 = 6$
7. $(1)(^{-}13) =$ ______
8. $(^{+}36)($______$) = {}^{+}36$
9. $^{-}26 +$ ______ $= 0$
10. ______ $+ {}^{-}4 = 0$
11. $^{-}2(^{-}7 \times {}^{-}3) = ($______ $\times {}^{-}7)(^{-}3)$
12. $(16)(7) = ($______$)(16)$

Give the general formulas for the following properties of integers.

13. Commutative property of addition ____________________

14. Associative property of addition ____________________

15. Identity-element property of addition ____________________

16. Additive-inverse property ____________________

17. Commutative property of multiplication ____________________

18. Associative property of multiplication ____________________

19. Identity-element property of multiplication ____________________

Key to Self-correcting Exercises in Lesson 26

Exercise 1, page 1

1. $0, 0$
2. $^{-}13, ^{-}13$
3. $^{+}5, ^{+}5$
4. $^{+}2, ^{+}2$
5. $^{+}7$
6. $^{+}68$
7. $^{-}86$
8. $^{-}37$

Exercise 2, page 4

1. $7 + {}^{-}6 = {}^{+}1$; $4 + {}^{-}3 = {}^{+}1$
2. $^{-}19 + {}^{+}8 = {}^{-}11$; $^{-}14 + {}^{+}3 = {}^{-}11$
3. $^{-}32 + {}^{-}19 = {}^{-}51$; $^{-}52 + {}^{+}1 = {}^{-}51$
4. $^{+}99 + 1 = 100$; $^{+}112 + {}^{-}12 = 100$
5. $^{-}8$
6. $^{-}1$
7. $^{-}19$
8. 37

Exercise 3, page 7

1. $^{+}16$
2. $3(^{-}42) = {}^{-}126$
3. $14 + {}^{-}19 = {}^{-}5$
4. $^{-}5$
5. 0
6. 0
7. $^{-}4$

End of Lesson 26

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number __________

Student's Questions and Comments

Apply Lesson Label Here

Name
Address
Postal Code

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned Teacher: ______

Lesson Grading: ______

Additional Grading E/R/P Code: ______

Mark: __________

Graded by: ________

Assignment Code: _____

Date Lesson Received:

____

Lesson Recorded _____

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do **not** enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

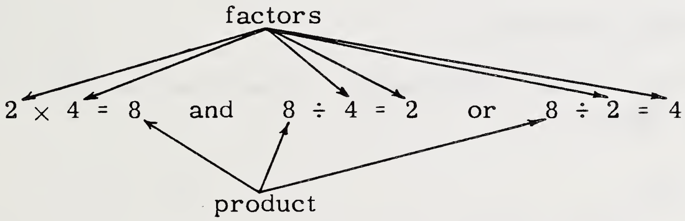
Formal Equation Solving

Topic One: Equations involving products

We developed a method of solving equations involving products when we studied ratios in Lesson 16. This method involved the relationship between the operations of multiplication and division. We learned that

factor × factor = product and
product ÷ factor = factor.

e.g.



Equations involving products are equations such as $2x = 12$, $14t = 7$ and $21n = 35$. To solve these equations, we must find the number that can be used to replace the placeholder so that the sentence will express a true statement.

e.g. In $2x = 12$ we cannot say that $x = 5$ because
$2(5) = 12$ does not express a true statement $(10 \neq 12)$.
In $2x = 12$ we can say that $x = 6$ because
$2(6) = 12$ does express a true statement $(12 = 12)$.

We can devise a step-by-step approach for finding the correct value of the placeholder.

EXAMPLE: Find the value of t in $14t = 7$.

Solution

$14t = 7$ means $14 \times t = 7$. Using the idea of product ÷ factor = factor we can say that $t = 7 \div 14$. (We use $t = 7 \div 14$ rather than $14 = 7 \div t$ because we wish to find the value of t. Therefore if we use $t = 7 \div 14$ we will immediately find the value of t.)

$t = 7 \div 14$ can be written $t = \frac{7}{14}$. ($\frac{7}{14}$ means $7 \div 14$.) By writing the fraction in its simplest form, we can say $\underline{t = \frac{1}{2}}$.

We can check to show that $t = \frac{1}{2}$ does satisfy $14t = 7$. Replace t in $14t = 7$ by $\frac{1}{2}$.

$$14(\frac{1}{2}) \overset{?}{=} 7$$

$$\frac{14}{2} \overset{?}{=} 7$$

$$7 = 7$$

Since $7 = 7$ is a true statement, we know that $t = \frac{1}{2}$ does satisfy **this** equation.

Self-correcting Exercise 1

Find the value of each placeholder. Check to see that you have **found** the correct value.

1. $^-7x = {}^-56$

Check:

2. $3x = {}^-19$

Check:

3. $.35x = 6.65$

Check:

4. $.25x = {}^-3.75$

Check:

Corrections are on page 15.

Assignment 1

Show how you would find the value of each placeholder. Check to see that you have found the correct value.

1. $.13h = 39$

$h = 39 \div .13$

$h = 3900 \div 13$

$h = 300$

$$\begin{array}{r} 300 \\ .13\overline{)39.00} \\ \underline{39} \\ 00 \\ \underline{0} \\ 00 \end{array}$$

Check:

$.13(300) \overset{?}{=} 39$

$39 = 39$

2. $.25n = {}^-1.5$

3. $6q = {}^-36$

4. ${}^-8.5p = 25.5$

5. $75y = 675$

6. ${}^-12r = 3.6$

Topic Two: Equations involving sums

Equations involving sums are equations like $m + 23 = {}^-19$, ${}^-6.4 + c = 8.9$ and $x + 4.6 = {}^-3.9$. To solve these equations we must find the number that can be used to replace the placeholder so that the sentence will express a true statement.

e.g. In $m + 23 = {}^-19$ we cannot say that $m = {}^-4$ because ${}^-4 + 23 = {}^-19$ does not express a true statement $(19 \neq {}^-19)$.
In $m + 23 = {}^-19$ we can say that $m = {}^-42$ because ${}^-42 + 23 = {}^-19$ does express a true statement $({}^-19 = {}^-19)$.

When solving equations which involve sums we can make use of the properties which were studied in Lesson 26. We can show step-by-step how we find the value of the placeholder. If we add the additive inverse of the number linked to the placeholder by addition, the resulting sum of numbers linked to the placeholder will be zero. This number must also be added to the side of the equation which does not have a placeholder.

EXAMPLE: Find the value of c in ${}^-6.4 + c = 8.9$.

Solution

In this case the number linked to c is ${}^-6.4$. The additive inverse of this number is ${}^+6.4$. If we add ${}^+6.4$ to each side of the equation, the value of the equation does not change. We put brackets around ${}^-6.4 + c$ because addition is a binary operation (only two numbers can be added at one time).

${}^+6.4 + ({}^-6.4 + c) = 8.9 + {}^+6.4$	Add ${}^+6.4$ to both sides of the equation.
$({}^+6.4 + {}^-6.4) + c = 8.9 + {}^+6.4$	Associative property of addition: We can add ${}^+6.4$ and ${}^-6.4$ but we cannot add ${}^-6.4$ and c because we do not know the value of c.
$0 + c = 15.3$	Additive inverse property $({}^-6.4 + {}^+6.4 = 0)$ and $8.9 + 6.4 = 15.3$
$c = 15.3$	Identity element for addition: $0 + c = c$

We can check to show that $c = 15.3$ does satisfy ${}^-6.4 + c = 8.9$. Replace c in ${}^-6.4 + c = 8.9$ by 15.3.

$${}^-6.4 + 15.3 \overset{?}{=} 8.9$$

$$8.9 = 8.9$$

Since $8.9 = 8.9$ is a true statement we know that $c = 15.3$ does satisfy the condition.

Self-correcting Exercise 2

The value of the placeholder in each equation has been found for you. Explain why each step was performed. (Name the property used, if possible.) Check to show that the value of the placeholder does satisfy the equation.

1. $x + 17 = 2$

$(x + 17) + {}^-17 = 2 + {}^-17$

a. ____________________

____

$x + (17 + {}^-17) = 2 + {}^-17$

b. ____________________

____

$x + 0 = {}^-15$

c. ____________________

____

$x = {}^-15$

d. ____________________

____

Check:

2. $15 + m = {}^-2$

${}^-15 + (15 + m) = {}^-2 + {}^-15$

a. ____________________

____

$({}^-15 + 15) + m = {}^-2 + {}^-15$

b. ____________________

____

$0 + m = {}^-17$

c. ____________________

____

$m = {}^-17$

d. ____________________

____

Check:

3. $x + 14 = {}^{-}19$

$(x + 14) + {}^{-}14 = {}^{-}19 + {}^{-}14$

a. ____________________
____

$x + (14 + {}^{-}14) = {}^{-}19 + {}^{-}14$

b. ____________________
____

$x + 0 = {}^{-}33$

c. ____________________
____

$x = {}^{-}33$

d. ____________________
____

Check:

Corrections are on page 15.

Assignment 2

Find the value of each placeholder. Explain why each step was performed. Check to show that the value of the placeholder does satisfy the equation.

1. $x + 4 = 7$

$(x + 4) + {}^{-}4 = 7 + {}^{-}4$ Add ⁻4 to each side of the equation.

$x + (4 + {}^{-}4) = 7 + {}^{-}4$ Associative property of addition

$x + 0 = 3$ additive inverse property $(4 + {}^{-}4 = 0)$ and $7 + {}^{-}4 = 3$

$x = 3$ Identity-element property of addition

Check:

$3 + 4 \overset{?}{=} 7$

$7 = 7$

2. $x + 3 = 11$

Check:

3. $x + 4 = 4$

Check:

4. $10 = x + 2$

$10 + {}^{-}2 = (x + 2) + {}^{-}2$

Check:

5. $13 = y + 11$

Check:

Equations involving differences can be expressed as equations involving sums. You will remember from Lesson 25 that differences can be expressed as sums. To do this we must use the additive inverse of the subtrahend.

e.g. $^{+}6 - {}^{-}3 = {}^{+}6 \overset{\text{add}}{\text{\textcircled{+}}} \overset{\text{Additive inverse of } ^{-}3}{\text{\textcircled{$^{+}3$}}}$ ($^{+}3$ is the additive inverse of $^{-}3$.)

We can use this idea to rewrite equations involving differences.

e.g. $a - 3 = 4$ can be written $a + {}^{-}3 = 4$.

Then we proceed to find the value of the placeholder as we did with equations involving sums.

Self-correcting Exercise 3

Rewrite each equation as an equation involving a sum rather than a difference.

1. $x - 2 = 7$ ____________________
2. $10 = a - 6$ ____________________
3. $3y - 13 = 7$ ____________________
4. $m - {}^{+}8 = 0$ ____________________

Corrections are on page 16.

Assignment 3

Find the value of each placeholder. Check to show that the value of the placeholder does satisfy the equation.. (Rewrite each subtraction equation as an addition equation first.)

1. $c - 5 = 1$

$c + {}^{-}5 = 1$ (addition equation)

$(c + {}^{-}5) + {}^{+}5 = 1 + {}^{+}5$

$c + ({}^{-}5 + {}^{+}5) = 6$

$c + 0 = 6$

$c = 6$

Check:

$6 - 5 \overset{?}{=} 1$

$1 = 1$

2. $x - 5 = 5$

3. $20 = a - 15$

4. $2 = a - 6$

5. $25 = n + 16$

6. $3x = 27$

Topic Three: Equations involving products and sums

Equations involving products and sums are equations such as $4x - 5 = 11$, $5y + 2 = {}^{-}13$ and $6x + 15 = {}^{-}21$. To solve these equations we must find the number that can be used to replace the placeholder so that the sentence will express a true statement.

e.g. In $4x - 5 = 11$, we cannot say that $x = 2$ because
$4(2) - 5 = 11$ does not express a true statement because
$8 - 5 = 11$ is not a true statement ($3 \neq 11$).

In $4x - 5 = 11$, we can say that $x = 4$ because
$4(4) - 5 = 11$ does express a true statement because
$16 - 5 = 11$ is a true statement ($11 = 11$).

When working with equations involving both products and sums, you must eliminate the sum part of the equation first so that you are left with an equation involving only a product.

For example, the equation $\underbrace{4x} - 5 = 11$ which can be written $4x + {}^{-}5 = 11$ means some amount plus ${}^{-}5$ is equal to 11. Therefore we can simplify this by thinking of it as $\square + {}^{-}5 = 11$. Using the rules for solving equations involving sums,

$$\begin{aligned} \square + {}^{-}5 &= 11 \text{ becomes} \\ (\square + {}^{-}5) + {}^{+}5 &= 11 + {}^{+}5 \\ \square + ({}^{-}5 + {}^{+}5) &= 11 + {}^{+}5 \\ \square + 0 &= 16 \\ \square &= 16. \end{aligned}$$

Since we used $\square$ to represent $4x$ we can change $\square = 16$ to $4x = 16$. Now we must use the rules for solving equations involving products to find the value of the placeholder in $4x = 16$.

$$\begin{aligned} 4x &= 16 \longleftarrow \text{This equation involves only a product.} \\ x &= 16 \div 4 \\ x &= 4 \end{aligned}$$

We can therefore conclude that when solving equations involving products and sums we must first simplify them to equations involving products only and then find the value of the placeholders.

EXAMPLE: Find the value of y in $5y + 2 = {}^{-}13$.

Solution

$(5y + 2) + {}^{-}2 = {}^{-}13 + {}^{-}2$	Add ${}^{-}2$ to each side of the equation.
$5y + (2 + {}^{-}2) = {}^{-}13 + {}^{-}2$	Associative property of addition
$5y + 0 = {}^{-}15$	Additive inverse property $(2 + {}^{-}2 = 0)$ and ${}^{-}13 + {}^{-}2 = {}^{-}15$
$5y = {}^{-}15$	Identity element property of addition
$y = {}^{-}15 \div 5$	Multiplication-division relationship
$y = {}^{-}3$	

Check:

$$\begin{aligned} 5({}^{-}3) + 2 &\overset{?}{=} {}^{-}13 \\ {}^{-}15 + 2 &\overset{?}{=} {}^{-}13 \\ {}^{-}13 &= {}^{-}13 \end{aligned}$$

Self-correcting Exercise 4

Change each equation to one containing only a product.

1. $4x + 3 = {}^-17$

2. $2a + 15 = 37$

3. $20 + 3x = 32$

4. $3x - .7 = 2$

Corrections are on page 16.

Assignment 4

Find the value of the placeholder in each equation. Check to show that the value of the placeholder does satisfy the equation.

1. $2x + 1 = 3$

$$(2x + 1) + {}^-1 = 3 + {}^-1$$
$$2x + (1 + {}^-1) = 3 + {}^-1$$
$$2x + 0 = 2$$
$$2x = 2$$
$$x = 2 \div 2$$
$$x = 1$$

Check: $2(1) + 1 \overset{?}{=} 3$
$$2 + 1 \overset{?}{=} 3$$
$$3 = 3$$

2. $3x + 5 = 11$

Check:

3. $2x - 1 = 5$

$2x + {}^{-}1 = 5$ ← (Rewrite as a sum first.)

$(2x + {}^{-}1) + {}^{+}1 = 5 + {}^{+}1$

4. $6c - 2 = {}^{-}10$

Check:

Check:

We can simplify the method used in solving equations involving addition. We can omit the step that uses the associative property because we know, without writing it down, that addition is a binary operation. We can also omit the step that shows that the sum of a number and its additive inverse is zero.

EXAMPLE: Find the value of w in $6w + 11 = 35$.

Solution

$$\begin{aligned} 6w + 11 &= 35 && \text{Add } {}^{-}11 \text{ to each side.} \\ 6w + 11 + {}^{-}11 &= 35 + {}^{-}11 \\ 6w &= 24 && \text{Find the value of } w. \\ w &= 24 \div 6 \\ w &= 4 \end{aligned}$$

Check: $6(4) + 11 \overset{?}{=} 35$

$24 + 11 \overset{?}{=} 35$

$35 = 35$

Assignment 5

Find the values of the placeholders in the equations.

1. $3x + {}^{-}5 = 19$

$3x + {}^{-}5 + {}^{+}5 = 19 + {}^{+}5$

$3x = 24$

$x = 24 \div 3$

$x = 8$

Check:

$3(8) + {}^{-}5 \overset{?}{=} 19$

$24 + {}^{-}5 \overset{?}{=} 19$

$19 = 19$

2. $4x - 11 = 5$

Check:

3. $7x + 8 = 15$

Check:

4. $3r + 17.8 = 24.7$

Check:

Key to Self-correcting Exercises in Lesson 27

Exercise 1, page 2

1. $^-7x = {}^-56$
   $x = {}^-56 \div {}^-7$
   $x = 8$

   Check: $-7(8) \overset{?}{=} -56$
   $-56 = -56$

2. $3x = -19$
   $x = -19 \div 3$
   $x = \frac{^-19}{3}$
   $x = {}^-6\frac{1}{3}$

   Check: $3(^-6\frac{1}{3}) \overset{?}{=} -19$
   $3(\frac{^-19}{3}) \overset{?}{=} -19$
   $^-19 = {}^-19$

3. $.35x = 6.65$
   $x = 6.65 \div .35$
   $x = 665 \div 35$
   $x = 19$

   Check: $.35(19) \overset{?}{=} 6.65$
   $6.65 = 6.65$

$$\begin{array}{r} 19 \\ \underline{.35} \\ 95 \\ \underline{570} \\ 6.65 \end{array}
\qquad
\begin{array}{r} 19 \\ 35\overline{)665} \\ \underline{35}^{\times} \\ 315 \\ \underline{315} \\ 0 \end{array}$$

4. $.25x = {}^-3.75$
   $x = {}^-3.75 \div .25$
   $x = {}^-375 \div 25$
   $x = {}^-15$

   Check: $.25\ (^-15) \overset{?}{=} -3.75$
   $^-3.75 = -3.75$

$$\begin{array}{r} 15 \\ \underline{.25} \\ 75 \\ \underline{300} \\ 3.75 \end{array}
\qquad
\begin{array}{r} 15 \\ 25\overline{)375} \\ \underline{25}^{\times} \\ 125 \\ \underline{125} \\ 0 \end{array}$$

Exercise 2, page 5

1. a. Add $^-17$ to both sides of the equation.
   b. Associative property of addition
   c. Additive inverse property $(17 + {}^-17 = 0)$ and $2 + {}^-17 = {}^-15$
   d. Identity element for addition

   Check: $^-15 + 17 \overset{?}{=} 2$
   $2 = 2$

2. a. Add ${}^{-}15$ to both sides of the equation.
   b. Associative property of addition
   c. Additive inverse property (${}^{-}15 + 15 = 0$) and ${}^{-}2 + {}^{-}15 = {}^{-}17$
   d. Identity element for addition

   Check:

$$\begin{aligned} 15 + {}^{-}17 &\overset{?}{=} {}^{-}2 \\ {}^{-}2 &= {}^{-}2 \end{aligned}$$

3. a. Add ${}^{-}14$ to each side of the equation.
   b. Associative property of addition
   c. Additive inverse property ($14 + {}^{-}14 = 0$) and ${}^{-}19 + {}^{-}14 = {}^{-}33$
   d. Identity element for addition

   Check:

$$\begin{aligned} {}^{-}33 + 14 &= {}^{-}19 \\ {}^{-}19 &= {}^{-}19 \end{aligned}$$

Exercise 3, page 9

1. $x + {}^{-}2 = 7$
2. $10 = a + {}^{-}6$
3. $3y + {}^{-}13 = 7$
4. $m + {}^{-}8 = 0$

Exercise 4, page 12

1.
$$\begin{aligned} 4x + 3 &= {}^{-}17 \\ (4x + 3) + {}^{-}3 &= {}^{-}17 + {}^{-}3 \\ 4x + (3 + {}^{-}3) &= {}^{-}17 + {}^{-}3 \\ 4x + 0 &= {}^{-}20 \\ 4x &= {}^{-}20 \end{aligned}$$

2.
$$\begin{aligned} 2a + 15 &= 37 \\ (2a + 15) + {}^{-}15 &= 37 + {}^{-}15 \\ 2a + (15 + {}^{-}15) &= 37 + {}^{-}15 \\ 2a + 0 &= 22 \\ 2a &= 22 \end{aligned}$$

3.
$$\begin{aligned} 20 + 3x &= 32 \\ {}^{-}20 + (20 + 3x) &= 32 + {}^{-}20 \\ ({}^{-}20 + 20) + 3x &= 32 + {}^{-}20 \\ 0 + 3x &= 12 \\ 3x &= 12 \end{aligned}$$

4.
$$\begin{aligned} 3x - .7 &= 2 \\ 3x + {}^{-}.7 &= 2 \\ (3x + {}^{-}.7) + {}^{+}.7 &= 2 + {}^{+}.7 \\ 3x + ({}^{-}.7 + {}^{+}.7) &= 2 + {}^{+}.7 \\ 3x + 0 &= 2.7 \\ 3x &= 2.7 \end{aligned}$$

End of Lesson 27

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned
Teacher: ________

Lesson Grading: ________

Additional Grading
E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________________

Lesson Recorded ______

Student's Questions and Comments

Apply Lesson Label Here

Name ____________________

Address ____________________

____

____

Postal Code ____________________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

St. Serv. 9-91

Problem Solving

Topic One: Problems

You have done a number of problems this year. For each problem, you had to decide what information was required. You used a placeholder to represent this unknown quantity in the equation. Then you were able to compute to find the value of the placeholder. Finally, you wrote a statement to answer the question asked in the problem.

Following are a few questions which will review the work you did on problems earlier in the course.

Assignment 1

For each problem:
- tell what the placeholder represents,
- write an equation,
- find an answer and
- write a statement answering the question asked.

1. A certain building is 475 ft. high. A television sending tower 352.1 ft. high was installed on the roof of the building. How many feet above the ground is the top of the television tower?

Placeholder: ______________________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ______________________________

____

2. Mr. Page bought 12.5 lb. of grass seed. After he seeded his lawn he had 5.25 lb. of seed left. How many pounds of seed did he use?

Placeholder: ______________________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ______________________________

____

3. Lake Champlain is 95 ft. above sea level. The elevation of Lake Superior is 6.09 times the elevation of Lake Champlain. What is the elevation of Lake Superior in feet?

Placeholder: ____________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

4. The distance from Vancouver to Fort William by air is 3 times the distance from Fort William to Sarnia by air. The distance from Vancouver to Fort William is 1512 miles. What is the distance from Fort William to Sarnia?

Placeholder: ____________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

5. The distance between two airports is 16 mi. A helicopter usually flies the distance in 9 min. At this rate, how many miles can the helicopter fly in 60 min.?

Placeholder: ____________________

Equation: ____________________ Computation

Answer: ____________________

Statement: ____________________

____

Topic Two: Hidden questions

The problems that you have done so far are very simple problems. In each problem you were given all the information you needed to write an equation and find the answer.

Some problems are more difficult. Sometimes information required in finding the answer is given indirectly. You have to make some calculations before you know all the information required to find the solution to the question asked in the problem. In other words the problems contain a hidden question which must be answered before you can find the answer to the question asked by the problems.

e.g. Bert and Eddie accepted summer jobs as box boys for the Consolidated Supermarket. They received $1.12 per hour. The first week Bert worked six hours a day for five days. How much did he earn the first week?

Before you can find how much Bert earned, you must calculate to find the total number of hours that Bert worked. Thus, the hidden question in this problem is: "How many hours did Bert work?"

Self-correcting Exercise 1

For each problem, state the hidden question.

1. Mrs. Wagner bought 5 pounds of tomatoes at 12¢ per pound. How much change did she receive from a dollar?

____

2. If you cut 6 pieces each 5 inches long from a brass rod 37 inches long, how long is the piece that is left?

____

3. Margaret works part-time as a check-out clerk in a supermarket. Last month she worked 15 hours during the first week, 22 hours the second, 18 hours the third and 15 hours the fourth. She is paid 90¢ per hour. How much did she earn last month?

____

4. Mr. Adams lives in a province where the tax on gasoline is 12¢ per gallon. His car can travel 20 miles on a gallon of gasoline. On a trip of 1200 miles, how much tax will Mr. Adams pay on the gasoline he uses?

____

5. Jane wants to buy a camera that will cost $25. She earns $1.25 an hour baby-sitting and can baby-sit 4 hours per week. How many weeks will it take her to earn enough to buy the camera?

____

Corrections are on page 10.

Since problems containing hidden questions involve two separate calculations, we must show two separate sets of problem-solving steps. We will use a small letter for the placeholder which represents the information required by the hidden question. We will use a capital letter for the placeholder which represents the information asked for by the question in the problem.

EXAMPLE: On a trip of 600 miles Mr. Fraser's car averaged 15 miles per gallon of gasoline. He paid 41.9¢ per gallon of gasoline. How much did the gasoline cost him?

Solution

Hidden question: How many gallons of gasoline did he use?

Placeholder 1: Let n be the number of gallons of gasoline he used.

Equation 1: $15 = \frac{600}{n}$ (Based on the average formula, $a = \frac{t}{n}$)

Computation 1

$$\begin{array}{r} 40 \\ 15\,\overline{)\,600} \\ \underline{60} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

Answer 1: $n = 40$

Statement 1: He used 40 gallons of gasoline.

Placeholder 2: Let C be the cost of the gasoline used.

Equation 2: $\frac{41.9}{1} = \frac{C}{40}$ $\left(\frac{\text{cost in cents}}{\text{gallons}}\right)$

$41.9 \times 40 = 1 \times C$

Computation 2

$$\begin{array}{r} 41.9 \\ \underline{40} \\ 1676.0 \end{array}$$

Answer 2: $1676 = C$

Statement 2: The cost of the gasoline is 1676¢ or $16.76.

Assignment 2

For each problem, fill in all the blanks with the required information.

1. Mike plans to build a bookshelf for his study. He purchased the following supplies: boards, $6.50; nails and hardware, $1.10; paint, $.95. How much change should he receive from a $10 bill?

Hidden question: What was the total cost of the materials?

Placeholder 1: Let t be the total cost of the materials.

Equation 1: t = 6.50 + 1.10 + .95

Computation 1

6.50
1.10
.95
______

Answer 1: t = ____________

Statement 1: The total cost of the material was ____________.

Placeholder 2: Let C be the change from the $10 bill.

Equation 2: C = 10.00 - _______

Computation 2

10.00
- _____

Answer 2: C = ___________

Statement 2: He received ______ change from the $10 bill.

2. Mr. Atchison's orchard produced 3200 bushels of apples last year. He sold them at $2.75 per bushel. Cost of labor and other expenses amounted to $4300. How much was earned from the orchard after these expenses were met?

Hidden question: How much did he receive from the sale of the apples?

Placeholder 1: Let s be the amount he received from the sale of the apples.

Equation 1: ____________________

Computation 1

Answer 1: s = ________________

Statement 1: He received ________ from the sale of the apples.

Placeholder 2: Let P be the amount earned after expenses were met.

Equation 2: P = ______ - ______ Computation 2

Answer 2: P = ______________

Statement 2: He earned _______ after expenses were met.

3. There are six 45-minute class periods at the Long School. How many hours does this amount to?

Hidden question: Six 45-minute class periods is a total of how many minutes?

Placeholder 1: Let m be the total number of minutes in 6 class periods.

Equation 1: __________________ Computation 1

Answer 1: m = ______________

Statement 1: Six 45-minute class periods is a total of _____ minutes.

Placeholder 2: Let H be _________________________________

Equation 2: __________________ Computation 2

Answer 2: H = ______________

Statement 2: Six 45-minute class periods is a total of _____ hours.

4. The length of a rectangle is $3\frac{1}{2}$ ft. The width of a rectangle is 64 in. What is the perimeter of a rectangle in inches?

Hidden question:

Placeholder 1:

Equation 1: Computation 1

Answer 1:

Statement 1:

Placeholder 2:

Equation 2: Computation 2

Answer 2:

Statement 2:

5. Mr. Gordon is reseeding a rectangular lawn 60 ft. long and 50 ft. wide. He uses 1 lb. of grass seed for 500 square feet of lawn. How many pounds of grass seed will he use in all?

Hidden question:

Placeholder 1:

Equation 1: Computation 1

Answer 1:

Statement 1:

Placeholder 2:

Equation 2: Computation 2

Answer 2:

Statement 2:

6. It cost Mr. Humphries $125 to have the floor of his kitchen covered with tile. If the rectangular floor is 14 ft. long and $11\frac{1}{2}$ ft. wide, what was the cost per square foot to the nearer cent?

Hidden question:

Placeholder 1:

Equation 1: Computation 1

Answer 1:

Statement 1:

Placeholder 2:

Equation 2: Computation 2

Answer 2:

Statement 2:

7. For a bazaar, Mrs. Hayes ordered 13 qt. of coffee ice cream, 11 gal. of vanilla ice cream and $8\frac{1}{2}$ gal. of chocolate ice cream. In all, she ordered how many gallons of ice cream? (Hint: There are 4 quarts in a gallon.)

Hidden question:

Placeholder 1:

Equation 1: Computation 1

Answer 1:

Statement 1:

Placeholder 2:

Equation 2: Computation 2

Answer 2:

Statement 2:

8. John had saved \$56.60 to buy sports equipment. He used some of this money to buy a bicycle that cost \$36.98 and a basketball that cost \$12.98. How much money did John have left?

Hidden question:

Placeholder 1:

Equation 1: Computation 1

Answer 1:

Statement 1:

Placeholder 2:

Equation 2: Computation 2

Answer 2:

Statement 2:

Key to Self-correcting Exercise in Lesson 28

Exercise 1, page 3

1. How much did the tomatoes cost?
2. What is the total length of the pieces?
3. What is the total number of hours she worked?
4. How many gallons of gasoline did Mr. Adams use?
5. How much does Jane earn in one week?

End of Lesson 28

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________

Time Spent on Lesson ____________

(If label is missing
or incorrect)

File Number ____________

Lesson Number ____________

FOR SCHOOL USE ONLY

Assigned Teacher: ________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ________

Date Lesson Received: ________

Lesson Recorded ________

Student's Questions and Comments

Apply Lesson Label Here

Name ____________

Address ____________

Postal Code ____________

Please verify that preprinted label is for correct course and lesson.

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to **mail each** lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

Sets

Topic One: Sets

In everyday life we use many different words to express the idea of a collection or group of objects. We speak of a herd of cattle, a pack of wolves, a flock of sheep, a swarm of bees, a fleet of ships, a stamp collection and a set of golf clubs. Each of these illustrations is a definite group containing only certain objects.

e.g. A horse in a herd of cattle is not considered to be part of the herd of cattle. Only cows, bulls, heifers, steers and calves can be part of a herd of cattle.

We call a well-defined collection of objects, like a herd of cattle, a SET.

A set must be very clearly defined. You must be able to decide whether or not specific objects are members or elements of a set.

e.g. A ruler is not a member of the set of books.

To be a member of a set, an object must belong to the set.

e.g. Treasure Island is a member of the set of books.

So far we have described sets. When describing a set we must make it clear which objects are members of the set.

Self-correcting Exercise 1

Following are descriptions of sets. Below each description is a list of objects. If the object is a member of the set, write "yes" on the line. If the object is not a member of the set, write "no" on the line.

1. The set of the months of the year

January ____________

Sunday ____________

Autumn ____________

July ____________

December ____________

May ____________

Halloween ____________

October ____________

2. The set of Prime Ministers of Canada

Sir John A. Macdonald ____________________

Thomas Edison ____________________

Louis St. Laurent ____________________

Mackenzie King ____________________

Maurice Richard ______________________

John Diefenbaker ______________________

3. The set of Canadian coins

nickel ____________ dime ______________

half-dollar ____________ cent ______________

franc ____________ mark ______________

Corrections are on page 11.

We have used one way of distinguishing between sets - by describing them. Another way of distinguishing between sets is by tabulation. To tabulate a set we must list all the names of the members of the set. This list is then enclosed in braces. The members of the set are separated by commas.

e.g. To tabulate the set of Great Lakes we would write:
{Michigan, Huron, Superior, Erie, Ontario}.

To read a tabulation we use the words "the set whose members are" for the braces and then read the names in the list.

e.g. {Michigan, Huron. Superior, Erie, Ontario}
The set whose members are Michigan, Huron, Superior, Erie, Ontario

When tabulating a set, each member is written only once.

EXAMPLE: Tabulate the letters in the word "correspondence".

Solution

The letters used in "correspondence" are c, o, r, e, s, p, n, d. Although the letters c, o, r, e. n appear more than once in "correspondence". they are not repeated in the tabulation.

Enclose the letters in braces. {c, o, r, e, s, p, n, d}

Also the order of the members in the set may be changed without affecting the set.

e.g. {c, o, r, e, s, p, n, d} is the same as {c, d. e, n, o,p, r. s}

Assignment 1

Decide whether the objects in the list are members of the set described.

1. The set of letters of the alphabet

a	yes	q	
b		7	
*		r	

2. The set of even numbers from 0 through 10 (This set includes 0 and 10.)

8		3	
5		4	
12		15	

Describe each of the sets that is tabulated below.

3. {North America, South America, Asia, Africa, Europe, Antarctica, Australia}

   The set of continents

4. {British Columbia, Alberta, Saskatchewan, Manitoba}

   ____

5. {0, 1, 2, 3}

   ____

6. {1, 3, 5, 7, 9}

   ____

7. {21, 22, 23, 24, 25, 26, 27, 28, 29}

   ____

Tabulate each of the sets that is described.

8. The set of numbers from 50 through 60

{50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60}

(Note that both 50 and 60 are included.)

9. The set of even numbers from 50 through 60

10. The set of letters in the word "tabulation"

11. The set of months having 30 days

12. The set of days in a week

13. Write the tabulation of {1, 2, 3} in as many different ways as possible by changing the order of the members of the set.

14. Is {Mary, Joe, John} the same set as {Joe, John, Mary}? ______

We can name sets by using capital letters. We can say that A = {1, 3, 5, 7, 9}. We can refer to this set as set A or as {1, 3, 5, 7, 9}.

Topic Two: Subsets

A subset is a set which consists of members each of which is also a member of another set.

e.g. The set of cats is a subset of the set of animals. All cats are animals. Therefore the set of cats is a part of the set of animals. Each member of the set of cats is also a member of the set of animals.

Every set is a subset of itself.

e.g. A = {0, 2, 4, 6} B = {0, 2, 4, 6}

A is a subset of B and B is a subset of A because each member of A is also a member of B.

Self-correcting Exercise 2

Decide whether each of the following sets is a subset of the given set.

1. $B = \{2, 4, 6, 8, 10, 12\}$

$\{8, 10, 12, 14\}$ ________ $\{2, 4, 6, 8\}$ ________

$\{1, 2, 3, 4\}$ ________ $\{10\}$ ________

2. $M = \{a, c, e, i, m\}$

$\{a, c, e, i, m\}$ ________ $\{a, b, c\}$ ________

$\{m, i, c\}$ ________ $\{e, a, n\}$ ________

3. The set of all fruits

{potatoes} ________ {lemons, limes, oranges} ________

{bananas, grapes} ________ {peas, corn, eggs} ________

Corrections are on page 12.

Topic Three: Types of sets

We can classify sets according to the number of elements they contain.

A. Finite sets

If you can count the members in a set, the set is a FINITE set.

e.g. $B = \{7, 8, 14, 16\}$

This set has 4 members. It is a finite set.

Sometimes you will find it inconvenient to tabulate all of the members in a set. This is usually because the set contains a great number of members. If the members occur in a regular pattern we need to list only the first three members of the set. Then make three dots. Finally write the last member in the set. Separate the three dots from the members by commas.

EXAMPLE: Tabulate the set of numbers from 1 through 1000.

Solution

It is inconvenient to list every member in this set. The pattern of the members is regular. They follow each other consecutively.
The first three members are 1, 2, 3.
Adding three dots, we get 1, 2, 3, ...
Including the last member of the set we get 1, 2, 3, ..., 1000.
To tabulate this set, we must enclose the members in braces.
{1, 2, 3, ..., 1000}

B. Infinite sets

Some sets follow a regular sequence or pattern but have an unlimited number of members. To tabulate such sets, we give the first three members of the set to show the pattern. Since there is no last member of the set, no member follows the three dots.

EXAMPLE: Tabulate the set of even numbers.

Solution

It is not possible to list every even number.
The first three even numbers are 0, 2, 4.
Add the three dots to show that the numbers continue in this pattern. 0, 2, 4, ...
Since there is no last member, no member follows the three dots.
To tabulate this set, we must enclose the members in braces.
{0, 2, 4, ...}

If it is not possible to count the members in a set, the set is an INFINITE set.

e.g. E = {0, 2, 4, ...}
This is the set of all even numbers. You cannot count the number of members in the set. It has no last member. The set is an infinite set.

Two special infinite sets of numbers used earlier in the course were the set of whole numbers and the set of natural numbers.
We can tabulate the set of whole numbers. W = {0, 1, 2, ...}
We can tabulate the set of natural numbers. N = {1, 2, 3, ...}

Remember to list only the first three members of the set. Then make the three dots.

Assignment 2

Tabulate each of the following sets. Then state whether the set is finite or infinite.

1. The set of odd numbers from 1 through 40

   $\{1, 3, 5, \ldots, 39\}$

   Finite

2. The set of natural numbers greater than 10

3. The set of odd numbers from 3 through 7

4. The set of odd natural numbers

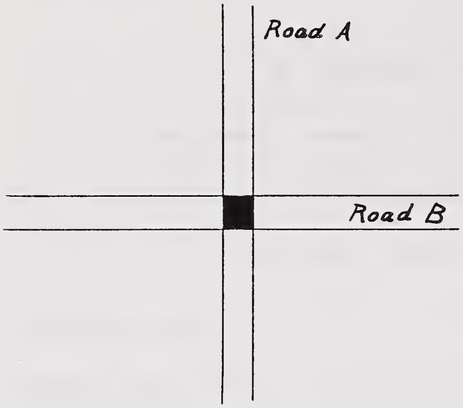
5. The set of even natural numbers

6. In $\{0, 1, 2, \ldots, 1000\}$, the three dots are used instead of what numbers?

7. In $\{0, 1, 2, \ldots\}$, the three dots are used instead of what numbers?

Topic Four: Intersection of sets

Consider the point at which two roads meet and cross.



Road A and Road B meet and cross. The shaded section belongs to both of these roads. We call the shaded section an intersection.

The word "intersection" has a similar meaning when it is applied to sets. The INTERSECTION of two sets is a third set which contains only the members which belong to both sets.

EXAMPLE: C = {1, 2, 3, 4} and D = {2, 4, 6}
Tabulate the intersection of C and D.

Solution

There are 2 members which belong to both C and D. They are 2 and 4. 2 and 4 are members of C. 2 and 4 are members of D. No other members of C and D are the same.

Therefore the intersection of C and D is {2, 4}.

We can use the symbol "$\cap$" to replace the term "intersection". In the above example we could have written $C \cap D = \{2, 4\}$ or $\{1, 2. 3, 4\} \cap \{2, 4, 6\} = \{2, 4\}$.

In some cases, the sets will have no members in common. We show this by writing set braces with nothing between them. When no members are written between the set braces, the set braces are called the EMPTY SET.

EXAMPLE: Tabulate the intersection of A and B.
A = {27} B = {36, 37, 38, ...}

Solution

Since B does not contain 27, A and B have no members in common. Therefore $A \cap B = \{\ \}$. (The intersection of A and B is the empty set.)

When the given sets contain three dots, remember which members are represented by the three dots. Consider those members when deciding on the intersection of the sets.

Assignment 3

Tabulate the intersection of the following pairs of sets.

1. $S = \{\text{Don, Ruth, Joe, Carl, Nancy, Jim}\}$
$C = \{\text{Don, Sally, Joe, Wayne}\}$

$S \cap C =$ {Don, Joe} ____________________

2. $A = \{1, 2, 3, 4, 6, 12\}$
$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$A \cap B =$ ____________________

3. $X = \{2, 4, 6, 8, 10\}$
$Y = \{1, 3, 5, 7, 9\}$

$X \cap Y =$ ____________________

4. $M = \{4, 8, 12, 16, 20, 24\}$
$N = \{6, 12, 18, 24, 30\}$

$M \cap N =$ ____________________

5. $F = \{0, 1, 2, \ldots\}$
$G = \{0, 2, 4, \ldots\}$

$F \cap G =$ ____________________

6. $K = \{1, 3, 5, \ldots\}$
$L = \{0, 2, 4, \ldots\}$

$K \cap L =$ ____________________

Topic Five: Union of sets

If two groups are united, they are combined into one group.

The union of two sets is related to the above idea. The UNION of two sets is a third set which contains all the members in the two sets.

EXAMPLE: Tabulate the union of C and D.
$C = \{5, 10\}$ $D = \{6, 8, 10\}$

Solution

The members in the two sets are 5, 10 and 6, 8, 10. Since each member is written only once when we are tabulating sets, the union of the two sets will include 5, 6, 8, 10.

Therefore the union of C and D is $\{\underline{5, 6, 8, 10}\}$.

We can use the symbol "$\cup$" to replace the term "union". In the above example we could have written $C \cup D = \{5, 6, 8, 10\}$ or $\{5, 10\} \cup \{6, 8, 10\} = \{5, 6, 8, 10\}$.

Assignment 4

Tabulate the union of the sets. (Remember that if an element appears in both sets, it is listed in the union set only once.)

1. A = {bluebird, crow, hawk, owl}
   B = {eagle, hawk, owl, vulture}

   $A \cup B$ = {bluebird, crow, hawk, owl, eagle, vulture}

2. $E = \{3, 6, 9, 12\}$
   $F = \{5, 10, 15\}$

   $E \cup F$ = ____________________

3. $J = \{0, 1, 2, \ldots\}$
   $K = \{1, 3, 5, \ldots\}$

   $J \cup K$ = ____________________

   (Hint: This union set will be an infinite set.)

4. $L = \{1, 2, 5, 9\}$
   $M = \{2, 9, 5, 1\}$

   $L \cup M$ = ____________________

5. $P = \{0, 2, 4, \ldots\}$
   $Q = \{1, 3, 5, \ldots\}$

   $P \cup Q$ = ____________________

6. $R = \{1, 5, 7, 9\}$
$S = \{1, 7, 13\}$

$R \cup S =$ ____

First tabulate the intersection of the sets. Then tabulate the union of the sets.

7. $F = \{a, b, c, d, e\}$
$G = \{d, e, f, i\}$

$F \cap G =$ ____

$F \cup G =$ ____

8. $A = \{1, 2, 3, 4, 6, 12\}$
$B = \{1, 2, 3, 5, 6, 10, 15, 30\}$

$A \cap B =$ ____

$A \cup B =$ ____

Key to Self-correcting Exercises in Lesson 29

Exercise 1, page 1

1.

January	Yes	December	Yes
Sunday	No	May	Yes
Autumn	No	Halloween	No
July	Yes	October	Yes

2.

Sir John A. Macdonald	Yes
Thomas Edison	No
Louis St. Laurent	Yes
Mackenzie King	Yes
Maurice Richard	No
John Diefenbaker	Yes

3.

nickel	Yes	dime	Yes
half-dollar	Yes	cent	Yes
franc	No	mark	No

Exercise 2, page 5

1. {8, 10, 12, 14} No
(14 does not belong to set B.)
{2, 4, 6, 8} Yes

{1, 2, 3, 4} No
(1 and 3 do not belong to set B.)
{10} Yes

2. {a, c, e, i, m} Yes
{a, b, c} No
(b does not belong to set M.)

{m, i, c} Yes
{e, a, n} No
(n does not belong to set M.)

3. {potatoes} No
(Potatoes are not fruits.)
{lemons, limes, oranges} Yes

{bananas, grapes} Yes
{peas, corn, eggs} No
(None of these elements are fruits.)

End of Lesson 29

LESSON RECORD FORM

5052 Mathematics 9 Modified
Revised 88/04

FOR STUDENT USE ONLY

Date Lesson Submitted ____________________

Time Spent on Lesson ____________________

(If label is missing
or incorrect)

File Number ____________________

Lesson Number ____________

Student's Questions and Comments

Apply Lesson Label Here

Name ____________
Address ____________
____
____
Postal Code ____________

Please verify that preprinted label is for correct course and lesson.

FOR SCHOOL USE ONLY

Assigned Teacher: ________

Lesson Grading: ________

Additional Grading E/R/P Code: ________

Mark: ________

Graded by: ________

Assignment Code: ______

Date Lesson Received: ____________

Lesson Recorded ______

Teacher's Comments:

____
Correspondence Teacher

St Serv 1-86

ALBERTA DISTANCE LEARNING CENTRE

MAILING INSTRUCTIONS FOR CORRESPONDENCE LESSONS

1. BEFORE MAILING YOUR LESSONS, PLEASE SEE THAT:

(1) All pages are numbered and in order, and no paper clips or staples are used.
(2) All exercises are completed. If not, explain why.
(3) Your work has been re-read to ensure accuracy in spelling and lesson details.
(4) The Lesson Record Form is filled out and the correct lesson label is attached.
(5) This mailing sheet is placed on the lesson.

2. POSTAGE REGULATIONS

Do not enclose letters with lessons.

Send all letters in a separate envelope.

3. POSTAGE RATES

First Class

Take your lesson to the Post Office and have it weighed. Attach sufficient postage and a green first-class sticker to the front of the envelope, and seal the envelope. Correspondence lessons will travel faster if first-class postage is used.

Try to mail each lesson as soon as it has been completed.

When you register for correspondence courses, you are expected to send lessons for correction regularly. Avoid sending more than two or three lessons in one subject at the same time.

REVIEW

THE END IS NEAR

This is the last lesson in Mathematics 9 (M) that you will do before you write your final test for this course under supervised conditions.

Lesson 30 is a review of the concepts taught in Lessons 1 to 29. You should use this lesson to help you prepare for the final test. Review the entire course very carefully. Then attempt to complete as much of this lesson as you can in one sitting. You must do this review lesson without any aids other than your geometry set.

If you have found that you had difficulty doing any of the questions in Lesson 30, review again the teaching notes and corrected exercises which you studied previously. Then go back and complete any parts of Lesson 30 that you had difficulty with and mail it in for correction and grading.

The supervised final test will make up 40% of your final grading in this course. It is an important part of this course.

Remember that when you write your final test you will not be able to refer to your lessons or obtain help from anyone when answering the questions. A supervisor will ensure that examination conditions prevail.

Please do continue to review your lessons until you write the final test. You will be required to understand the contents of Mathematics 9 (M) very well. Best of luck in your efforts.

DIRECTIONS

1. Attempt all questions.
2. Work carefully and do your best.

Marks:

1 each **Section A**

Complete the blank spaces below:

1. In the numeral 237,543,809 the numeral 7 is in the
   ____________________ place.
2. In the numeral 1,605,394 the numeral 0 is in the
   ____________________ place.
3. The least common multiple (L.C.M.) of 8 and 6 is
   ____________________.
4. The product of 1200 and 5 is ____________________.
5. The quotient of 1200 and 240 is ____________________.
6. A quadrilateral has _______________ sides.
7. The number of pairs of parallel sides in a parallelogram is
   ____________________ pair(s).
8. The number of pairs of parallel sides in a rhombus is
   ____________________ pair(s).
9. An octagon has _______________ sides.
10. The prime numbers between 20 and 30 are ____________________
    and ____________________.

Marks:

1 each **Section B**

Write the formula for the following:

1. The cost formula $c = np$
2. The interest formula ________
3. The formula for the perimeter of a square ________
4. The formula for the perimeter of a rectangle ________
5. The formula for the area of a square ________
6. The formula for the area of a rectangle ________
7. The formula for the volume of your classroom ________

1 each **Section C**

From the list following each question, choose the symbol, word or words that correctly complete the sentence. Write the letter of your choice in the blank at the right.

1. We have learned how to find the volume of a ________.

   a. rectangle c. rhombus
   b. square d. rectangular prism ________

2. 5 - 6 is the same as ______.

   a. -5 + -6 c. 5 + 6
   b. -5 + 6 d. 5 + -6 ________

3. Of the four numbers below, _____ is the least.

a. 24.8352
b. 24.8325
c. 24.8301
d. 248.3526 ________

4. Of the numbers named below, _____ is composite.

a. 23
b. 39
c. 47
d. 17 ________

5. A trapezoid is made up of four _____.

a. segments
b. half-lines
c. rays
d. lines ________

6. It is true that _____.

a. $-3 < -3$
b. $0 < -3$
c. $3 < -3$
d. $-3 < 0$ ________

7. $\frac{22}{4}$ as a mixed numeral is _____.

a. $2\frac{2}{4}$
b. $5\frac{1}{2}$
c. $5\frac{1}{4}$
d. $4\frac{1}{2}$ ________

8. A(n) _____ triangle is a triangle which has three sides that are congruent.

a. right
b. scalene
c. equilateral
d. isosceles ________

9. Every rectangle is not a _____.

a. polygon
b. parallelogram
c. quadrilateral
d. square ________

10. The set whose members are _____ is an infinite set.

a. all the dogs in the world.
b. the whole numbers greater than 500.
c. the natural numbers from 2 through 1,000,000.
d. the number of motor cars on the North American continent. ________

11. The symbol _____ expresses "less than".

a. $<$ c. $\cong$
b. $>$ d. $\cup$ _____

12. Segments of the same size are said to be _____.

a. lines c. congruent
b. rays d. infinite _____

13. The symbol _____ expresses "union".

a. $\cup$ c. $>$
b. $<$ d. $\cap$ _____

14. The fraction _____ is equal to the fraction $\frac{8}{26}$.

a. $\frac{26}{8}$ c. $\frac{4}{12}$

b. $\frac{20}{65}$ d. $\frac{24}{46}$ _____

15. $\frac{3}{4} \div \frac{2}{3}$ is the same as _____.

a. $\frac{4}{3} \times \frac{2}{3}$ c. $\frac{4}{3} \div \frac{2}{3}$

b. $\frac{3}{4} \div \frac{3}{2}$ d. $\frac{3}{4} \times \frac{3}{2}$ _____

16. A charge for the use of money is called _____.

a. per cent c. average
b. interest d. principal _____

17. _____ represents 60%

a. $\frac{6}{100}$ c. $\frac{3}{4}$

b. $\frac{60}{50}$ d. $\frac{60}{100}$ _____

18. The reciprocal of $2\frac{1}{4}$ is ______.

a. $2\frac{4}{1}$ c. $\frac{4}{9}$
b. $\frac{9}{4}$ d. $\frac{8}{1}$ ________

19. The greatest common factor of 16 and 144 is ______.

a. 2^3 c. 2
b. 2^2 d. 2^4 ________

20. (3)(5)(7) is the complete factorization of _____.

a. 210 c. 15
b. 105 d. 21 ________

21. When you round 287 to the next greater ten, you obtain ______.

a. 70 c. 290
b. 280 d. 300 ________

22. When you find the product of 37.4532 and 8.06759, the product will have ______ digits to the right of the decimal point.

a. 3 c. 4
b. 5 d. 9 ________

23. If $\frac{9}{4} = \frac{36}{n}$, then ______.

a. $n \times 9 = n \times 4$ c. $9 \times n = 36 \times 4$
b. $36 \times 9 = n \times 4$ d. $4 \times 9 = n \times 36$ ________

Marks:

1 each

Section D

Write in each blank the number, symbol, or word that correctly completes each sentence.

1. If you add ________ to $x + -5$, it can be simplified to $x + 0$.
2. $a = \frac{t}{n}$ is called the ________________ formula.
3. $\{ 0, 1, 2, 3 \} \cap \{ 1, 3 \} =$ ____________.
4. Name the factors of 72. ________________
5. The numbers 2^1, 2^2, 2^3, and 2^4 are powers of __________.
6. 70,000 expressed in scientific notation is ____________________.
7. One hundred thirty-four trillion, forty-seven thousand, three hundred twenty-eight is written as the numeral ________________.
8. The vertex of an angle is the common ________________ of the two rays that form the angle.
9. The digits ________________ form the repetend of $3.25\overline{794}$.
10. The standard name for 5^3 is ________________.
11. We say 2 is a factor of 8 but 8 is a ________________ of 2.
12. The standard name for 6.52×10^3 is _____________________.
13. The ratio showing kilometres to metres is ___________________.

Marks:

Section E

2 1. The expanded form of 683.25 is

3 2. Convert 507_8 to a base-ten number.

2 3. $\frac{7}{9}$ as a decimal is __________________ .

2 4. $\frac{5}{8}$ as a decimal is ____________________ .

2 5. 2.64 as a fraction in its simplest form is ____________________ .

3 6. 24 is ________________% of 60.

3 7. ________________ is 35% of 180.

3 8. 8 is 2% of ________________.

2 9. Name the factors of 128. ________________________________

2 10. Find the value of x in $\frac{.4}{8} = \frac{x}{24}$.

Marks:

1 each Section F

Perform the required computation.

1. $\frac{3}{7} \times \frac{4}{5} =$ ________

2. $\frac{1}{2} + \frac{2}{3} =$ ________

3. $\frac{11}{12} - \frac{5}{8} =$ ________

4. $27.3 + 5.924 + 182.96 =$ ________

5. $8.0069 \times 3.47 =$ ________

6. $13\frac{5}{9} - 5\frac{2}{3} =$ ________

7. $2\frac{3}{4} \times 2\frac{4}{5} =$ ________

8. $8 \div \frac{3}{7} =$ ________

9. 31.7 ÷ 5.01 = ________

10. 74.29 ÷ 3.4 = ________

11. -9 + +6 = ________

12. -3 + -3 = ________

13. +7 - +3 = ________

14. -12 - +8 = ________

15. (4)(-9) = ________

16. (-39)(-18) = ________

17. -636 + +120 = ________

18. -756 ÷ -27 = ________

Marks:

5 each Section G

For each problem:

- tell what the placeholder represents,
- write an equation,
- give the answer, and
- write a statement answering the question asked.

1. Mr. Andrews worked in his office for $4\frac{3}{4}$ hours one Saturday morning. He worked a total of $6\frac{1}{3}$ hours during the morning and afternoon. How many hours did he work that afternoon?

2. Central School has $100.00 to buy textbooks, each textbook being listed at $8.25 each. What number of textbooks can be bought?

3. Sam can pluck 6 chickens in $\frac{1}{4}$ of an hour. At this rate, how long will it take him to pluck 102 chickens?

4. The width of a rectangle is 19 metres and the length is 24 metres. What is the perimeter of the rectangle in metres?

5. If a jet plane flies at the rate of 965 km per hour, how far will it fly in 6 hours?

6. Mr. Hardy bought a used car for $2000. He overhauled it and sold it one month later for $2375. His expenses in repairing the car were $90. How much money did Mr. Hardy make on the car?

Total Marks
125

END OF LESSON 30

N.L.C. - B.N.C.
3 3286 10937189 4